Explanatory Notes on Liability Rules

Ram Singh[*]

September 17, 2015

1 Unilateral care accidents

Suppose only the injure can take care. In that case the total accident cost (TAC) will be

$$x + \pi(x)D(x) = x + L(x) \tag{1}$$

Suppose TAC is minimized at x^*, assume that $x^* > 0$. From social efficiency perspective, we want the injurer to choose x^* as care level. However, injurer will act to minimize her costs. So, depending on liability rule she may or may not choose x^*.

We say that rule is efficient if it encourages the injurer to choose x^* in her own interest. For example, under the rule of strict liability she indeed will choose x^*. Similarly, under the rule of negligence, when the due care standard is set at x^*, the injure will find in her interests to choose x^*. So, both of these rules are efficient. In contrast, under the rule of no-liability she will spend nothing on care, i.e., will choose 0 as care level. So, the rule of no-liability is inefficient.

2 Bilateral care accidents

Suppose both the injure as well as the victim can take care. In that case the total accident cost (TAC) will be

[*]Department of Economics, Delhi School of Economics.

$$x + y + \pi(x, y)D(x, y) = x + y + L(x, y) \tag{2}$$

Suppose TAC are uniquely minimized at (x^*, y^*), assume that $x^* > 0$ and $y^* > 0$.

Remark 1 *Note that if it happens that x opted by the injurer is not equal to x^* or y opted by the victim is not equal to y^* or both, i.e., $x \neq x^*$ or $y \neq y^*$ or both, then*

$$x + y + L(x, y) > x^* + y^* + L(x^*, y^*) \tag{3}$$

That is, the actual TAC will be greater than the minimum possible TAC.

From social efficiency perspective, we want the injurer to choose x^* as care level. Simultaneously, we want the victim to choose y^* - only then the TAC will be minimized. However, injurer will act to minimize her costs. So, depending on liability rule she may or may not choose x^*. Similarly, for the victim.

The injure would want to choose x to minimize her total costs

$$x + s(x, y)L(x, y)$$

And, the victim would choose y to minimize his total costs

$$y + [1 - s(x, y)]L(x, y)$$

As you can see their costs and therefore their behaviour would depend on $s(x, y)$ which is determined by the liability rule.

We say that a liability rule is efficient if it encourages the injurer to choose x^* in her own interest. At the same time, the rule should encourage the victim to choose x^* in his own interest. We can check whether the rule is efficient or not by finding out the Nash equilibrium under the rule. The rule will be called efficient if and only if (x^*, y^*) is a unique Nash equilibrium under the rule.

For example, under the rule of negligence, when the due care standard for the injurer is set at x^*, we have seen that (x^*, y^*) is a unique Nash equilibrium.

That, the injure finds in her interests to choose x^*, and simultaneously the victim finds in her interest to opt for y^*. In contrast, under the rule of no-liability, in equilibrium, the injurer will spend nothing on care, i.e., will choose 0 as care level. So, assuming that $x^* > 0$, (x^*, y^*) is NOT a Nash equilibrium. the rule of no-liability is inefficient. Similarly, assuming that $y^* > 0$, (x^*, y^*) is NOT a Nash equilibrium under the rule of strict liability. So this rule is also inefficient.

3 Detailed Proofs

Definition 1 *Rule of Negligence:*

$$\begin{aligned} x \geq x^* \quad &\Rightarrow \quad s(x,y) = 0 \\ x < x^* \quad &\Rightarrow \quad s(x,y) = 1 \end{aligned}$$

Proposition 1 (x^*, y^*) *is N.E. under the Rule of Negligence (RON).*

Proof: Note that under the Rule of Negligence, $s(x^*, y^*) = 0$. To prove that (x^*, y^*) is N.E. under RON, we have to show that if the victim opts for y^*, the injurer will minimize her cost by choosing x^*, and *vice-versa*. So, suppose the victim has opted for y^*. Now, note that

- if the injurer opts for x^*, his total cost will be $x^* + s(x^*, y^*)L(x^*, y^*) = x^*$, and
- if he opts for some $x < x^*$ his total cost will be

  $$\begin{aligned} x + s(x, y^*)D(x, y^*)\pi(x, y^*) \quad &= \quad x + s(x, y^*)L(x, y^*) \\ &= \quad x + L(x, y^*) \end{aligned}$$

  since $s(x, y^*) = 1$ under the RON.

Injurer will choose $x < x^*$ over x^*, only if it is a lower cost choice for her, that is, only if

$$\begin{aligned} x + L(x, y^*) \quad &< \quad x^*, i.e., \text{only if} \\ x + y^* + L(x, y^*) \quad &< \quad x^* + y^*, i.e., \text{only if} \\ x + y^* + L(x, y^*) \quad &< \quad x^* + y^* + L(x^*, y^*) \qquad (4) \end{aligned}$$

That is, a choice of x less than x^* can be better for the injurer only if (4) is true; formally speaking, (4) has to be necessarily true if a choice of $x < x^*$ is better for the injurer.

However, (4) cannot be true - in fact, it is a contradiction. To see why note that since $x < x^*$, therefore $(x, y) \neq (x^*, y^*)$. Therefore, in view of the above Remark, total accident costs (TAC) at (x, y) are greater than TAC at (x^*, y^*): That is,

$$\bar{x} + \bar{y} + L(\bar{x}, \bar{y}) > x^* + y^* + L(x^*, y^*) \tag{5}$$

is definitely true. In that case (4) cannot be true.

(4) is the result of assuming that a choice of $x < x^*$ rather than the choice of x^* is better for the injurer. So, this assumption was wrong. This means x^* rather than some $x < x^*$ is better choice for the injurer.

Next, consider a choice of $x > x^*$ by the injurer (assuming that the victim is still spending y^* on care). Note that under the RON, the injurer can avoid liability simply by choosing x^*; so why to choose a $x > x^*$ that will increase her costs. So, we have proved that:

- If the victim opts for y^*, the injurer will opt for x^*
- Similarly, we can prove that if the injurer opts for x^*, the victim will opt for y^*
- (x^*, y^*) is a N.E.

Proposition 2 (x^*, y^*) *is a* unique *N.E. under the Rule of Negligence.*

Proof: Let $(\bar{x}, \bar{y})$ be a (any) N.E. under the Rule of Negligence (RON).

Note that $(\bar{x}, \bar{y})$ is a N.E. means that -given the choice of $\bar{y}$ by the victim - the injure's costs are minimum at $\bar{x}$, and *vice-versa*. However, under the RON, the injurer can avoid liability simply by opting x^*, so there is no need for her to spend more on care. This means that if $(\bar{x}, \bar{y})$ is a N.E. then $\bar{x} > x^*$ cannot be true; otherwise the injurer can reduce her cost be choosing x^* instead. This leaves us with two possible cases.

Case 1: $\bar{x} = x^*$. Recall, we have seen that (x^*, y^*) is a N.E. and that if the injurer opts for x^*, then the victim's costs are uniquely minimum at y^*. Therefore, if $(\bar{x}, \bar{y})$ is a N.E., and $\bar{x} = x^*$, this means $\bar{y} = y^*$. That is,

$$(\bar{x}, \bar{y}) = (x^*, y^*).$$

Case 2: $\bar{x} < x^*$.

Now $(\bar{x}, \bar{y})$ is a N.E. implies that given the choice of $\bar{y}$ by the victim, the injurer's costs are minimum at $\bar{x}$. That is, if $(\bar{x}, \bar{y})$ is a N.E, then the following should be true

$$\begin{aligned} \bar{x} + s(\bar{x}, \bar{y})L(\bar{x}, \bar{y}) &\leq x^* + s(x^*, \bar{y})L(x^*, \bar{y}), i.e., \\ \bar{x} + L(\bar{x}, \bar{y}) &\leq x^*, \end{aligned} \tag{6}$$

since when $\bar{x} < x^*$, $s(\bar{x}, \bar{y}) = 1$ under the RON. Note that the RHS denotes the injurer's costs if she opts for x^* while the victim stays put at $\bar{y}$.

$(\bar{x}, \bar{y})$ is a N.E. also implies that given the choice of $\bar{x}$ by the injurer, the victim's costs are minimum at $\bar{y}$. That is, if $(\bar{x}, \bar{y})$ is a N.E, then the following should also be true

$$\begin{aligned} \bar{y} + (1 - \bar{s}(\bar{x}, \bar{y}))L(\bar{x}, \bar{y}) &\leq y^* + [1 - s(\bar{x}, y^*)]L(\bar{x}, y^*), i.e., \\ \bar{y} &\leq y^*. \end{aligned} \tag{7}$$

But (6) and (7) together imply

$$\begin{aligned} \bar{x} + \bar{y} + L(\bar{x}, \bar{y}) &\leq x^* + y^*, i.e., \\ \bar{x} + \bar{y} + L(\bar{x}, \bar{y}) &\leq x^* + y^* + L(x^*, y^*), \end{aligned}$$

To sum up, the assumption that $(\bar{x}, \bar{y})$ is a N.E. implies that

$$\bar{x} + \bar{y} + L(\bar{x}, \bar{y}) \leq x^* + y^* + L(x^*, y^*) \tag{8}$$

However, (8) cannot be true. To see why, note $\bar{x} < x^*$. Therefore, $(\bar{x}, \bar{y}) \neq (x^*, y^*)$. Therefore, in view of the above Remark, the TAC at $(\bar{x}, \bar{y})$ are greater than TAC at (x^*, y^*), that is,

$$\bar{x} + \bar{y} + L(\bar{x}, \bar{y}) > x^* + y^* + L(x^*, y^*) \tag{9}$$

is true. That is (8) cannot be true. Since (8) follows from the assumption that [$\bar{x} < x^*$ and $(\bar{x}, \bar{y})$ is a N.E.]

So, either $\bar{x} < x^*$ is not true or $(\bar{x}, \bar{y})$ is NOT a N.E. This means that if $\bar{x} < x^*$ as is being considered in this case, then $(\bar{x}, \bar{y})$ CANNOT be a N.E.