Comparative Negligence and Vigilance

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Standard Liability Rules I

Let

- $\hat{x}$ be the due care level set for party X, i.e., the injurer
- $\hat{y}$ be the due care level set for party Y, i.e., the victim

Let

- s be the fraction of loss put on party X, i.e., the injurer
- $(1-s)$ be the fraction of loss put on party Y, i.e., the victim

Recall $s(x,y)$ and $1-s(x,y)$ are functions:

$$s,(1-s) : X \times Y \Rightarrow [0,1]$$

such that $(\forall(x,y))[s+(1-s)=1]$

Standard Liability Rules II

The standard liability rules are such that

$$x < \hat{x} \ \& \ y \geq \hat{y} \quad \Rightarrow \quad s = 1$$
$$x \geq \hat{x} \ \& \ y < \hat{y} \quad \Rightarrow \quad s = 0$$

Further,

$$[s(\hat{x}, \hat{y}) = \hat{s}] \Rightarrow (\text{for all } x \geq \hat{x} \text{ and } y \geq \hat{y})[s = \hat{s}]$$

Moreover, (For all $x \geq \hat{x}$ and $y \geq \hat{y}$):

$$[s = \hat{s} \in \{0, 1\} \text{ and } (1 - s) = (1 - \hat{s}) \in \{0, 1\}]$$

In the literature, the due care levels are assumed to be set efficiently. That is,

Standard Liability Rules III

Axiom

Wherever relevant

- for the injurer, the legal (due care) standard is x^*.
- for the victim, the legal (due care) standard is y^*.

In view of this assumption, the standard liability rules satisfy what we call 'Property P1', i.e., are such that

Axiom

$$\begin{aligned} x < x^* \ \& \ y \geq y^* &\Rightarrow s = 1 \\ x \geq x^* \ \& \ y < y^* &\Rightarrow s = 0 \end{aligned}$$

Standard Liability Rules IV

Moreover, standard liability rules are assumed to satisfy the following

Axiom

$$s(x^*, y^*) = s^*] \Rightarrow (\text{for all } x \geq x^* \text{and } y \geq y^*)[s = s^*]$$

Further,

$$s^* \in \{0, 1\}$$

Under the above assumptions, we get the following results:

Proposition

The care levels (x^*, y^*) is N.E.

Proposition

If $(\bar{x}, \bar{y})$ is a N.E, then $(\bar{x}, \bar{y}) \in M$

Critique of Economic Models

Standard modeling of liability rules has been criticized

- Kahan (1989): Focused on Negligence Rule and argued that
  - Sudden jump in liability is inconsistent with "Causation-doctrine"
  - Causation-doctrine implies liability for loss ‘caused by the negligence’
  - it does NOT imply liability for the entire loss
- Calabresi and Cooper (1996): Argued that
  - Juries and courts split loss when both parties are negligent or both are found to be vigilant
  - Awards are sensitive of degree of negligence as well as degrees of vigilance

Comparative Negligence and Vigilance I

Let

$$\Delta x = x - x^* \text{ and } \Delta y = y - y^*$$

Definition

Comparative Negligence: when $x < x^*$ & $y < y^*$:

- Injurer's comparative negligence is $\frac{x^*-x}{(x^*-x)+(y^*-y)} = \frac{-\Delta x}{-\Delta x - \Delta y}$
- Victim's comparative negligence is $\frac{y^*-y}{(x^*-x)+(y^*-y)} = \frac{-\Delta y}{-\Delta x - \Delta y}$

Rule of Comparative Negligence:

$$\begin{aligned} x \geq x^* \quad &\Rightarrow \quad s = 0 \\ x < x^* \;\&\; y \geq y^* \quad &\Rightarrow \quad s = 1 \\ x < x^* \;\&\; y < y^* \quad &\Rightarrow \quad s = \frac{x^* - x}{(x^* - x) + (y^* - y)} \end{aligned}$$

Comparative Negligence and Vigilance II

Definition

Comparative Vigilance: When $x \geq x^*$ & $y \geq y^*$, with $x > x^*$ or $y > y^*$:

- Injurer's Comparative Vigilance is

$$\frac{x - x^*}{(x - x^*) + (y - y^*)} = \frac{\Delta x}{\Delta x + \Delta y}$$

- Victim's Comparative Vigilance is

$$\frac{y - y^*}{(x - x^*) + (y - y^*)} = \frac{\Delta y}{\Delta x + \Delta y}$$

Pure Comparative Vigilance I

Definition

Rule of Pure Comparative Vigilance:

$$\begin{aligned} x \geq x^* \;\&\; y < y^* \quad &\Rightarrow \quad s = 0 \\ x < x^* \;\&\; y \geq y^* \quad &\Rightarrow \quad s = 1 \\ x \geq x^* \;\&\; y \geq y^* \quad &\Rightarrow \quad s = 1 - \frac{\Delta x}{\Delta x + \Delta y} = \frac{\Delta y}{\Delta x + \Delta y} \end{aligned}$$

Let $s(x^*, y^*) = s^*$

Proposition

(x^*, y^*) is NOT a N.E. under the Rule of Pure Comparative Vigilance.

Pure Comparative Vigilance II

Example

Let $\pi(x, y) = \frac{1}{(1+x)(1+y)}$, and let $D = 216 = 6^3$. So, the SOP is

$$\min_{x,y} \left\{ x + y + \frac{1}{(1 + x)(1 + y)} D \right\}$$

For this SOP, it can be shown that

- the efficient point is at $x^* = y^* = L^{1/3} - 1 = 5$.
- However, $(6.3646, 6.3646)$ is a N.E. under the rule of Pure Comparative Negligence.

Super-Symmetric Rule I

Let $s(x^*, y^*) = s^*$. Suppose,

- When $x = x^* \& y = y^*$: $s(x^*, y^*) = s^* \in [0, 1]$, and $(1 - s(x^*, y^*)) = (1 - s^*) \in [0, 1]$
- When $x \geq x^* \& y \geq y^*$, with $x > x^*$ or $y > y^*$:

$$s = s^* \frac{\pi(x^*, y^*)}{\pi(x, y)} - \frac{\Delta x}{\Delta x + \Delta y} \left(\frac{\pi(x^*, y^*)}{\pi(x, y)} - 1 \right)$$

$$(1 - s) = (1 - s^*) \frac{\pi(x^*, y^*)}{\pi(x, y)} - \frac{\Delta y}{\Delta x + \Delta y} \left(\frac{\pi(x^*, y^*)}{\pi(x, y)} - 1 \right)$$

Super-Symmetric Rule II

So, at $x \geq x^{*} \& y \geq y^{*}$, with $x > x^{*}$ or $y > y^{*}$, the Injurer's costs are

$$x + s\pi(x, y)D = x + s^{*}\pi(x^{*}, y^{*})D - \frac{\Delta x}{\Delta x + \Delta y}[\pi(x^{*}, y^{*})D - \pi(x, y)D]$$

But, the Victim's costs are

$$y + (1 - s)\pi(x, y)D = y + (1 - s^{*})\pi(x^{*}, y^{*})D - \frac{\Delta y}{\Delta x + \Delta y}[\pi(x^{*}, y^{*})D - \pi(x, y)D]$$

Next, suppose,

Super-Symmetric Rule III

- When $x < x^* \& y \geq y^*$:

$$s = s^* \frac{\pi(x^*, y^*)}{\pi(x, y)} + \frac{\pi(x, y) - \pi(x^*, y)}{\pi(x, y)}$$

$$(1 - s) = (1 - s^*) \frac{\pi(x^*, y^*)}{\pi(x, y)} + \frac{\pi(x^*, y) - \pi(x^*, y^*)}{\pi(x, y)}$$

So,the Injurer's costs become

$$\begin{aligned} x + s\pi(x, y)D &= x + s^*\pi(x^*, y^*)D \\ &\quad + [\pi(x, y)D - \pi(x^*, y)D] \end{aligned}$$

Similarly, the Victim's costs become

$$\begin{aligned} y + (1 - s)\pi(x, y)D &= y + (1 - s^*)\pi(x^*, y^*)D \\ &\quad + [\pi(x^*, y)D - \pi(x^*, y^*)D] \end{aligned}$$

Super-Symmetric Rule IV

Further, suppose,

- When $x \geq x^* \& y < y^*$:

$$s = s^* \frac{\pi(x^*, y^*)}{\pi(x, y)} + \frac{\pi(x, y^*) - \pi(x^*, y^*)}{\pi(x, y)}$$

$$(1 - s) = (1 - s^*) \frac{\pi(x^*, y^*)}{\pi(x, y)} + \frac{\pi(x, y) - \pi(x, y^*)}{\pi(x, y)}$$

So,

$$\begin{aligned} x + s\pi(x, y)D &= x + s^*\pi(x^*, y^*)D \\ &\quad + [\pi(x, y^*)D - \pi(x^*, y^*)D] \end{aligned}$$

$$\begin{aligned} y + (1 - s)\pi(x, y)D &= y + (1 - s^*)\pi(x^*, y^*)D \\ &\quad + [\pi(x, y)D - \pi(x, y^*)D] \end{aligned}$$

Finally, suppose

Super-Symmetric Rule V

- When $x < x^* \& y < y^*$:

$$s = s^* \frac{\pi(x^*, y^*)}{\pi(x, y)} + \frac{\Delta x}{\Delta x + \Delta y} \left(1 - \frac{\pi(x^*, y^*)}{\pi(x, y)} \right)$$

$$(1 - s) = (1 - s^*) \frac{\pi(x^*, y^*)}{\pi(x, y)} + \frac{\Delta y}{\Delta x + \Delta y} \left(1 - \frac{\pi(x^*, y^*)}{\pi(x, y)} \right)$$

So, we have

$$\begin{aligned} x + s\pi(x, y)D &= x + s^* \pi(x^*, y^*)D \\ &+ \frac{\Delta x}{\Delta x + \Delta y} [\pi(x, y)D - \pi(x^*, y^*)D] \end{aligned}$$

$$\begin{aligned} y + (1 - s)\pi(x, y)D &= y + (1 - s^*)\pi(x^*, y^*)D \\ &+ \frac{\Delta y}{\Delta x + \Delta y} [\pi(x, y)D - \pi(x^*, y^*)D] \end{aligned}$$

Super-Symmetric Rule VI

Proposition

(x^*, y^*) is a N.E. under the Super-Symmetric Rule.

Proof: Suppose, $y = y^*$. Injurer's costs at x^* are

$$x^* + s^*\pi(x^*, y^*)D$$

At any $x < x^*$, Injurer's costs are

$$\begin{aligned} x + s\pi(x, y)D &= x + s^*\pi(x^*, y^*)D \\ &\quad + [\pi(x, y^*)D - \pi(x^*, y^*)D], \textit{i.e.}, \end{aligned}$$

$$x + \pi(x, y^*)D + \text{a constant term}$$

So, Injurer's costs at any $x < x^*$ are greater than his costs at x^*.

At any $x > x^*$, Injurer's costs are

Super-Symmetric Rule VII

$$x + s\pi(x, y^*)D = x + s^*\pi(x^*, y^*)D - \frac{\Delta x}{\Delta x + 0}\left[\pi(x^*, y^*)D - \pi(x, y^*)D\right], i.e.,$$

$$x + \pi(x, y^*)D + \text{a constant term}$$

So, Injurer's costs at any $x > x^*$ are greater than his costs at x^*.

Similarly, it can be verified that if $x = x^*$, the Victim's costs are minimum at y^*.

Proposition

(x^*, y^*) *is a Unique N.E. under the super-symmetric rule.*

Proof: Feldman and Singh (2009) *American Law & Econ Review*.