Bilateral Care Externality and Efficient Liability

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Externality and Liability I

Question

To achieve efficient outcome

- Is it necessary to make I liable for accident loss?
- Is it necessary to make I liable for accident loss for all choices of care levels by her?
- Is it necessary to make V liable for a part of the accident loss?
- Is it possible to achieve efficient outcome and at the same time compensate the V fully?

Efficient Liability Rules I

Consider a bilateral care accident context: X, Y, $L(x, y)$ and (x^*, y^*), where

- X is the set of possible care levels for I; $X \subset \Re_+$
- Y is the set of possible care levels for V; $Y \subset \Re_+$
- $L(x, y)$ expected cost technology function, and
- (x^*, y^*) uniquely solves

$$\min_{x,y}\{x + y + L(x, y)\}$$

Property P1: A liability rule satisfies property P1, if

$$\begin{aligned} x < x^* \;\&\; y \geq y^* \quad &\Rightarrow \quad s(x, y) = 1 \\ x \geq x^* \;\&\; y < y^* \quad &\Rightarrow \quad s(x, y) = 0 \end{aligned}$$

Efficient Liability Rules II

Under P1:

$$x \geq x^* \;\&\; y \geq y^* \quad \Rightarrow \quad s(x, y) = ?$$
$$x < x^* \;\&\; y < y^* \quad \Rightarrow \quad s(x, y) = ?$$

Property P2: A liability rule satisfies property P2, if

$$[s(x^*, y^*) = s^*] \Rightarrow (\text{for all } x \geq x^* \text{and } y \geq y^*)[s(x, y) = s^*]$$

$0 \leq s^* < 1$.

Efficient Liability Rules III

Definition

Nash Equilibrium (N.E.): A pair of care levels $(\hat{x}, \hat{y})$ is N.E. under a liability rule, if

- Given $\hat{x}$ opted by the injurer, $\hat{y}$ is total cost minimizing care level for the victim
- Given $\hat{y}$ opted by the victim, $\hat{x}$ is total cost minimizing care level for the injurer
- Injurer believes that the victim will choose $\hat{y}$, and the victim believes that injurer will choose $\hat{x}$

Proposition

If a liability rules satisfies Properties, P1 and P2, care levels (x^*, y^*) is N.E. under the liability rule.

Efficient Liability Rules IV

Proof: Suppose,

- $s(x^*, y^*) = s^*$, $s^* \in [0, 1]$, and
- the victim has opted for y^*

So,

- if the injurer opts for x^*, his total cost is $x^* + s^* L(x^*, y^*)$, and
- if he opts for some $x < x^*$ his total cost is

$$\begin{aligned} x + s(x, y^*)D(x, y^*)\pi(x, y^*) &= x + s(x, y^*)L(x, y^*) \\ &= x + L(x, y^*) \end{aligned}$$

since $s(x, y^*) = 1$

Efficient Liability Rules V

Injurer will choose $x < x^*$ only if

$$\begin{aligned} x + L(x, y^*) &< x^* + s^* L(x^*, y^*), i.e., \text{only if} \\ x + y^* + L(x, y^*) &< x^* + y^* + s^* L(x^*, y^*), i.e., \text{only if} \\ x + y^* + L(x, y^*) &< x^* + y^* + L(x^*, y^*) \qquad (0.1) \end{aligned}$$

But, (0.2) cannot be true in view of (**??**). That is, for the injurer choice of x^* is better than choice on any $x < x^*$.

Next, consider a choice of $x > x^*$ by the injurer (assuming that the victim is still spending y^* on care).

Note that when $x > x^*$

$$x + s(x, y^*)L(x, y^*) = x + s^* L(x, y^*)$$

So, injurer will choose $x > x^*$ only if

Efficient Liability Rules VI

$$\begin{aligned} x + s^* L(x, y^*) &< x^* + s^* L(x^*, y^*), \textit{i.e.}, \text{only if} \\ x + y^* + s^* L(x, y^*) &< x^* + y^* + s^* L(x^*, y^*) \end{aligned} \tag{0.2}$$

Also, note that $x > x^* \Rightarrow$

$$\begin{aligned} L(x, y^*) &\leq L(x^*, y^*), \textit{i.e.}, \\ (1 - s^*) L(x, y^*) &\leq (1 - s^*) L(x^*, y^*) \end{aligned} \tag{0.3}$$

Now, (0.2) and (0.3) imply that

$$\begin{aligned} x + y^* + [s^* + (1 - s^*)] L(x, y^*) &< x^* + y^* + [s^* + (1 - s^*)] L(x^*, y^*), \textit{i.e.}, \\ x + y^* + L(x, y^*) &< x^* + y^* + L(x^*, y^*) \end{aligned} \tag{0.4}$$

Efficient Liability Rules VII

But, (0.4) cannot be true, in view of (**??**). This implies that (0.3) cannot be true.

So, we have proved that:

- If the victim opts for y^*, the injurer will opt for x^*
- Similarly, we can prove that if the injurer opts for x^*, the victim will opt for y^*
- (x^*, y^*) is a N.E.

Efficient Liability Rules VIII

Properties, P1 and P2, together are sufficient condition a liability rule to be efficient

Proposition

If a liability rules satisfies Properties, P1 and P2, (x^*, y^*) *is a* unique *N.E. under the Rule.*

Proof: Let $s(x^*, y^*) = s^*$, $0 \leq s^* \leq 1$. Due to P2,

$$(\text{for all } x \geq x^* \text{and } y \geq y^*)[s(x, y) = s^*]$$

Let $(\bar{x}, \bar{y})$ be a (any) N.E. under the Rule. Note that $(\bar{x}, \bar{y})$ be a N.E. means

$$\begin{aligned} \bar{x} + s(\bar{x}, \bar{y})L(\bar{x}, \bar{y}) &\leq x^* + s(x^*, \bar{y})L(x^*, \bar{y}) \\ \bar{y} + (1 - s(\bar{x}, \bar{y}))L(\bar{x}, \bar{y}) &\leq y^* + (1 - s(\bar{x}, y^*)L(\bar{x}, y^*) \end{aligned}$$

That is,

$$\bar{x} + \bar{y} + L(\bar{x}, \bar{y}) \leq x^* + y^* + s(x^*, \bar{y})L(x^*, \bar{y}) + (1 - s(\bar{x}, y^*)L(\bar{x}, y^*) \quad (0.5)$$

Efficient Liability Rules IX

Case 1: $\bar{x} \geq x^*$ & $\bar{y} \geq y^*$**:**

From (0.5) and P2, it follows that

$$\begin{aligned} \bar{x} + \bar{y} + L(\bar{x}, \bar{y}) &\leq x^* + y^* + s^* L(x^*, \bar{y}) + (1 - s^*) L(\bar{x}, y^*), i.e., \\ \bar{x} + \bar{y} + L(\bar{x}, \bar{y}) &\leq x^* + y^* + s^* L(x^*, y^*) + (1 - s^*) L(x^*, y^*), i.e., \\ \bar{x} + \bar{y} + L(\bar{x}, \bar{y}) &\leq x^* + y^* + L(x^*, y^*). \end{aligned} \tag{0.6}$$

However, (0.6) can hold only if $(\bar{x}, \bar{y}) = (x^*, y^*)$.

Case 2: $\bar{x} < x^*$ & $\bar{y} < y^*$**:**

From (0.5) and P1, it follows that

$$\begin{aligned} \bar{x} + \bar{y} + L(\bar{x}, \bar{y}) &\leq x^* + y^*, i.e., \\ \bar{x} + \bar{y} + L(\bar{x}, \bar{y}) &\leq x^* + y^* + L(x^*, y^*). \end{aligned} \tag{0.7}$$

However, when $\bar{x} < x^* \ \& \ \bar{y} < y^*$, (0.7) cannot hold, i.e., $(\bar{x}, \bar{y})$ cannot be a N.E.

Case 3: $\bar{x} \geq x^* \ \& \ \bar{y} < y^*$**:**

From (0.5), P1 and P2, it follows that

$$\begin{aligned} \bar{x} + \bar{y} + L(\bar{x}, \bar{y}) &\leq x^* + y^* + (1 - s^*)L(\bar{x}, y^*), i.e., \\ \bar{x} + \bar{y} + L(\bar{x}, \bar{y}) &\leq x^* + y^* + (1 - s^*)L(x^*, y^*), i.e., \\ \bar{x} + \bar{y} + L(\bar{x}, \bar{y}) &\leq x^* + y^* + L(x^*, y^*). \end{aligned} \tag{0.8}$$

However, when $\bar{x} \geq x^* \ \& \ \bar{y} < y^*$ (0.8) cannot hold. That is, $(\bar{x}, \bar{y})$ cannot be a N.E.

Case 4: $\bar{x} < x^* \ \& \ \bar{y} \geq y^*$**:**
You can show that $(\bar{x}, \bar{y})$ cannot be a N.E. That is,

$(\bar{x}, \bar{y})$ can be a N.E. only if $(\bar{x}, \bar{y}) = (x^*, y^*)$

General Result: OPTIONAL

Consider a bilateral care accident context: X, Y, $L(x, y)$.

Definition

$$M = \{(x, y) | \min_{x,y}\{x + y + L(x, y)\}\}$$

- So far we have assumed that M is singleton, and $M = \{(x^*, y^*)\}$.
- Now, suppose M can have more than one elements; $\sharp M > 1$

Still, the above conditions are sufficient for efficiency of a liability rule. Formally,

Proposition

Suppose a liability rule satisfies Properties, P1 and P2. If $(\bar{x}, \bar{y})$ is a N.E., then $(\bar{x}, \bar{y}) \in M$.

See Jain and Singh (2001), *Journal of Economics*

Efficient Rules: Examples I

Definition

Rule of Negligence with Defense of Contributory negligence:

$$\begin{aligned} x \geq x^* &\Rightarrow s(x,y) = 0 \\ x < x^* \;\&\; y < y^* &\Rightarrow s(x,y) = 0 \\ x < x^* \;\&\; y \geq y^* &\Rightarrow s(x,y) = 1 \end{aligned}$$

Definition

Rule of Strict Liability with Defense of Contributory negligence:

$$\begin{aligned} y \geq y^* &\Rightarrow s(x,y) = 1 \\ y < y^* &\Rightarrow s(x,y) = 0. \end{aligned}$$