# Eminent Domain: Compensation and Incetives 

Ram Singh

Lecture 3-4
July 29, 2015

## Efficient Outcome? I

Let

- $C(x)=\bar{C}$ be the Compensation paid by the G to the $\mathrm{O} ; C^{\prime}(x)=0$ Suppose, the G will acquire land iff $B \geq m \bar{C}$. So, acquisition set

$$
A S(x)=\{B \mid B>m \bar{C}\}
$$

Now, the owner will choose $x$, to solve

$$
\max _{x}\left\{\int_{0}^{m \bar{C}} V(x) f(B) d B+\int_{m \bar{C}}^{\bar{B}} \bar{C} f(B) d B+\int_{m \bar{C}}^{\bar{B}}[b-t] f(B) d B-x\right\}
$$

That is, O solves:

$$
\max _{x}\left\{F(m \bar{C}) V(x)+(1-F(m \bar{C})) \bar{C}+\int_{m \bar{C}}^{\bar{B}}[b-t] f(B) d B-x\right\}, \text { i.e., }
$$

## Efficient Outcome ? II

$x^{F C}$ solves

$$
F(m \bar{C}) V^{\prime}(x)-1=0
$$

Suppose:

- $C=V\left(x^{*}\right)=V^{*}$, where $x^{*}$ is the FB level of $x$.

Now, we get

$$
A S(x)=\left\{B \mid B>m C=m V\left(x^{*}\right)\right\} .
$$

That is, O solves:
$\max _{x}\left\{F\left(m V\left(x^{*}\right)\right) V(x)+\left(1-F\left(m V\left(x^{*}\right)\right)\right) V\left(x^{*}\right)+\int_{m V\left(x^{*}\right)}^{\bar{B}}[b-t] f(B) d B-x\right\}, i$
$x^{F C}$ solves

$$
F\left(m V\left(x^{*}\right)\right) V^{\prime}(x)-1=0 .
$$

That is, the $O$ will make FB choice of $x$.

## Efficient Outcome? III

Suppose:

- $C=0$. But
- The $G$ will acquire land iff $B>m V\left(x^{*}\right)$

Now, we get

$$
A S(x)=\left\{B \mid B>m V\left(x^{*}\right)\right\} .
$$

O will solve:
$\max _{x}\left\{F\left(m V\left(x^{*}\right)\right) V(x)+\left(1-F\left(m V\left(x^{*}\right)\right)\right) 0+\int_{m V\left(x^{*}\right)}^{\bar{B}}[b-t] f(B) d B-x\right\}$, i.e.,
Again, O will choose $x$ to solve

$$
F\left(m V\left(x^{*}\right)\right) V^{\prime}(x)-1=0
$$

That is, $x^{F C}=x^{*}$.

## Incentives for the Govt I

## Let

- G provide several public goods for a population of $n$ individuals
- m parcels/acres are needed
- per-person benefit from the public goods is $b(m)$; such that $b^{\prime}(m)>0$, $b^{\prime \prime}(m)<0$.
- $\pi(m)$ be the probability that land of an owner will NOT be required; $\pi^{\prime}(m)<0$
- $1-\pi(m)$ be the probability that land of an owner will be required;


## Incentives for the Govt II

TWO Possible interpretations of $m$ :

- There are $n$ identical individuals in the society, and $m$ is number of randomly chosen owners whose land will be acquired. In this case

$$
1-\pi(m)=\frac{m}{n}
$$

- $m$ is the total number of land parcels needed for all public goods. There is uncertainty about location of projects.
- In either case, $\pi^{\prime}(m)<0$ holds.

We assume:

- $\pi(m)$ is the same across all individuals.


## First Best I

Note: At date $t=2$, G should acquire $m$ acres iff

$$
n b(m)>m V(x)
$$

So, given $x$, opted by the owner, the $G$ should chose $m$ that solves:

$$
\max _{m}\{n b(m)-m V(x)\}
$$

That is, $m^{*}$, solves

$$
\begin{equation*}
n b^{\prime}(m)-V(x)=0 \tag{1}
\end{equation*}
$$

Assumptions: At $t=1$, each owner takes the following as given

- $m$ chosen by the G
- $x$ opted by the other owners


## First Best II

For given $m$, the owners should choose $x$ to solve

$$
\begin{gather*}
\max _{x}\{b(m)+\pi(m) V(x)-x\}, \text { i.e. } \\
\pi(m) V^{\prime}(x)-1=0 \tag{2}
\end{gather*}
$$

Let $m^{*}$ and $x^{*}$ solve (1) and (2), simultaneously. Assume $m^{*}>0$ and $x^{*}>0$.

Question
What is the sign of $\frac{\partial x^{*}}{\partial m}$, where $x^{*}$ solves (2)?

## Question

Is the assumption about behaviour of owners plausible?

## Outcome Under Eminent Domain I

Let

- $C(x)$ be the Compensation paid by the G to the O , if land is acquired
- $t$ be the lump sum tax per-person to finance the public goods.

Given $x$ opted by the owners, and given the compensation rule $C(x)$, suppose $G$ will acquire $m$ acres iff

$$
n b(m)>m C(x)
$$

In that case, for given $x$ opted by the owners, the $G$ will solve:

$$
\max _{m}\{n b(m)-m C(x)\}
$$

That is, $m^{S B}$, solves

$$
\begin{equation*}
n b^{\prime}(m)-C(x)=0 \tag{3}
\end{equation*}
$$

## Outcome Under Eminent Domain II

Taking $m$ as given, the owners will choose $x$ to solve

$$
\begin{gather*}
\max _{x}\{\pi(m) V(x)+(1-\pi(m)) C(x)+b(m)-t-x\}, \text { i.e., } \\
\pi(m) V^{\prime}(x)+(1-\pi(m)) C^{\prime}(x)-1=0 \tag{4}
\end{gather*}
$$

CASE 1: $C(x)=V(x)$ :
In this case, the owners will choose $x$ that solves

$$
\begin{equation*}
V^{\prime}(x)-1=0 . \tag{5}
\end{equation*}
$$

That is,

- the govt will choose $m^{*}(x)$
- however, $x$ opted by the owners will be excessive, i.e., $x>x^{*}$.


## Outcome Under Eminent Domain III

CASE 2: $C(x)=\bar{C}$, where $\bar{C} \geq 0$ is a constant. In this case,

- Given $x$, opted by the owner, the $G$ will solve

$$
\max _{m}\{n b(m)-m \bar{C}\}, \text { i.e. }
$$

$m^{S B}$ will solve

$$
n b^{\prime}(m)-\bar{C}=0
$$

## Outcome Under Eminent Domain IV

- however, the owners will choose $x$ to solve

$$
\begin{gathered}
\max _{x}\{\pi(m) V(x)+(1-\pi(m)) \bar{C}+b(m)-T-x\}, \text { i.e., } \\
\pi(m) V^{\prime}(x)-1=0 .
\end{gathered}
$$

## Question

Is there a trade-off between higher (ex-post) full compensation and incentives of $O$ and $G$ ?

## Question

What will be outcome if $\bar{C}=V\left(x^{*}\right)$ ?

