202: Dynamic Macroeconomic Theory

Inequality, Credit Market Imperfection & Growth: Galor-Zeira (1993)

Mausumi Das

Lecture Notes, DSE

April 20, 2015

Inequality & Growth:

- In the last class we have seen that inequality can adversly affect growth - working through the political economy channel (Alesina-Rodrik(1994)).
- Recall that in this paper inequality per se is not bad for growth. It becomes bad for growth only if one allows the poor people to have a say in the decision making process!
- In fact if one removes the majority voting part of the paper altogether, then a re-distributive taxation (i.e., an increase in τ) may actually lower growth.
- This is because taxation does two job here - enhances productivity (by increasing g); and redistributes income from rich to poor.
- The fisrt channel becomes inefficient beyond τ^*, but the redistrbutive mechanism still operates beyond τ^*.
- But any such redistributive taxation would hamper the incentive to accumulate capital. Since the rich have higher incentive to accumulate capital, it adversely affects the overall process of capital accumulation and hence growth.

Inequality & Growth: (Contd.)

- Thus the same equity-efficiency trade-off (which was present in the earlier developement literature, e.g., Mirlees) shows up again: in the absence of majority-voting, **a re-distribution of wealth from rich to poor (beyond τ^*) is actually inefficient!**

Inequality, Credit Market Imperfection & Growth: Galor-Zeira (1993)

- An alternative mecahnism was highlighted by O. Galor and J. Zeira ("Income Distribution and Macroeconomics", RES, 1993), who argued that the negative link between inequality and growth/development may exist quite independent of the political economy channel.
- Their arguement operates through the credit market inperfection channel.
- They argue that when credit markets are either absent or do not work properly, this leads to inefficient allocation of resources across various productive activities.
- So it is clear that imperfect credit market would lower growth.
- But how does inequality get into the picture?
- Inequality would get into the picture if for some reason **less wealthy** people are also 'potentially' **more productive** at the margin.

Inequality, Credit Market Imperfection & Growth: (Contd.)

- This can happen very easily in a world where **technolgy is convex** (implying **a concave production function**):
  - Consider an economy where households differ in terms of their initial wealth/capital holding.
  - Each household is endowed with an identical technology which uses only capital and the associated production function is concave:

$$y^h = f(k^h); \; f(0) = 0; f' > 0; f'' < 0.$$

  - One can think of each household as owning a firm.
  - Notice if there was a market for capital, households/firms would lend or borrow from one another, and there will be a flow of funds from the capital-rich to the capital-poor so that ultimately all will be able to enjoy the same rate of return at the margin.
  - In the process all will benefit and the economy would operate at an efficient level.
  - But such free flow of funds would not happen if the credit market does not function properly. Then the households as well as the economy would be stuck in a sub-optimal or inefficient situation, leading to lower level of output.

Inequality, Credit Market Imperfection & Growth: (Contd.)

- But this can even in a world where **technology is non-convex** (implying that the **production function is not concave**):
  - Consider the same economy as before.
  - But now suppose that the technology entails a threshold effect:

$$y^h = \begin{cases} \underline{y} \text{ for } k^h < \hat{k}; \\ \bar{y} \text{ for } k^h \geqq \hat{k}. \end{cases} \; ; \text{ where } \bar{y} > \underline{y}.$$

  - Notice that marginal return to investment around $\hat{k}$ is infinitely high; and is zero before and after.
  - Thus if there was a market for capital where households/firms could lend or borrow from one another, once again there would be a flow of funds from the capital-rich households (with $k^h > \hat{k}$) to the capital-poor households (with $k^h < \hat{k}$) so that all households would be operating with $k^h = \hat{k}$;consequently the economy would be at its most efficient level.
  - But this will not happen if the capital market is imperfect.
- Notice that in either example, **a redistribution of wealth from the rich to the poor is efficient (increases total output.)**

Galor & Zeira (1993):

- Galor & Zeira (1993) actually exploits the second channel.
- This is because in the first channel, poorer people enjoy higher return to capital; so even in the absence of credit market, their income would grow at a faster rate and eventually in the long run income of the rich and the poor would converge (as happens in the Solow model).
- In the second channel however, the actual return to capital for the poor people is low (though potential return is high - if they can be brought to the level $\hat{k}$); so absence of the credit market here implies that the income difference bewteen the rich and the poor could persist even in the long run.

Galor & Zeira (RES, 1993): Production Side Story

- Consider a small open economy which takes the world interest rate (r^*) as given.
- A single final commodity is produced - using two possible technologies: **a traditional cottage technology** (associated with low productivity); **a modern technology** (associated with high productivity).
- The traditional technology uses **only unskilled labour** and is assoacited with a constant marginal as well as average product:

$$Y_t^N = AL_t^N.$$

- The modern technology uses **capital and skilled labour** and is assoacited with a neoclassical production function exhibiting positive but diminishing marginal as product of each factor:

$$\begin{aligned} Y_t^S &= F(K_t^S, H_t^S) \\ F_K, F_H &> 0; \; F_{KK}, F_{HH} < 0 \end{aligned}$$

Production Side Story: (Contd.)

- Since $F(K_t^S, H_t^S)$ obeys all neoclassical properties, including CRS:

$$\frac{Y_t^S}{H_t^S} = f\left(\frac{K_t^S}{H_t^S}\right); \; f(0) = 0; f' > 0; f'' < 0.$$

- The modern tecnology is operated by profit-maximizing firms, operating under perfect competition. Thus the skilled wage rate and the market interest rate are given respectively by:

$$\begin{aligned} w_t^s &= f\left(\frac{K_t^S}{H_t^S}\right) - f'\left(\frac{K_t^S}{H_t^S}\right)\left(\frac{K_t^S}{H_t^S}\right); \\ r_t^s &= f'\left(\frac{K_t^S}{H_t^S}\right). \end{aligned}$$

- A small open economy with perfect capital mobility implies:

$$r_t^s = r^* \Rightarrow f'\left(\frac{K_t^S}{H_t^S}\right) = r^*.$$

Production Side Story: (Contd.)

- This fixes the **domestic** capital to skilled labour ratio at:

$$\frac{K_t^S}{H_t^S} = f'^{-1}(r^*) = \bar{k}.$$

- This in turn implies that the wage rate for skilled labour is fixed at:

$$\bar{w}^S = f(\bar{k}) - f'(\bar{k})(\bar{k}).$$

- The traditional technology is operated by family firms angaged in own production.
- Wage earnings of an unskilled labour engaged in traditional sector:

$$\bar{w}^N = A.$$

- If skill formation is costly, then of course, $\bar{w}^S > \bar{w}^N$. We shall make a more precise assumption about these two wage rates later.

Skill Formation Technology:

- One unit of unskilled labour can be converted into one unit of skilled labour by investing

  (i) one full time period (in acquiring skills);

  (ii) a fixed investment of h units of the final commodity.

- The fixed investment h has to be paid upfront - before skill formation begins.
- Notice that since skill formation also require a time investment of one full period, those who engage in skill formation cannot work for one time period.

Household Side Story:

- A two-period overlapping generations economy with constant population.
- There are $\bar{L}$ households - each consisting of a young member and an old member at any point of time t.
- An agent lives exactly for two periods - youth and maturity, and has an offspring at the beginning of the second period of his life.
- He dies at the end of the 2nd period but the dynastic link is carried forward over time by his progeny.
- Each agent is born with **an endowment of one unit of unskilled labour.**
- The young agent also receives **an endowment of final goods as bequest** from parent.
- Agents differ in terms of the bequest received.

Household Side Story: (Contd.)

- All agents born the beginning of period t will be called 'generation t'.
- The life cycle of an agent belonging to generation t is as follows:
- In the first period of his life:
  - He is endowed with one unit of unskilled labour and some inherited wealth (x_t).
  - In the first period, he consumes nothing in the first period and only takes the decision as to whether to acquire skill or not.
  - If he decides not to acquire skill then he works as unskilled labour to earn $\bar{w}^N$.
  - If he decides to to acquire skill then he spends the first period of his life in schooling.
  - In the latter case he also spends a fixed amount h is school fees.
- In the second period of his life:
  - Depending on his first period decision to go for skill formation on not, he works either as a skilled labour, (earning $\bar{w}^S$) or as an unskilled labour (again earning $\bar{w}^N$).
  - He spends his entire second period income in own consumption and in leaving a bequest for his child.

Preferences of an Agent:

- Consider an agent with a second period income y.
- The agent spends this income in own consumption (c) and in bequest for his child (b).
- His preference is represented by the following utility function (identical for all agents):

$$U(c, b) = \alpha \log c + (1 - \alpha) \log b; \;\; 0 < \alpha < 1.$$

- The agent maximises the above utility function subject to his second period budget constraint:

$$c + b = y.$$

- Optimal solution:

$$\begin{aligned} c &= \alpha y; \\ b &= (1 - \alpha) y. \end{aligned}$$

- Corresponding indirect utility:

$$\hat{U} = \alpha \log(\alpha y) + (1 - \alpha) \log((1 - \alpha) y)$$

Occupational/Skill Formation Decision of an Agent:

- Notice that the indirect utility of an agent depends on his second period income:

$$\begin{aligned} \hat{U} &= [\alpha \log \alpha + (1-\alpha)\log(1-\alpha)] + \log y \\ &= M + \log y. \end{aligned}$$

- The second period earning on the other hand depends on the skill formation decidion undertaken by the agent in the first period of his life.
- Let us now analyse the optimal occupational/skill formation choice of an agent.
- Given his inherited wealth level x_t, a forward-looking agent endowed with perfect foresight will choose his skill and therefore occupation so as to maximise his second period utility $\hat{U}$.

Skill Formation Decision of an Agent: (Contd.)

- In deciding whether to go for skill formation or not, there are two possibilities:
  - Case I: $x_t \geqq h$
    In this case the agent can invest in skill formation directly from his inherited wealth. The rest of the wealth can then be invested in the capital market to earn an additional interest income in the next period (over and above the skilled wage rate that he would be earning).
  - Case II: $x_t < h$
    In this case the agent can invest in skill formation only if he borrows from the capital market an amount $(h - x_t)$. If he does so, then he has to repay the loan with interest in the next period from the skilled wage rate that he would be earning.
- We shall now bring in **credit market impefection** into the story, which will affect the skill formation decision of the rich agents (with $x_t \geqq h$) vis-a-vis the poor agents (with $x_t < h$).

Credit Market Imperfection:

- Credit market imperfection can show up in an economy in three possible ways:
  1. **Complete absence of credit (missing credit market)**: In this case there is no borrowing possible. Thus only rich agents can invest in skill formation; the poor agents are completely excluded form acquiring skill and therefore working in the modern sector.
  2. **Collateral Requirement & Credit Rationing**: In this case the amount of credit obtained depends on the wealth level of an agent (which can be pledged as collateral). Once again agents who are too poor (such the inherited wealth + limited credit fall short of h) will be excluded form acquiring skill and therefore working in the modern sector.
  3. **A Wedge Between Borrower's and Lender's Rate:** In this case the agents who are who are borrowing from the credit market face a higher interest rate (i) than the interest rate recieved by the lenders (r). If the borrower's rate is too high, this again will deter the poorer agents to avail the credit and go for skill formation, since they won't be able to pay back that high an interest.

Reason for Imperfect Credit Market:

- Typically the credit market does not work perfectly because of assocaited moral hazard problem.
- One can take the loan and run away. In order to prevent this, the financial institution will have to incur some cost either in monitoring or in tracing the borrower who has ran away.
- Such costly activities would either lead to credit rationing or would create a wedge between borrower's and lender's rate.
- Galor & Zeira adopts a credit market imperfection story that entails the latter.

Credit Market Impefection Story a la Galor-Zeira:

- Imagine a world where borrowing and lending are done by a number of competing financial institutions/banks - each taking deposits from the lenders and lending it out to the borrowers.
- Since the lender has the option of directly investing in the international capital market (small open economy with perfect capital mobility), he has to be given the world interest rate r^*.
- The borrower on the other hand can run away with the loan amount.
- The bank can spend some amount z is checking his whereabouts; but the borrower can still evade by spending a higher amount βz ($\beta > 1$).
- Suppose a borrower has asked for a loan amount d.
- The bank now has to decide what interest to charge the borrower (i) and how much to spend in checking him (z).
- **The bank's participation constraint**:

$$(1+i)d \geqq (1+r^*)d + z \tag{1}$$

Credit Market Impefection Story a la Galor-Zeira: (Contd.)

- Given it's participation constraint, the bank will choose i and z so that the borrower has no incentive to run away.
- Notice that if the borrower does not renege, then he has to pay back: $(1+i)d$
- On the other hand if he evades, then his cost is: βz
- Hence **the incentive compatibility constraint for the borrower set by the bank:**

$$\beta z \geqq (1+i)d \tag{2}$$

- When there are many banks competing with one another, all financial intermediaries will eventually earn zero profit, whch implies that the above two conditions will hold with equality.
- Solving, we get:

$$i = \frac{1+\beta r^*}{\beta - 1} > r^*$$

Skill Formation Decision of an Agent: (Contd.)

- Recall that an agent with an inherited wealth level x_t, will go for skill formatio if his second period utility $\hat{U}$ is higher under skill formation than under no-skill.
- His indirect utility in turn is an increasing function of his second period income:

$$\hat{U} = M + \log y.$$

- So let us now calculate the second period income of an agent under skill formation and no-skill.
- Two possible cases:
    - **Case I:** $x_t \geqq h$
        - second period income of the agent under skill formation:

$$\left(y_t^{skill}\right)_{\text{Case I}} = (1 + r^*)(x_t - h) + \bar{w}^S$$

        - second period income of the agent under no skill:

$$\left(y_t^{noskill}\right)_{\text{Case I}} = (1 + r^*)(x_t + \bar{w}^N) + \bar{w}^N$$

Skill Formation Decision of an Agent: (Contd.)

- **In Case I, agents will go for skill formation iff**

$$\begin{aligned}(1+r^*)(x_t - h) + \bar{w}^S &\geqq (1+r^*)(x_t + \bar{w}^N) + \bar{w}^N \\ \text{i.e., } \bar{w}^S - \bar{w}^N &\geqq (1+r^*)\left(\bar{w}^N + h\right)\end{aligned}$$

- Now let us consider the other case:
  - **Case II:** $x_t < h$
    - second period income of the agent under skill formation:

$$\left(y_t^{skill}\right)_{\text{Case II}} = \bar{w}^S - (1+i)(h - x_t)$$

    - second period income of the agent under no skill:

$$\left(y_t^{noskill}\right)_{\text{Case II}} = (1+r^*)(x_t + \bar{w}^N) + \bar{w}^N$$

- **In Case II, agents will go for skill formation iff**

$$\begin{aligned}\bar{w}^S - (1+i)(h - x_t) &> (1+r^*)(x_t + \bar{w}^N) + \bar{w}^N \\ \text{i.e., } x_t &> \frac{(1+r^*)\left(2+\bar{w}^N\right) + (1+i)h - \bar{w}^S}{i - r^*} \equiv f\end{aligned}$$

Skill Formation Decision of an Agent: (Contd.)

- **Assumption 1:**

$$(1+r^*)\left(2+\bar{w}^N\right)+(1+i)h > \bar{w}^S > (1+r^*)\left(2+\bar{w}^N\right)+(1+r^*)h$$

- The latter part of Assumption 1 implies that **rich agents (with $x_t \geqq h$) would always prefer to go for skill formation.**
- The first part of Assumption 1 ensures that **there exists a positive cut-off wealth level** f $(< h)$ such that
  - **agents with $f < x_t < h$ would prefer to go for skill formation by borrowing;**
  - **but for agents with $0 \leqq x_t \leqq f$, the cost of repayment is too high and they prefer to remain unskilled.**
- In any time period t , given the distribution of wealth (among the current youth), we can break up the entire distribution in three distinct intervals and analyse the correspondimg optimal occuptional and other choices.

Wealth Distribution and Skill Formation:

- 1. For any x_t such that $0 \leqq x_t \leqq f$
    - The agent works as unskilled labour in the traditional sector:
    - Income: $y_t = (1 + r^*)(x_t + \bar{w}^N) + \bar{w}^N$
    - Bequest left to his child:
      $b_t = (1 - \alpha)y_t = (1 - \alpha)\left[(1 + r^*)(2 + \bar{w}^N) + (1 + r^*)x_t\right]$
  2. For any x_t such that $f < x_t < h$
    - The agent acquires skills by borrowing and works as skilled labour in the modern sector:
    - Income: $\bar{w}^S - (1 + i)(h - x_t)$
    - Bequest left to his child:
      $b_t = (1 - \alpha)y_t = (1 - \alpha)\left[\bar{w}^S - (1 + i)h + (1 + i)x_t\right]$
  3. For any x_t such that $x_t \geqq h$
    - The agent acquires skills from own inherited wealth and works as skilled labour in the modern sector:
    - Income: $(1 + r^*)(x_t - h) + \bar{w}^S$
    - Bequest left to his child:
      $b_t = (1 - \alpha)y_t = (1 - \alpha)\left[\bar{w}^S - (1 + r^*)h + (1 + r^*)x_t\right]$

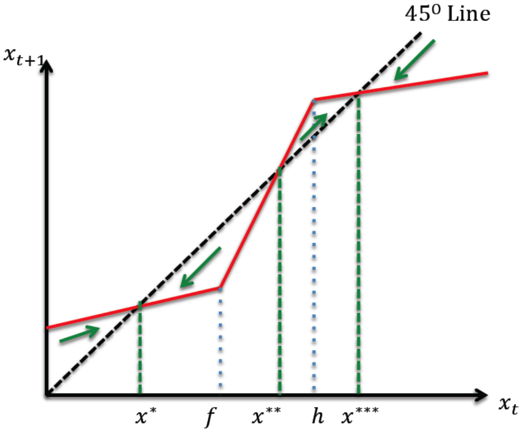
Intergenerational Wealth Dynamics:

- Notice that for any dynasty, b_t (bequest left by the agent of generation t for his child) is nothing by x_{t+1}(the inherited wealth of the next generation in the same dynasty).
- Thus we can represent the bequest/wealth dynamics for any dynasty by the following difference equation:

$$x_{t+1} = \begin{cases} (1-\alpha)\left[(1+r^*)(2+\bar{w}^N) + (1+r^*)x_t\right] & \text{for } 0 \leqq x_t \leqq f; \\ (1-\alpha)\left[\bar{w}^S - (1+i)h + (1+i)x_t\right] & \text{for } f < x_t < h; \\ (1-\alpha)\left[\bar{w}^S - (1+r^*)h + (1+r^*)x_t\right] & \text{for } x_t \geqq h. \end{cases}$$

Corresponding Phase Diagram:

- Under *suitable parametric conditions*, one can draw the corresponding phase diagram as follows:



Identification of Steady States and Stability:

- Given the above phase diagram, one can identify there steady states for this economy: x^*, x^{**} and x^{***}.
- Out of these, **the middle one is** (locally) **unstable**, the **other two are** (locally) **stable**.
- This implies that the long run wealth position of any dyanstic household h will be determined by its initial level of inherited wealth: x_0^h :
  - If $x_0^h > x^{**}$ then the wealth level of the dynasty in the long run approches x^{**};
  - If $x_0^h < x^{**}$ then the wealth level of the dynasty in the long run approches x^*.
- This has implication for the occupational pattern of the dyansties in the long run:
  - If $x_0^h > x^{**}$ then the members of the dynasty **in the long run** are skilled and engaged in modern production;
  - If $x_0^h < x^{**}$ then the members of the dynasty **in the long run** are unskilled and engaged in cottage production.

Identification of Steady States and Stability: (Contd.)

- Thus the middle steady state (x^{**}) can be interpreted as a poverty trap or a low-skill trap or an occupational trap.
- Notice that Assumption 1 alone does not gaurantee that there would always be three steady states.
- There are other possible parametric conditions where there is no trap - there is uniform convergence to a single steady state irrespective of the initial wealth position of the households .
- We need some additional assumptions to ensure that there are indeed three intersection points in the phase diagram.
  (**Derive a set of sufficient conditions in the terms of the parameters such that this is the case**).

Wealth Dynamics & The Macroeconomy in the Long Run:

- Interestingly, the initial distribution of wealth and the corresponding wealth dynamics have important implications for the overall position of the macroeconomy in the long run.
- Recall that total number of households in the economy is $\bar{L}$, which is also the number (measure) of young population at any point of time t.
- Let the $D_0(x_0^h)$ denote the initial distribution of wealth (inheritence) across all the $\bar{L}$ young agents, such that

$$\int_0^\infty dD_0(x_0^h) = \bar{L}$$

- Out of these $\bar{L}$ young agents, let $L_0^{x^{**}}$ denote that measure of young agents at time 0 who have an inherited wealth level $x_0^h \leqq x^{**}$:

$$\int_0^{x^{**}} dD_0(x_0^h) = L_0^{x^{**}}$$

Initial Distribution, Wealth Dynamics & The Macroeconomy:

- From the intergenerational wealth dyanamics , we know that the children of all these $L_0^{x^{**}}$ agents will get trapped in the low skill-low income cottage sector in the long run.
- Children of the rest $(\bar{L} - L_0^{x^{**}})$ on the other hand will be skilled in the long run and will get employed in the high skill-high income modern sector.
- Then the **long run average wealth** in this economy:

$$\begin{aligned} \bar{x} &= \frac{L_0^{x^{**}} x^* + \left(\bar{L} - L_0^{x^{**}}\right) x^{***}}{\bar{L}} \\ &= x^* - \frac{L_0^{x^{**}}}{\bar{L}} \left(x^{**} - x^*\right) \end{aligned}$$

- It is easy to see that the long run average wealth of the entire economy depends on the initial distribution of wealth:
  - the higher is the proprtion of people below x^{**}, the lower in the long run average wealth.

Initial Distribution, Wealth Dynamics & The Macroeconomy: (Contd.)

- Similar conclusion would hold even for the **long run average income**. (**Verify**)
- Notice that a direct redistributive policy that taxes the wealth of the rich and gives it to the poor such that post- redistribution the proportion of people with wealth level $> x^{**}$ goes up, would make the economy better off in the long run (in an average sense).
- Also notice that any random redistribution can actually make the economy worse off in the long run!!

- Now what about inequality?
- Again consider two economies, **A** and **B**, which are identical in every respect except in terms of initial wealth distribution.
- In particular suppose the average initial wealth holding is the same in both economies, but wealth in economy **A** is more unequally distributed than in economy **B** such that

$$\left(\frac{L_0^{x^{**}}}{\bar{L}}\right)_A > \left(\frac{L_0^{x^{**}}}{\bar{L}}\right)_B.$$

- Then economy **A** will experience a **lower** average income in the long run than economy **B.**

Whither Growth?

- But what about growth?
- The Galor-Zeira model exhibits no growth - in the long run all agents reach their respective steady state **levels** of income and wealth and so does the economy.
- However one can derive similar conclusions in terms of the long run growth rate of the economy if we slightly modify the model in the following way:
  - Assume that credit market in completely missing;
  - Allow for a Harrod-Neutral technical progress in both sectors such that labour productivity in the cottage sector grows at the constant rate n while labour productivity in the modern sector grows at the constant rate m ($m > n$);
  - Assume that fixed investment requirement for skill formation also grows at the same rate as modern sector productivity: m
  - In this modified structure one can now show that the higher inequality implies lower average growth in the long run.
    (**Try this exercise**)