Litigation: Market Value Vs Awards

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Lecture 9

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Market Value Vs Awards I

OPTIONAL: For the general case, i.e., when

$$E(r^c \mid r, x, y) \quad = \quad \phi(r)(ax^{\frac{1}{k}} - by^{\frac{1}{j}}),$$

where $j, k > 1$. x^* and y^* solve the following FOCs:

$$M(\frac{a\phi(r)}{k})x^{\frac{1-k}{k}} = x \tag{0.1}$$

$$-M\lambda(\frac{-b\phi(r)}{j})y^{\frac{1-j}{j}} = y \tag{0.2}$$

We get

$$y^* \quad = \quad \left(\frac{b\lambda\phi(r)M}{j}\right)^{\frac{j}{2j-1}}$$

$$x^* \quad = \quad \left(\frac{aM\phi(r)}{k}\right)^{\frac{k}{2k-1}}$$

Market Value Vs Awards II

Further

$$\frac{dx^*}{dr} = \left(\frac{aM}{k}\right)^{\frac{k}{2k-1}} \left(\frac{k}{2k-1}\right) (\phi(r))^{\frac{1-k}{2k-1}} \phi^{'}(r)$$

$$\frac{dy^*}{dr} = \left(\frac{b\lambda M}{j}\right)^{\frac{j}{2j-1}} \left(\frac{j}{2j-1}\right) (\phi(r))^{\frac{1-j}{2j-1}} \phi^{'}(r)$$

$$\begin{aligned}\frac{dE^*}{dr} &= (\phi(r))^{\frac{1}{2k-1}} \phi^{'}(r) \left(\frac{a}{k}\right)^{\frac{2k}{2k-1}} \left(\frac{k}{2k-1}\right) \\ &- (\phi(r))^{\frac{1}{2j-1}} \phi^{'}(r) (\lambda)^{\frac{1}{2j-1}} \left(\frac{b}{j}\right)^{\frac{2j}{2j-1}} \left(\frac{j}{2j-1}\right). \end{aligned} \quad (0.3)$$

Market Value Vs Awards III

Proposition

$$[(1 < k \leq j \text{ and } a > b) \text{ or } (1 < k < j \text{ and } a \geq b)] \Rightarrow \frac{dE^*}{dr} > 0.$$

From (0.3) note that

- when λ is small $\frac{dE^*}{dr} > 0$ will hold, for a wide range of a, b, j and k.
- In fact, when λ is sufficiently small $\frac{d[\frac{E^*}{r}]}{dr} > 0$ will hold.

Special Case I

NOT OPTIONAL:
Consider a special case of $E(r^c \mid r, x, y) = \phi(r)(ax^{\frac{1}{k}} - by^{\frac{1}{j}})$, such that

- $\phi(r) = \delta r$, $\delta > 0$
- $a = b$ and $j = k$

That is,

$$E(r^c \mid r, x, y) = \phi(r)(ax^{\frac{1}{k}} - by^{\frac{1}{j}}) = \delta r(ax^{\frac{1}{k}} - ay^{\frac{1}{k}}).$$

So, given y and r, the O will solve:

$$\max_x \left\{ M[\delta r(ax^{\frac{1}{k}} - ay^{\frac{1}{k}})] - \psi(x) - x_0 \right\}, i.e., \tag{0.4}$$

For given x, G solves:

$$\min_y \left\{ \lambda \left[M[\delta r(ax^{\frac{1}{k}} - ay^{\frac{1}{k}})] + y_0 \right] + \psi(y) \right\}, i.e., \tag{0.5}$$

Special Case II

So, x^* and y^* solve the following FOCs:

$$M(\frac{a\delta r}{k})x^{\frac{1-k}{k}} = x$$

$$-M\lambda(\frac{-a\delta r}{k})y^{\frac{1-k}{k}} = y$$

We get

$$x^* = \left(\frac{aM\delta r}{k}\right)^{\frac{k}{2k-1}} \quad (0.6)$$

$$y^* = \left(\frac{a\lambda\delta rM}{k}\right)^{\frac{k}{2k-1}} \quad (0.7)$$

Note that:

$$\frac{ME(r^c \mid r, x, y)}{Mr} = \delta(ax^{\frac{1}{k}} - ay^{\frac{1}{k}}). \quad (0.8)$$

Special Case III

Therefore, from (0.8), (0.6) and (0.7), the equilibrium ratio is

$$\frac{E^*(r^c \mid r, x, y)}{r} = \frac{E(r^c \mid r, x^*, y^*)}{r} = \delta a(x^{*\frac{1}{k}} - y^{*\frac{1}{k}}). \tag{0.9}$$

Proposition

$$\lambda < 1 \Rightarrow \frac{d}{dr}\left(\frac{E^*(r^c \mid r, x, y)}{r}\right) > 0.$$

Show that:

Proposition

$$\lambda < 1 \Rightarrow \frac{d}{dM}\left(\frac{E^*(r^c \mid r, x, y)}{r}\right) > 0.$$

Multiplier Vs Awards I

Assume $\lambda = 0$. So, $x^*(M, r)$ and $y^*(M, 0, r)$ satisfy

$$M \frac{\partial E(r^c \mid r, x^*)}{\partial x} = x^*, \tag{0.10}$$

and

$$y^*(M, 0, r) = \underline{y}, \tag{0.11}$$

respectively. From (0.10) and (0.11) it can be seen that

$$\frac{dy^*(M, 0, r)}{dM} = 0 \tag{0.12}$$

$$\frac{dx^*(M, r)}{dM} = \frac{\frac{\partial E(r|x^*, y^*)}{\partial x}}{1 - M \frac{\partial^2 E(r|x^*, y^*)}{\partial x^2}} > 0. \tag{0.13}$$

Therefore, we can make the following claim.

Multiplier Vs Awards II

Lemma

$(i)\ \frac{dx^*(M,y^*)}{dM} > 0$, *and* $(ii)\ \frac{dy^*(M,0,x^*)}{dM} = 0$.

Proposition

$(i)\ \frac{dE(r^c|r,x^*,y^*)}{dM} > 0$, *and* $(ii)\ \frac{dV^*}{dM} > 0$.

Proof: (i) Note that

$$\frac{dE(r^c \mid r, x^*, y^*)}{dM} = E_x(r^c \mid r, x^*, y^*)\frac{dx^*}{dM} + E_y(r^c \mid r, x^*, y^*)\frac{dy^*}{dM}.$$

Now, the claim follows immediately, in view of (0.12) and (0.13).

Multiplier Vs Awards III

(ii) The optimum value function is

$$V^* = ME(r^c \mid r, y^*(M,0,x^*), x^*(M,y^*)) - \frac{{x^*}^2(M,y^*)}{2} - x_0. \tag{0.14}$$

Therefore, we get[1]

$$\frac{dV^*}{dM} = E(r^c \mid r, y^*(M,0,x^*), x^*(M,y^*)) > 0.$$

The claim follows from the envelope theorem; alternatively, use (**??**) and (0.12).

□

Proposition

For any given r, $\frac{d\left(\frac{E(r^c|r,x^*,y^*)}{r}\right)}{dM} > 0.$

Proof is left as an exercise.

[1] Note that the litigation is feasible only if $E(r^c \mid r, x^*, y^*) - r^o > 0$ and therefore $E(r \mid x^*, y^*) > 0$ hold.

Payoffs: Symmetric Uncertainty

Let,
V_O^* denoted the expected net gains for O from litigation.

Let

$$r^a M = V_O^* = ME(r^c \mid r, x^*(r), y^*(r, \lambda)) - \psi(x^*(r, y^*)) - x_0.$$

The owner will accept the offer r^o only if

$$r^o \geq r^a.$$

Clearly, r^a depends on r. Whenever $\frac{dV_O^*}{dr} > 0$,

$$\frac{dr^a}{dr} > 0. \tag{0.15}$$

If there are no constraints to bargaining:

- The parties will bargain successfully .
- Payoffs of the O will increase with market value of property.

Prohibition of Reformatio in Peius I

- The legal doctrine applies to the decision of appeal courts, especially in the civil law countries.
- The court decision should not put the appellant in a position worse than his position before appeal.
- As a result, it is the principle of 'appeal without fear'.

In India, Section 25 of LAA 1894 (amendment, 1984)

- mandates that the court award cannot be less than the LAC awarded compensation.
- litigation by the affected parties is risk-free venture.

Prohibition of Reformatio in Peius II

Formally, let

r_{LAC} denote the compensation rate offered by the LAC.

Now, is the expected value of the compensation rate, per-square meter, awarded by the court can be written as

$$E(r \mid x, y) = \int_{r_{LAC}}^{\bar{r}^c} rf(r^c \mid r, x, y)dr.$$

It is immediate that

- $E(r^c \mid r, x, y) > r_{LAC}$. Therefore,
- litigation is always profitable, as long as the cost of legal efforts is small.