002: Introductory Mathematical Economics
Midterm 1, September 2014

Maximum Marks: 15. Please try to be brief and precise. This is meant to be a short exam, but the time allowed is the standard 70 minutes.

(1) Let $f : \Re^2 \to \Re$ be defined by $f(x, y) = x + y^2$. We wish to maximize f subject to the constraints: $y^2 \leq x$, $x \leq 2 - y$, and $y \geq 0$. Let S be the set of all points in $\Re^2$ that satisfy all three constraints.
(i) Make a sketch of the set S.
(ii) Let $((x_k, y_k))$ be any sequence of points that converges to a point (x, y). Using this sequence, show that f is continuous at (x, y). Take as given any known, intermediate, convergence results that you need.
(iii) Show formally that S is closed and bounded.
(iv) By (ii) and (iii), a maximum for f on the set S exists. Using *any* method that you wish, supported by rigorous arguments, find all points at which f attains this maximum on the set S. Marks:$(3, 1, 2, 6)$

(2) Prove or disprove the following statements. (To disprove, a counterexample suffices).
(i) If S is a bounded subset of $\Re$ and $f : S \to \Re$ is a continuous function, then $f(S)$ is bounded.
(ii) If S is a closed subset of $\Re$ and $f : S \to \Re$ is a continuous functions, then $f(S)$ is closed. (2 marks if you do either 2(i) or 2(ii) correctly; 3 marks for both correct).