Product Liability

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Lecture 18

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Basics I

- x the cost of care taken by the firm, $x \geq 0$, $x \in X$,
- y the cost of care taken by the consumer, $y \geq 0$, $y \in Y$,
- π the probability of occurrence of an accident,
- D the loss in case an accident actually materializes, $D \geq 0$,
- L the expected loss due to the accident. $L = \pi D$,
- L_c the expected accident loss as perceived by the consumer,
- n number of firms in the industry,
- q number of units of the product produced by a firm,

Basics II

DAC the direct accident costs - the sum of costs of care and the expected accident loss, i.e., $DAC = x + y + L(x, y)$.
Function $x + y + L(x, y)$ is uniquely minimized at $(x^*, y^*) >> (0, 0)$. As a result, for all $(x, y) \neq (x^*, y^*)$, we have

$$x + y + L(x, y) > x^* + y^* + L(x^*, y^*).$$

- Neither party observes the care taken by the other party.
- Firms are completely informed about the expected loss function $L(x, y)$.
- Consumers, on the other hand, are not completely informed about the expected loss;
- when the expected loss is $L(x, y)$, a consumer perceives it to be $L_c(x, y)$, where $L_c(.)$ may not be equal to $L(.)$. However, $L_c(x, y) \geq 0$

Basics III

Remark

x, y, π, D, and L are defined *per unit* of the product.

DAC = $x + y + L(x, y)$. Therefore, the direct accident costs per firm are

$$q[x + y + L(x, y)],$$

and the direct accident costs for the entire industry are

$$nq[x + y + L(x, y)].$$

Social Objective I

- $u_i(.)$ marginal consumption benefit to the i'th consumer from the product; $u_i'(.) < 0$
- p the *market* price of one unit of the product,
- $C(.)$ the cost of production function for a firm. $C(q)$, is such that $C(q)/q$ has a unique minima.
- $P(.)$ the inverse demand function for the industry,

Social Objective II

The social objective is to choose x, y, q and n so as to maximize the social surplus

$$\int_0^{nq} P(z)dz - nC(q) - nq[x + y + L(x, y)]. \tag{0.1}$$

The focs for q and n are given by (2) and (3), respectively.

$$P(nq) = C'(q) + x + y + L(x, y) \tag{0.2}$$

$$P(nq) = \frac{C(q)}{q} + x + y + L(x, y). \tag{0.3}$$

Let

- $\bar{q}(x, y)$ and $\bar{n}(x, y)$ uniquely solve (0.2) and (0.3) simultaneously.
- $\bar{q}(x^*, y^*) = q^*$ and $\bar{n}(x^*, y^*) = n^*$.

Product Liability Rules I

A PLR can be defined as a function

$$w_X : X \times Y \mapsto [0, 1], \text{such that: } w_X \in [0, 1]$$

where $w_X \geq 0$ [$w_Y \geq 0$] is the proportion of loss that the firm [the consumer] is required to bear. Clearly, $w_X + w_Y = 1$.

The firm's expected accident costs are:

$$x + w_X(x, y)\pi(x, y)D(x, y) = x + w_X(x, y)L(x, y).$$

As far as the consumer is concerned, his his expected accident costs are:

$$y + L_c(x, y) - w_X(x, y)L_c(x, y) = y + w_Y(x, y)L_c(x, y)$$

Competitive Equilibrium I

The 'perceived' full price *per unit* of product is

$$p + y + w_Y(x, y)L_c(x, y)$$

Therefore, a consumer i's problem is to choose the quantity q_i and the level of care y to maximize his utility

$$\int_0^{q_i} u_i(z)dz - pq_i - q_i[y + w_Y(x, y)L_c(x, y)] \tag{0.4}$$

the foc is

$$u_i(q_i) = p + y + w_Y(x, y)L_c(x, y).$$

Competitive Equilibrium II

Alternatively, consumers' problem is equivalent to that of choosing the quantity Q and the care y to maximize

$$\int_0^Q P(z)dz - pQ - Q[y + w_Y(x,y)L_c(x,y)]. \tag{0.5}$$

The foc for Q is

$$P(Q) = p + y + w_Y(x,y)L_c(x,y). \tag{0.6}$$

Similarly, given the PLR a firm's problem is to choose the quantity q and the level of care x so as to maximize

$$pq - C(q) - q[x + w_X(x,y)L(x,y)]. \tag{0.7}$$

The foc for q is

$$p = C'(q) + x + w_X(x,y)L(x,y). \tag{0.8}$$

Competitive Equilibrium III

Free entry assumption implies that

$$\begin{aligned} pq &= C(q) + q[x + w_X(x,y)L(x,y)], \textit{i.e.}, \\ p &= \frac{C(q)}{q} + x + w_X(x,y)L(x,y) \end{aligned} \tag{0.9}$$

Remark

A consumer chooses y to minimizes $y + w_Y(x,y)L_c(x,y)$, regardless of his level of consumption.
Analogously, the firm chooses x to minimizes $x + w_X(x,y)L(x,y)$.

Competitive Equilibrium IV

An *equilibrium* is defined as a tuple $< \hat{Q}, \hat{p}, \hat{q}, \hat{n}, \hat{x}, \hat{y} >$ such that:

$$\hat{Q} = \hat{n}\hat{q};$$

and $\hat{Q}$, $\hat{p}$, and $\hat{q}$ satisfy (6), (8) and (9), respectively;

$$\hat{x} = arg\ min_{x \in X}\{x + w_X(x, y)L(x, y)\}$$

and

$$\hat{y} = arg\ min_{y \in Y}\{y + w_Y(x, y)L_c(x, y)\}.$$

Competitive Equilibrium V

Now, from (0.6)&(0.8) in equilibrium we have

$$\begin{aligned} P(Q) &= P(nq) \\ &= C'(q) + y + x + w_Y(x,y)L_c(x,y) + w_X(x,y)L(x,y) \end{aligned} \tag{0.10}$$

and from (0.6) and (0.9) we get

$$\begin{aligned} P(Q) &= P(nq) \\ &= \frac{C(q)}{q} + y + x + w_Y(x,y)L_c(x,y) + w_X(x,y)L(x,y). \end{aligned} \tag{0.11}$$

Remark

Generally the solution to (0.10)&(0.11) will be different from that of (0.2)&(0.3). (0.10)&(0.11), however, imply that $C'(q) = C(q)/q$, i.e., in equilibrium output per firm is efficient.

Efficient PLRs: Unilateral Care I

- Only the firm can take care
- Formally, $y > 0$ is not possible; you can also assume $y^* = 0$.
- So, x^* solves

$$\min_x \{x + L(x)\}$$

So, (0.10) and (0.11) become

$$\begin{aligned} &= C'(q) + x + w_Y(x, y)L_c(x, y) + w_X(x, y)L(x, y) \\ &= \frac{C(q)}{q} + x + w_Y(x, y)L_c(x, y) + w_X(x, y)L(x, y). \qquad (0.12) \end{aligned}$$

Also, a firm will choose x to minimizes

$$x + w_X(x)L(x).$$

Efficient PLRs: Unilateral Care II

Exercise

Find out the equilibrium outcome under the Rule of Strict Liability, Rule of Negligence, and the Rule of No-liability. Assuming that

1. consumers know $L(x)$ function and can observe x.
2. consumers know $L(x)$ function but cannot observe x.
3. consumers do NOT know $L(x)$ function correctly, moreover they cannot observe x.

Efficient PLRs: Bilateral Care I

Definition

A PLR is efficient if for every given Y, X, L, (x^*, y^*), and $C(q)$, iff: (x^*, y^*) is a unique Nash equilibrium (N.E.); and in equilibrium q^* and n^* solve (0.10) and (0.11), simultaneously.

Lemma

A PLR is efficient for every possible choice of Y, X, L, (x^*, y^*), L_c, and $C(q)$ only if

$$y \geq y^* \Rightarrow w_X = 1.$$

Efficient PLRs: Bilateral Care II

Definition

Condition of Negligent Consumer's Liability (NCL): A PLR satisfies condition NCL if:

$$\begin{aligned} y \geq y^* &\Rightarrow w_X = 1 \\ x \geq x^* \& \; y < y^* &\Rightarrow w_X = 0 \end{aligned}$$

Rule of Strict Liability with Defense:

$$\begin{aligned} y \geq y^* &\Rightarrow w_X = 1 \\ y < y^* &\Rightarrow w_X = 0 \end{aligned}$$

Efficient PLRs: Bilateral Care III

Another Strict Liability based rule:

$$\begin{aligned} y \geq y^* &\Rightarrow w_X = 1 \\ x \geq x^* \,\&\, y < y^* &\Rightarrow w_X = 0 \\ x < x^* \,\&\, y < y^* &\Rightarrow w_X = 1 \end{aligned}$$

Property P1: If a PLR specifies the due care standards for both the parties, L_c is such that: $(\forall (x, y) \in X \times Y)$

$$\begin{aligned} x^* + y^* + L_c(x^*, y^*) &\leq x + y + L_c(x, y), \\ (x, y) \neq (x^*, y^*) &\Rightarrow [x^* + y^* + L_c(x^*, y^*) < x + y + L_c(x, y)] \end{aligned}$$

Lemma

If a PLR satisfies the condition NCL then for every possible choice of X, Y, L, (x^*, y^*), and L_c satisfying (P1), (x^*, y^*) is a N.E.

Efficient PLRs: Bilateral Care IV

Lemma

If a PLR satisfies condition NCL then for every possible choice of X, Y, L, (x^*, y^*), and L_c satisfying (P1),

$$(\forall(\bar{x}, \bar{y}) \in X \times Y)[(\bar{x}, \bar{y}) \text{ is a N.E.} \rightarrow (\bar{x}, \bar{y}) = (x^*, y^*)].$$