Delhi School of Economics
Course 001: Microeconomic Theory
Solutions to Midterm exam, 2014.

1. (a) For interior solution

$$MRS = \frac{1}{2}\left(\frac{x_2+1}{x_1-1}\right) = \frac{p_1}{p_2}$$

Using this in the budget constraint, we get

$$x_1 = \frac{y+2p_1+p_2}{3p_1}$$
$$x_2 = \frac{2y-2p_1-p_2}{3p_2}$$

For a corner solution at $x_2 = 0$, we need $|MRS| > \frac{p_1}{p_2}$ at that point. This is also equivalent to the expression for x_2 derived above being positive, i.e.,

$$y \geq \frac{2p_1+p_2}{2}$$

The condition for interior solution is the opposite inequality. There will never be a corner solution at $x_1 = 0$, since choices that generate positive utility are within the budget set (given $y > p_1$).

(b) The parametric condition required for $x_2 = 0$ is captured in the last equation.

2. (a) The firm's supply curve is obtained by applying, first, the $P = MC$ rule, which gives

$$4y = p \Rightarrow y = \frac{p}{4}$$

The firm's average cost curve is given by

$$AC(y) = \frac{8}{y} + 2y$$

Setting $AC'(y) = 0$ and solving, we find that the minimum average cost is attained at $y = 2$ and this minimum value is $AC_{\min} = 8$. Therefore, the supply is positive iff $p \geq 8$. Now for this range of prices and for 20 firms, the aggregate supply is

$$Y = \frac{p}{4}.20 = 5p$$

Using the demand function, we have market clearance when

$$60 - p = 5p \Rightarrow p = 10$$

Since this price is above $AC_{\min}$, firms will be willing to supply the required quantity. Equilibrium $q = 50$.

(b) In the long run equilibrium, price $p^* = AC_{\min} = 8$. Hence equilibrium quantity $q^* = 60 - 8 = 52$. Each firm supplies $y^* = 2$. Total number of firms $n^* = \frac{q^*}{y^*} = 26$.

(c) Let the tax be t per unit. Now we have $AC_{\min} = 8 + t$. In a long run equilibrium, since firms make zero profits, we must have $p = 8 + t$. However, since the target quantity is 20, from the demand function, we also have $20 = 60 - p$, or, $p = 40$. Equating the two, we get $t = 32$.

(d) For any tax t, the long run equilibrium price will be $p = 8 + t$. From the demand function, total sales is $q = 60 - 8 - t$. Then the revenue maximizing government sets

$$t^* = \arg\max_t t(52 - t) = 26$$

The revenue maximizing tax rate is lower.