Efficiency of Liability Rules: Re-Examination

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Lecture 17

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Equilibrium under Standard Liability Rules I

Miceli (1997 p. 29), Cooter and Ulen (2004, pp. 332-33), Dari Mattiacci (2002), Parisi and Fon (2004), among others, assume $L(s, x, t, y) = stl(x, y)$.

These studies argue that:
the injurer and the victim opt for x^* and y^*, respectively

- under the rule of negligence
- under the rule of negligence with the defense of contributory negligence
- under the rule of strict liability with the defense of contributory negligence

Equilibrium under Standard Liability Rules II

Lemma

Suppose a liability rule satisfies Properties (P1) and (P2) with $w_X^* \in \{0, 1\}$: Under the rule:
$((s, x), (t, y))$ *is a N.E.* $\Rightarrow (x \neq x^*$ *or* $y \neq y^*)$

Proposition

Suppose a liability rule satisfies Properties (P1) and (P2). If $w_X^* = 0$, then for some $y > y^*$ & $t < t^*$, $((s_p^*, x^*), (t, y))$ is a N.E. ($s_p^* > s^*$).

Let the equilibrium be denoted by $((\hat{s}, \hat{x}), (\hat{t}, \hat{y}))$. We know that $\hat{x} \geq x^*$ and $\hat{y} \geq y^*$ holds.

When $w_X^* = 0$, $\hat{x} \geq x^*$ and $\hat{y} \geq y^*$
$u(s) - sx^*$ attains a unique maximum at (s_p^*, x^*), where s_p^* solves

$$u'(s) = x^*, i.e.,$$

Equilibrium under Standard Liability Rules III

$s_p^* > s^*$. Therefore, when $w_X^* = 0$, the injurer will choose the pair (s_p^*, x^*). That is, $(\hat{s}, \hat{x}) = (s_p^*, x^*)$

Now, given (s_p^*, x^*) opted by the injurer, the problem facing the victim is

$$\max_{(t,y)\in T\times Y} \quad v(t) - ty - s_p^* tl(x^*, y).$$

Therefore, the victim will choose $\hat{t} \in T$ and $\hat{y} \in Y$ that simultaneously satisfy

$$v'(t) = y + s_p^* l(x^*, y) \tag{0.1}$$

$$1 + s_p^* l_y(x^*, y) = 0. \tag{0.2}$$

Now in view of the fact that $s_p^* > s^*$ and that $l_{yy}(.) > 0$, implies that $\hat{y} > y^*$.

Equilibrium under Standard Liability Rules IV

Note (t^*, y^*) uniquely solves

$$\max_{(t,y)\in T\times Y}\{v(t)+u(s^*)-ty-s^*x^*-s^*tl(x^*,y)\}, i.e.,$$

$$\max_{(t,y)\in T\times Y}\{v(t)-ty-s^*tl(x^*,y)\}, i.e.,$$

y^* uniquely solves

$$\max_{y}\{v(t^*)-t^*y-s^*t^*l(x^*,y)\}, i.e.,$$

y^* uniquely solves

$$\min_{y}\{t^*[y+s^*l(x^*,y)]\}, i.e.,$$

y^* uniquely solves

$$\min_{y}\{y+s^*l(x^*,y)\}.$$

Equilibrium under Standard Liability Rules V

Therefore

$$(\forall y \in Y)[y + s^* l(x^*, y) \geq y^* + s^* l(x^*, y^*)] \tag{0.3}$$

Moreover,

$$(\forall y \in Y)[y + s_p^* l(x^*, y) > y + s^* l(x^*, y)]$$

Therefore,

$$(\forall y \in Y)[y + s_p^* l(x^*, y) > y^* + s^* l(x^*, y^*)] \tag{0.4}$$

Recall, t^* solves

$$v'(t) = y^* + s^* l(x^*, y^*)$$

but, equilibrium choice $\hat{t}$ will solve

$$v'(t) = y + s_p^* l(x^*, y)$$

Which gives us $\hat{t} < t^*$.

Equilibrium under Standard Liability Rules VI

Remark

- When For $(s_p^*, x^*, \hat{t}, \hat{y})$ to be a N.E., the following must hold: for all (s, x) such that $x < x^*$,

$$u(s) - sx - s\hat{t}l(x, \hat{y}) < u(s_p^*) - s_p^* x^*$$

Proposition

Suppose a liability rule satisfies Properties (P1) and (P2). If $w_X^* = 1$, then for some $x > x^*$ & $s < s^*$, $((s, t), (t_p^*, y^*))$ is a N.E.

The Second Best Liability Rules I

Suppose the Social Planner can

- fix legal standards at x^* and y^* and can implement them with sufficient penalty for deviations
- choose $w_X^* = w_X(x^*, y^*)$ and allows parties to choose ONLY their activity levels

Question

- Is it possible to induce an equilibrium in which injurer chooses (s^*, x^*), and the victim opts for y^* along with some t?
- Is it possible to induce an equilibrium in which injurer chooses x^* along with some s, and the victim opts for (t^*, y^*) ?

The Second Best Liability Rules II

By assumption the Social Planner can induce x^* and y^*.

Let $w_Y^* = 1 - w_X^*$. Now, given the choice of x^* by the injurer and of y^* by the victim, the injurer will choose s to solve

$$\max_s \{u(s) - sx^* - w_X^* stl(x^*, y^*)\},$$

Similarly, the victim will choose t that satisfies

$$\max_t \{v(t) - ty^* - (1 - w_X^*)stl(x^*, y^*).\}$$

So, in equilibrium, $\hat{s}$ and $\hat{t}$ will satisfy (0.5) and (0.6), respectively.

$$u'(s) = x^* + w_X^* tl(x^*, y^*) \quad (0.5)$$

$$v'(t) = y^* + (1 - w_X^*)sl(x^*, y^*), \quad (0.6)$$

The Mis-specified Optimization Problem I

It turns out that the social benefit function considered above, leads to a mis-specified optimization problem. To show this let us use the following example.

- $u(s) - sx = \sqrt{s} - sx$
- $v(t) - ty = \sqrt{t} - ty$
- $l(x, y) = \pi D = \frac{D}{1+x+y}$, where D is constant

so the SOP is

$$\sqrt{s} + \sqrt{t} - sx - ty - \frac{stD}{1 + x + y}$$

Assuming $D = 576$, we get

$$(s^*, x^*, t^*, y^*) = (\frac{1}{64}, 1, \frac{1}{64}, 1)$$

Moreover, the TSB at (s^*, x^*, t^*, y^*)

The Mis-specified Optimization Problem II

is

$$0.172$$

Suppose the Social Planner

- sets the legal standards at $x^* = 1$ and $y^* = 1$ and implements them
- choose $w_X^* = 0$ and allows parties to choose there care levels

Now, $\hat{s}$ will maximize

$$\sqrt{s} - sx^*$$

$$\frac{1}{2}s^{\frac{-1}{2}} = 1, i.e., \tag{0.7}$$

$\hat{s} = \frac{1}{4}$.

But $\hat{t} \in T$ will be such that

$$\frac{1}{2}t^{\frac{-1}{2}} = 1 + 1\frac{sD}{3} \tag{0.8}$$

The Mis-specified Optimization Problem III

$\hat{t} = \frac{1}{(98)^2}$
Moreover, the TSB at $(\hat{s}, x^*, \hat{t}, y^*) = (\frac{1}{4}, 1, \frac{1}{(98)^2}, 1)$ is

$$0.255$$

Reason???
Let $x = y = t = 0$, now the TSB is given by

$$\sqrt{s}$$

which is unbounded!

The standard SOP is

$$u(s) + v(t) - sx - ty - stl(x, y)$$

has the same problem as long as $u(s)$ or $v(t)$ is monotonic.