Activity Levels

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Extended Model I

- x care level as well as the cost of care for the injurer,
- y care level as well as the cost of care for the victim,
- s activity level for the injurer,
- t activity level for the victim,
- $X = \{x \mid x$ is some feasible level of care for the injurer $\}$,
- $Y = \{y \mid y$ is some feasible level of care for the victim$\}$,
- $S = \{s \mid s$ is some feasible level of activity for the injurer $\}$,
- $T = \{t \mid t$ is some feasible level of activity for the victim$\}$,
- $u(s, x)$ the benefit function for the injurer,
- $v(t, y)$ the benefit function for the victim,
- π the probability of accident,

Extended Model II

- D the loss suffered by the victim in the event of an accident, $D \geq 0$.
- L the expected accident loss.
- Social benefits from the activity of a party are fully internalized by that party.
- $u(s,x) = u(s) - sx$, $v(t,y) = v(t) - ty$, and $L(s,x,t,y) = st\pi(x,y)D(x,y) = stl(x,y)$.

So, the social optimization problem is given by:

$$\max_{(s,x,t,y) \in S \times X \times T \times Y} \quad u(s,x) + v(t,y) - L(s,x,t,y), i.e.,$$

$$\max_{(s,x,t,y) \in S \times X \times T \times Y} \quad u(s) - sx + v(t) - ty - stl(x,y). \tag{0.1}$$

Let

- $((s^*, x^*), (t^*, y^*))$ uniquely solve (0.1)

Extended Model III

- $((s^*, x^*), (t^*, y^*)) >> ((0, 0), (0, 0))$

Therefore, s^*, x^*, t^*, and y^* simultaneously and respectively solve the following necessary and sufficient first order conditions:

$$
\begin{aligned}
u'(s) - x - tl(x, y) &= 0 \quad (0.2)\\
1 + tl_x(x, y) &= 0 \quad (0.3)\\
v'(t) - y - sl(x, y) &= 0 \quad (0.4)\\
1 + sl_y(x, y) &= 0 \quad (0.5)
\end{aligned}
$$

That is, s^*, x^*, t^*, and y^* simultaneously and respectively satisfy the following:

$$
\begin{aligned}
u'(s) &= x^* + t^* l(x^*, y^*) \quad (0.6)\\
0 &= 1 + t^* l_x(x, y^*) \quad (0.7)\\
v'(t) &= y^* + s^* l(x^*, y^*) \quad (0.8)\\
0 &= 1 + s^* l_y(x^*, y) \quad (0.9)
\end{aligned}
$$

Extended Model IV

- x and y are verifiable but s and t are not
- The legal due care standard (i.e., the negligence standard) for the injurer, is set at x^*. Similarly, the legal negligence standard of care for the victim, is set at y^*.

A liability rule is a function w_X:

$$w_X : X \times Y \mapsto [0, 1]$$

such that; $0 \leq w_X(x, y) \leq 1$, and $w_X(x, y) + w_Y(x, y) = 1$, i.e., $w_Y(x, y) = 1 - w_X(x, y)$.

Impossibility of Efficient Outcome I

For given $(t, y) \in T \times Y$ opted by the victim and the liability rule in force, the problem facing the injurer is

$$\max_{(s,x)\in S\times X} \quad u(s) - sx - w_X(x,y)stl(x,y). \tag{0.10}$$

Likewise, given $(s, x) \in S \times X$ opted by the injurer, the problem facing the victim is

$$\max_{(t,y)\in T\times Y} \quad v(t) - ty - (1 - w_X(x,y))stl(x,y), \tag{0.11}$$

To see why no liability rule is efficient. Consider a liability rule. Suppose it induces an equilibrium in which the injurer opts for x^* and the victim opts for y^* - otherwise there is nothing to prove.

Now, given the equilibrium choice of x^* by the injurer and of y^* by the victim, the injurer will choose s to solve

$$\max_{s}\{u(s) - sx^* - w_X(x^*, y^*)stl(x^*, y^*)\},$$

Impossibility of Efficient Outcome II

Similarly, the victim will choose t that satisfies

$$\max_t \{v(t) - ty^* - (1 - w_X(x^*, y^*))stl(x^*, y^*).\}$$

So, in equilibrium, s and t will satisfy (0.12) and (0.13), respectively.

$$u'(s) = x^* + w_X(x^*, y^*)tl(x^*, y^*) \quad (0.12)$$

$$v'(t) = y^* + (1 - w_X(x^*, y^*))sl(x^*, y^*), \quad (0.13)$$

In view of (0.6) and (0.8), s^* and t^* will solve (0.12) and (0.13) only if $w_X(x^*, y^*) = 1$ and simultaneously $(1 - w_X(x^*, y^*)) = w_Y(x^*, y^*) = 1$ holds. However, under a liability rule this is impossible.

An Efficient Mechanism I

Consider the following mechanism:

$$(\forall (x, y) \in X \times Y)[w_X(x, y) = 1 \text{ and } w_Y(x, y) = 1]$$

That is, regardless of the care choice made by the two parties, both are required to bear full accident loss. For example,

- the injurer is required to deposit a fine equal to accident loss
- at the same time, the victim is not provided any compensation

Under such a mechanism, the injurer will solve

$$\max_{s,x}\{u(s) - sx - w_X(x, y)stl(x, y)\},$$

Similarly, the victim will choose t that satisfies

$$\max_{t}\{v(t) - ty - w_Y(x, y)stl(x, y).\}$$

An Efficient Mechanism II

So, the equilibrium is characterized by the following first order conditions:

$$u'(s) - x - tl(x, y) = 0 \quad (0.14)$$

$$1 + tl_x(x, y) = 0 \quad (0.15)$$

$$v'(t) - y - sl(x, y) = 0 \quad (0.16)$$

$$1 + sl_y(x, y) = 0 \quad (0.17)$$

which are same as the system (0.2)- (0.5). So, s^*, x^*, t^*, and y^* will be opted simultaneously.

Predicting the Outcome I

Property (P1):

$$[(x \geq x^* \& \;\; y < y^* \Rightarrow \;\; w_X = 0) \text{ and } (x < x^* \& \;\; y \geq y^* \Rightarrow \;\; w_X = 1)].$$

Lemma

Under a liability rule satisfying (P1),
$(\forall((s,x),(t,y)))\ [x < x^* \ \& \ y < y^* \Rightarrow ((s,x),(t,y))$ *cannot be a N.E.* $]$.

Take any $((s,x),(t,y))$ such that $x < x^*$ and $y < y^*$. Suppose, the injurer opts for (s,x) and the victim for (t,y).

At at $((s,x),(t,y))$, the expected payoff of the victim is

$$v(t) - ty - (1 - w_X(x,y))stl(x,y).$$

On the other hand, if the victim instead opts for (t^*, y^*), then his payoff will be $v(t^*, y^*)$.

Predicting the Outcome II

Similarly, at $((s,x),(t,y))$ the expected payoff of the injurer is $u(s) - sx - w_X(x,y)\ stl(x,y)$. But, if the injurer instead opt for (s^*,x^*), his payoff will be $u(s^*,x^*)$.

At $((s,x),(t,y))$ if

$$u(s^*,x^*) > u(s) - sx - w_X(x,y)\ stl(x,y),$$

$((s,x),(t,y))$ cannot be a N.E. Therefore, assume that

$$u(s) - sx - w_X(x,y)\ stl(x,y) \geq u(s^*,x^*). \tag{0.18}$$

Since $((s,x),(t,y)) \neq ((s^*,x^*),(t^*,y^*))$, by assumption, we know that

$$u(s^*,x^*) + v(t^*,y^*) - L(s^*,x^*,t^*,y^*) > u(s) - sx + v(t) - ty - stl(x,y). \tag{0.19}$$

Predicting the Outcome III

Subtracting $u(s^*, x^*)$ from the LHS and $u(s) - sx - w_X(x,y)stl(x,y)$ from the RHS of (5), in view of (4), we get

$$v(t^*, y^*) - L(s^*, x^*, t^*, y^*) > v(t) - ty - (1 - w_X(x,y))stl(x,y). \qquad (0.20)$$

Now, since $L(s^*, x^*, t^*, y^*) \geq 0$, from (6) we have

$$v(t^*, y^*) > v(t) - ty - (1 - w_X(x,y))stl(x,y).$$

Again, $((s,x),(t,y))$ cannot be a N.E.

Lemma

Under a liability rule satisfying (P1),
$(\forall((s,x),(t,y)))$ $[x > x^* \;\&\; y < y^* \Rightarrow ((s,x),(t,y))$ *cannot be a N.E.* $]$.

Predicting the Outcome IV

Lemma

Under a liability rule satisfying (P1),
$(\forall((s,x),(t,y)))$ $[x < x^* \ \& \ y > y^* \Rightarrow ((s,x),(t,y))$ *cannot be a N.E.* $]$.

Remark

The above results hold for any general $u(s,x)$, $v(t,y)$ and $L(s,x,t,y)$ functions, subject to usual assumptions.

Theorem

Suppose $L(s,x,t,y) = stl(x,y)$. *Under a liability that satisfies Property* $(P1)$,
$(\forall((s,x),(t,y)))$ $[((s,x),(t,y))$ *is a N.E.* $\Rightarrow (x \geq x^* \& \ y \geq y^*)]$.

Property (P2):

$$(\forall x \in X)(\forall y \in Y)[x \geq x^* \& y \geq y^* \Rightarrow w_X(x,y) = w_X(x^*,y^*) = w_X^* \ ].$$

Equilibrium under Standard Liability Rules I

Miceli (1997 p. 29), Cooter and Ulen (2004, pp. 332-33), Dari Mattiacci (2002), Parisi and Fon (2004), among others, assume $L(s, x, t, y) = stl(x, y)$.

These studies argue that:
the injurer and the victim opt for x^* and y^*, respectively

- under the rule of negligence
- under the rule of negligence with the defense of contributory negligence
- under the rule of strict liability with the defense of contributory negligence

Equilibrium under Standard Liability Rules II

Lemma

Suppose a liability rule satisfies Properties (P1) and (P2) with $w_X^* \in \{0, 1\}$*:*
Under the rule:
$((s, x), (t, y))$ *is a N.E.* $\Rightarrow (x \neq x^*$ *or* $y \neq y^*)$

Lemma

Suppose a liability rule satisfies Properties (P1) and (P2) with $w_X^* \in \{0, 1\}$*,*
Under the rule: when $w_X^* = 0$*, for some* $y > y^*$ *&* $t < t^*$*,* $((s_p^*, x^*), (t, y))$ *is a N.E.* $(s_p^* > s^*)$

Lemma

Suppose a liability rule satisfies Properties (P1) and (P2) with $w_X^* \in \{0, 1\}$*,*
Under the rule:
when $w_X^* = 1$*, for some* $s < s^*$ *&* $x > x^*$*,* $((s, x), (t_p^*, y^*))$ *is a N.E.* $(t_p^* > t^*)$

Equilibrium under Standard Liability Rules III

Suppose, $((s,x),(t,y))$ is a N.E. under the rule.

- if $x \neq x^*$, there is nothing to prove.
- so, let $x = x^*$.

$w_X^* = 0$, in view of Properties (P1)-(P2), implies that if the injurer opts for a pair (s,x^*), his payoff is $u(s,x^*)$, regardless of the care level and activity level chosen by the victim.

Let, $u(s,x^*) = u(s) - sx^*$ attains a unique maximum at (s_p^*, x^*). Clearly, $s_p^* > s^*$. Therefore, when $w_X^* = 0$,

$$[((s,x),(t,y)) \text{ is a N.E. and } x = x^*] \Rightarrow ((s_p^*, x^*),(t,y)) \text{ is a N.E.}$$

Now, given (s_p^*, x^*) opted by the injurer and $w_X^* = 0$, the problem facing the victim is

$$\max_{(t,y)\in T\times Y} \quad v(t) - ty - s_p^* t l(x^*, y).$$

Equilibrium under Standard Liability Rules IV

Therefore, the victim will choose $t \in T$ and $y \in Y$ that simultaneously satisfy

$$v'(t) = y + s_p^* l(x^*, y) \tag{0.21}$$

$$1 + s_p^* l_y(x^*, y) = 0. \tag{0.22}$$

Now in view of the fact that $s_p^* > s^*$ and that $l(.)$ is strictly convex, implies that $y > y^*$.

This means that regardless of the $t \in T$ opted by the victim, $((s_p^*, x^*), (t, y^*))$ cannot be a N.E.

When $w_X^* = 1$, an analogous argument shows that $((s, x^*), (t, y^*))$ cannot be a N.E.
Therefore, $[((s, x), (t, y))$ is a N.E. $\Rightarrow (x \neq x^*$ or $y \neq y^*)]$.