Economic Analysis of Contract Law: Mathematical Analysis

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Students should supplement this note with the classroom discussions. Please bring typos to my attention.

1 Contracts and Economic Efficiency

Consider an example of a contractual setting:

Example 1 *Individual B needs a customized piece of furniture. S can prepare it for B. However, if B does not buy it, the piece will fetch little or price in the market. B and S sign a contract. The contract is signed at time $t = 0$. At this time the contract price $P = 170$ is also decided. S is required to deliver the good in future, at $t = 1$, say one month from now. If the good is supplied by the S to B, the benefit to B is V. Let, $V = 200$.*

Assume, at $t = 0$, there is uncertainty about the cost of production to the Seller, C, of the good. Assume that at $t = 0$, C is unknown to both the parties. C can take any of the values in the set

$$\{50, 90, 175, 205, 235\}$$

each with probabilities, $1/5$. The uncertainty w.r.t. C gets resolved and both the parties get to know its value at $t = \frac{1}{2}$. At $t = 1$, S decides whether to produce and supply the good or not. B will pay the contracted price only if S supplies the good.

Question 1 *What production decision will make both parties better off, if the contract were to be signed at $t = \frac{1}{2}$.*

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Question 2 *At $t = \frac{1}{2}$, what is the* **K-H efficient** *production decision w.r.t. C? That is, from efficiency perspective for which values of S should produce?*

The answer to question 2 is same as for question 1. Can you explain why?

Question 3 *Suppose, the contract is signed ex-ante, i.e., at $t = 0$. For which values of C, an ex-ante Pareto efficient contract will require S to produce?*

The following claim about Pareto Efficient Contracts is left to you as an exercise.

Proposition 1 *If contracting is costless, at $t = 0$ parties will sign a Complete Pareto Efficient.*

In particular, for the above example a Pareto Efficient contract will allow production only iff cost turns out to be less than 200. That is, have the following 'Breach Set'

$$\{205, 235\},$$

and will lead to the following set as the 'Performance Set':

$$\{50, 90, 175\}$$

Example of a Pareto Efficient Contract:

$$\begin{cases} \text{If } C \in \{50, 90, 175\} & \text{S will supply for } P\text{, otherwise will pay } K; \\ \text{If } C \in \{205, 235\} & \text{No trade,} \end{cases} \tag{1}$$

where P denotes the payment to be made by the B to S, and K is a large number (amount) that S will have to pay to B if he does not supply. K can be interpreted as the penalty of breach by S.

1.1 Contracts and Damages

Definition 1 *A contract can be defined as specification of actions that parties are supposed to take under various contingencies that may unfold in the future.*

Definition 2 *Damages-measure: is a rule that specifies the entitlement of the victim of the breach, when there is 'breach' of the contract.*

Let,

D denote the damages provided by the contract law. That is,

- D is amount the promisor has to pay to the promisee as compensation, in the event of non- performance. D can be a function of several contract characteristics. For example, we will see that D be equal to the buyers valuation of the contract, or contract price, etc.

- $\bar{D}$ is the total payment made by promisor to the promisee as compensation, in the event of non- performance. E.g., if P' is the advance payment made by B to S them $\bar{D} = D + P'$.

Assumption 1 *Contract price is to be paid on delivery at $t = 1$.*

So, $\bar{D} = D$.

1.2 Damages and Incentive to Perform

Let,

- V be the (gross) valuation of the contract to the Buyer; $V = 200$.

- P be the contract price; $P = 170$.

Let $D = 30$. What will S do if at $t = \frac{1}{2}$, C turns out to be 175.? If S performs, his profit is $P - C = 170 - 175 = -5$. However, if he does not perform, he will have to pay damages D to B. So, his profit will be $-D = -30$. Clearly, S wants to perform.

For any cost C, S will perform iff

$$\begin{aligned} P - C &\geq -D, i.e., iff \\ 170 - C &\geq -D. \end{aligned}$$

Question 4 *What will S do if at $t = \frac{1}{2}$, C turns out to be 225.*

If S performs, his profit is

$$P - C = 170 - 225 = -55$$

If he does not perform, his profit will be

$$-D = -30$$

For any cost C, S will breach iff

$$\begin{aligned} P - C &< -D, i.e., iff \\ 170 - C &< -D. \end{aligned}$$

So, the breach set induced by $D = 30$, is

$$\{205, 235\}$$

i.e., it is P.E.

Note that

$$D = 30 = 200 - 170 = V - P$$

Remark 3 *When $D = V - P$:*

- *B is indifferent between performance/delivery by S, on one hand, and breach by S on the other hand.*
- *the damages-measure meets the expectations of the Promisee, i.e., B. So,*
- *It is known as the* Expectation Damages*.*
- *The breach or performance decision of S does not depend on the contract price $P(.)$.*

1.3 Expectation Damages and Social Efficiency

Question 5 *Does the expected production cost for S depend on the damages measure?*

Proposition 2

- *If $D = v - P$, the outcome will be socially (Kaldor-Hicks) efficient.*
- *If $D \neq v - P(C)$, generally, the outcome will NOT be socially (Kaldor-Hicks).*

Example 1 *Assuming $P = 170$, suppose $D \neq 30$, say $D = 60$, you can show that: Now,*

- *the Breach Set will be $\{235\}$, i.e., it is NOT P.E.*
- *the Performance Set is $\{50, 90, 175, 205\}$, i.e., there will be excessive performance by S.*

Please supplement this section with the other examples discussed in the class

2 Mathematical Analysis of Contract Law

2.1 Modeling the Contract Law

Let,

r = the reliance investment made by the Buyer;
V = the value of the performance to the Buyer, given the reliance investment made by him; $V = V(r)$ $V'(r) > 0$, $V''(r) < 0$,
C = the cost of performance to the Seller;

We assume that C is a random variable; $C \in [0, \infty)$. Let, $F(C)$ and $f(C)$, respectively, be the distribution and density functions of C.

2.1.1 The First Best

(Shavell 1980, BJE)

For given r, the seller should perform as long as $C \leq V(r)$. Therefore, for given r, the efficient breach set

$BS^*(r) = \{C|C > V(r)\}$ and the efficient performance set

$\tilde{BS}^*(r) = \{C|V(r) \leq C\}$.

Clearly for given r, $BS^*(r)$ is the Pareto efficient breach set.

Now, given efficient breach decision, i.e., when S perform if and only of $C \leq V(r)$, the social value of the given level of reliance, $Z(r, BS^*(r))$, is

$$\int_0^{V(r)} [V(r) - C] dF(C) - r.$$

Note that when S decides whether to perform or not, investment r has become a sunk cost. So, for any given realization of C, if S produces and delivers then the net social gains would be $V(r) - C - r$; and if he does not do so the net social gains will be $-r$. However,

$$C \leq V(r) \Rightarrow V(r) - C - r \geq -r$$

and

$$C > V(r) \Rightarrow -r > V(r) - C - r.$$

Therefore, decision to perform if and only of $C \leq V(r)$ means choosing $\max\{V(r) - r - C, -r\}$. So, the above social surplus can be expressed as

$$Z(r, BS^*(r)) = \int_0^\infty \max\{V(r) - r - C, -r\}\, dF(C). \tag{2}$$

Assuming that performance decision will be efficient, at $t = \frac{1}{2}$ the socially efficient reliance level maximizes

$$\int_0^{V(r)} [V(r) - C] dF(C) - r, i.e.,$$

$$F(V(r))V(r) - E(C|C \leq V(r)) - r,$$

$$F(V(r))V(r) - \int_0^{V(r)} C dF(C) - r, \ i.e.,$$

In particular, the optimal r, denoted by r^*, solves

$$\max_r \{\int_0^{V(r)} [V(r) - C] dF(C) - r\}, i.e.,$$

$$\max_r \{\int_0^{V(r)} V(r) f(C) dC - \int_0^{V(r)} C f(C) dC - r\}$$

Using Leibniz Rule,[1] differentiation of $\int_0^{V(r)} V(r)f(C)dC$ gives us $\int_0^{V(r)} V'(r)f(C)dC + V(r)f(V(r))\frac{\partial V'(r)}{\partial r} - V(0)f(V(0))\frac{\partial 0}{\partial r}$, the last term is equal to zero. Similarly, differentiation of $\int_0^{V(r)} Cf(C)dC$ gives $V(r)f(V(r)).V'(r)$. So, the FOC is given by

$$\begin{aligned} \int_0^{V(r)} V'(r)f(C)dC &+ V(r)f(V(r))V'(r) \\ &- V(r)f(V(r)).V'(r) - 1 = 0 \end{aligned}$$

Clearly, the FOC reduces to

$$\int_0^{V(r)} V'(r)f(C)dC - 1 = 0, i.e.,$$

$$F(V(r))V'(r) - 1 = 0. \tag{3}$$

Let r^* solve (3). So the First Best is given by $(r^*, BS^*(r^*))$, where $BS^*(r^*) = \{C|C > V(r^*)\}$.

[1] $\frac{\partial}{\partial z}\int_{a(z)}^{b(z)} f(x,z)dx = \int_{a(z)}^{b(z)} \frac{\partial}{\partial z}(f(x,z))dx + f(b(z),z)\frac{\partial}{\partial z}(b(z)) - f(a(z),z)\frac{\partial}{\partial z}(a(z)$. Leibniz Rule is generally used when one or both of limits integration are function of the variable with which we are differentiating, e.g., r in this context.

2.1.2 Complete Contingent Contract

OPTIONAL Section

Complete Contingent Contract (CCC) explicitly determines the set BS and r that is to be opted by the Buyer. e.g., if a CCC sets $BS = BS^*(r^*)$ and $r = r^*$, then clearly it achieves the first best.

Consider a CCC, say (BS, r). Let,

$E^B(BS, r)$ be the expected benefit to the Buyer, exclusive of the price, from the CCC (BS, r).

$E^S(BS, r)$ be the expected benefit to the Seller, exclusive of the price, from the CCC (BS, r).

P the (expected) price fixed under the CCC (BS, r).

Therefore, the expected payoff to the Buyer and the Seller are $E^B(BS, r) - P$ and $E^S(BS, r) + P$, respectively.

Let, Total Social Surplus $TSS = B(BS, r) - P + S(BS, r) + P = B(BS, r) + S(BS, r) = Z(BS, r)$

Note, under a CCC TSS, $Z(.)$ is a function of BS and r and note of P.

Proposition 3 *A CCC is Pareto efficient iff it maximizes TSS*

Proof: Take any contract, say (BS, r, P). Suppose it does not maximizes the TSS. This means that there are BS' and r' such that $Z(BS, r) < Z(BS', r')$. That is, for some $\delta > 0$, $Z(BS', r') - Z(BS, r) = \delta$.

Now, consider the contract (BS', r', P'), where

$P' = P + E^B(BS', r') - E^B(BS, r) - \frac{\delta}{2}$.

This means, $E^B(BS', r') - P' = E^B(BS, r) - P + \frac{\delta}{2} > E^B(BS, r) - P$, i.e., the Buyer is strictly better off under (BS', r', P') rather than under (BS, r, P).

Also, $E^S(BS', r') + P' = E^S(BS', r') + P + E^B(BS', r') - E^B(BS, r) - \frac{\delta}{2}$, i.e.,

$E^S(BS', r') + P' = Z(BS', r') + P - E^B(BS, r) - \frac{\delta}{2}$, i.e.,

$E^S(BS', r') + P' = Z(BS', r') - Z(BS, r) + E^S(BS, r) + P - \frac{\delta}{2}$, i.e.,

$E^S(BS', r') + P' = E^S(BS, r) + P + \frac{\delta}{2} > E^S(BS, r) + P$, i.e.,

the Seller is strictly better off under (BS', r', P') rather than under (BS, r, P). Q.E.D.

Why a Contract (may) can not be a CCC?

(i) It may be very costly and even impossible to write about and negotiate over all the possible contingencies.

(ii) Even when a contract may be a CCC ex-ante, a contingency may not be verifiable ex-post. e.g., if in a contingency $P < C < V(r)$ holds and the C is non-observable to the Buyer then the Buyer may think that the Seller is bluffing and the transaction may not take place.

Therefore, contracts are not complete in general.

END OF THE OPTIONAL Section

2.2 A Legal Contract:

Generally, contracts provide for remedies to the victim of the breach of the contract.

Generally, a Legal Contract is a tuple $(P, D(r, P))$; where P is the price (to be) paid by the Buyer to the Seller, D is the Damages (compensatory payments) paid by the Seller to the Buyer, if the Seller breaches the contract.

Note a contract may or may not specify r for the Buyer; in the former case a contract is a tuple $(P, D(r, P), r)$.

Consider the contract $(P, D(r, P)$; where $D(r, P) = V(r^*) - P$, and $\int_0^{V(r^*)} CdF(C) \leq P \leq V(r^*)$.

What is the outcome under the contract $(P, V(r^*) - P)$?

We assume that the price is paid at the time of delivery. Under this contract, the Seller will breach iff $P - C < -(V(r^*) - P)$, i.e., iff $C > V(r^*)$, therefore the breach set is

$BS(r) = \{C|C > V(r^*)\}$, i.e., $BS(r) = BS^*(r^*)$, and

the performance set $\overline{BS(r)} = \{C|C \leq V(r^*)\}$.

Given the breach set, the Buyer opts for r that maximizes

$$F(V(r^*))[V(r) - P] + (1 - F(V(r^*)))[V(r^*) - P] - r, \text{ i.e.,}$$

the r opted by the Seller, solves

$$F(V(r^*))V'(r) - 1 = 0. \tag{4}$$

That is, r^* is opted by the Seller.

Since the contract $(P, V(r^*) - P)$ maximizes the total social surplus (TSS), it achieves the first best and obviously it is Pareto efficient.

Should damages provided by the contract law matter?

Shouldn't the renegotiation between the parties result in efficient breach always?

Why the parties will not settle for a first best contract in the first place?

2.3 Damages

To start with, assume that there are no Ex-post negotiations

2.3.1 Expectation Damages:

The reference point is the performance of the contract, i.e., the damages restore the victim of the breach to his position in the event of performance, i.e., $D_{ED}(r, P) = V(r) - P$.

Therefore, the breach set $BS_{ED}(r) = \{C | C > V(r)\}$, i.e., for the given reliance, the breach set is Pareto efficient. This also means that when Buyer chooses r, she knows that probability of performance is $F(V(r))$; with remaining probability S will not deliver but will have to pay damages to B. So The Buyer chooses r that maximizes

$$F(V(r))[V(r) - P] + (1 - F(V(r)))[V(r) - P] - r, \ i.e.,$$

$$V(r) - P - r, \ i.e.,$$

the r opted by the Buyer, r_{ED}, solves

$$V'(r) - 1 = 0. \tag{5}$$

That is, r_{ED} is opted by the Buyer is such that $r_{ED} > r^*$ (Compare (4) and (5)). Note that $BS_{ED}(r) \subset BS^*(r^*)$.

Question: Does the contract price affect the reliance and breach decisions?

2.3.2 Reliance Damages:

The reference point is the situation of no contract, i.e., the damages restore the victim of the breach to the position he will be in if he does not enter into the contract at all, i.e., $D_R(r, P) = r$.

Therefore, the Seller will perform iff $P - C \geq -r$, i.e., the breach set $BS_R(r)$ is $BS_R(r) = \{C|C > P + r\}$, i.e., for the given level of reliance the breach set is *not* Pareto efficient; assuming that $V(r) > P + r$. Now, when Buyer chooses r, she knows that probability of performance is $F(P+r)$; with remaining probability S will not deliver but will have to pay damages of r to B. So, the Buyer chooses r that maximizes

$$F(P + r)[V(r) - P] + (1 - F(P + r))r - r, \ i.e.,$$

$$F(P + r)[V(r) - P - r], \ i.e.,$$

the r opted by the Buyer, r_R, solves

$$V'(r) - 1 = -\frac{f(P + r)[V(r) - P - r]}{F(P + r)}. \tag{6}$$

That is, r_R is opted by the Buyer is such that $r_R > r_{ED} > r^*$. Why? Note that $BS_R(r_R) \supset BS^*(r_R)$.

Question: Does the contract price affect the reliance and breach decisions? If yes, how?

2.3.3 Restitution Damages:

Restitution damages restore to the victim of the breach whatever he had paid to the promisor. Therefore under our assumption restitution damages means no damages, i.e., $D_N(r, P) = 0$.

Therefore, under restitution damages the Seller will perform iff $P - C \geq 0$, i.e., the breach set $BS_N(r)$ is $BS_N(r) = \{C|C > P\}$, i.e., given reliance the breach set is *not* Pareto efficient. The Buyer chooses r that maximizes

$$F(P)[V(r) - P] + (1 - F(P))0 - r, \ i.e.,$$

$$F(P)[V(r) - P] - r, \ i.e.,$$

the r_N opted by the Buyer, solves

$$F(P)V'(r) - 1 = 0. \tag{7}$$

Since $P < V(r)$, r_N is opted by the Buyer is such that $r_N < r^* < r_{ED}$. Also note that $P < V(r_N) - r_N < V(r_{ED}) - r_{ED} < V(r_{ED})$. Therefore, $BS_N(r_N) \supset BS_{ED}(r_{ED})$ holds.

Question: Does the contract price affect the reliance and breach decisions? If yes, how?

2.3.4 Specific Performance

Under this measure, the breach set $BS(SP) = \emptyset$.

Therefore, the probability of performance is 1, and the Buyer chooses r that maximizes

$$V(r) - P - r, \ i.e.,$$

the r_{SP} opted by the Buyer, solves

$$V'(r) - 1 = 0. \tag{8}$$

That is, r_{SP} is opted by the Buyer is such that $r_{SP} = r_{ED} > r^*$.

What will be the contract price under Specific Performance?

3 Comparative Efficiency: Without Ex-post Renegotiations

Proposition 4 *Expectation damages are K-H superior to Reliance damages*

Proof: Take a contract under Reliance damages $(D(r), P) = (r, P)$. Suppose under this contract, the outcome is $(BS(r_R), r_R)$. Clearly $(BS(r_R) = \{C|C > P + r_R\}$. First of all notice that for given r, $Z(r, BS^*(r)) \geq Z(r, BS(r))$. In particular, $Z(r_R, BS^*(r_R)) \geq Z(r_R, BS_R(r_R))$.

Also as we demonstrated earlier, given that the breach set is efficient, for any given r, the total social surplus is equal to $\int max\{V(r) - r - C, -r\}dF(C)$, i.e.,

$Z(r, BS^*(r)) = \int_0^\infty max\{V(r) - r - C, -r\}dF(C)$.

Notice that r_{ED} solves $\max_r\{V(r) - r\}$. So, regardless of C, for all $r > r_{ED}$, $V(r_{ED}) - r_{ED} > V(r) - r$ and $-r > -r_{ED}$. So, for any given r such that $r > r_{ED}$, $max\{V(r_{ED}) - r_{ED} - C, -r_{ED}\} > max\{V(r) - r - C, -r\}$. Since $r_R > r_{ED}$, we have $max\{V(r_{ED}) - r_{ED} - C, -r_{ED}\} > max\{V(r_R) - r_R - C, -r_R\}$. This means,

$$\int_0^\infty max\{V(r_{ED}) - r_{ED} - C, -r_{ED}\} > \int_0^\infty max\{V(r_R) - r_R - C, -r_R\}$$

, i.e., $Z(r_{ED}, BS^*_{ED}(r_{ED})) > Z(r_R, BS^*_R(r_R))$ But, $Z(r_R, BS^*_R(r_R)) \geq Z(r_R, BS_R(r_R))$. However, $Z(r_{ED}, BS_{ED}(r_{ED})) = Z(r_{ED}, BS^*_{ED}(r_{ED}))$. Therefore, $Z(r_{ED}, BS_{ED}(r_{ED})) > Z(r_R, BS_R(r_R))$ Q.E.D.

4 With Ex-post Negotiations

(Rogerson 1984, RJE)

We assume that whenever ex-post negotiations take place, the Buyer gets α times the *potential* surplus, where $\alpha \in [0, 1]$. We do not model α.

4.1 Expectation Damages:

Notice that under Expectation Damages in the absence of ex-post negotiations, the breach set $BS_{ED}(r) = \{C|C > V(r)\}$, i.e.,$BS_{ED}(r) = BS(r)$, i.e., for given level of reliance, the breach set is Pareto efficient. Therefore, there cannot be Pareto improving negotiations. As a result, even when costless renegotiations are possible, the breach set remains $BS_{ED}(r) = \{C|C > V(r)\}$.

Moreover, the Buyer, as before, chooses r that maximizes

$$F(V(r))[V(r) - P] + (1 - F(V(r)))[V(r) - P] - r, \; i.e.,$$
$$V(r) - P - r, \; i.e.,$$

the r_{ED} opted by the Buyer, solves

$$V'(r) - 1 = 0. \tag{9}$$

That is, the outcome remains the same.

4.2 Reliance Damages:

Under Reliance Damages in the absence of ex-post negotiations, the breach set $BS_{RD}(r) = \{C|C > P + r\}$, i.e., for given reliance the breach set is not Pareto efficient; assuming $P + r < V(r)$. Why? Note that when $P + r < C < V(r)$ holds, the parties will re-negotiate the contract - In the absence of negotiation, S will not deliver. But if he delivers, the total social gains will be $V(r) - C$, which parties can divide between themselves leading to Pareto improvement. However, as before, when $V(r) > C$ or $C < P+r$ holds, there cannot be Pareto improving negotiations.

Therefore, in the presence of ex-post negotiations, the breach set $BS_{RD}(r) = \{C|C > V(r)\}$. We assume that during negotiations, parties engage in Nash Bargaining. Under Nash Bargaining each parties get its disagreement payoff (i.e., whatever this party can ensure itself without engaging in negotiations) PLUS a fraction α of the surplus from negotiations. So, the Buyer chooses r that maximizes

$$\int_0^{P+r} [V(r) - P]dF(C) + \int_{P+r}^{V(r)} [r + \alpha[V(r) - C]]dF(C) + \int_{V(r)}^{\infty} rdF(C) - r.$$

The first term follows from the fact that when $C < P + r$ no renegotiation will take place, and S will deliver. Second term captures the negotiation payoffs - remember if S does not deliver he will have to pay r to B. But, renegotiations take place when $P + r < C < V(r)$. As a result, B gets her disagreement payoff r PLUS α fraction of surplus $[V(r) - C]$. The third term follows from the fact that when $C > V(r)$ no renegotiates are possible and S will not deliver. Re-writing the 2nd and 3rd terms, we get

$$\int_0^{P+r} [V(r) - P]dF(C) + \alpha[\int_{P+r}^{V(r)} [V(r) - C]dF(C)] + \int_{P+r}^{\infty} rdF(C) - r.$$

$$\int_0^{P+r} [V(r) - P - r]dF(C) + \alpha[\int_{P+r}^{V(r)} [V(r) - C]dF(C)],\ i.e.,$$

$$(1-\alpha)\int_0^{P+r} [V(r)-P-r]dF(C)+\alpha\int_0^{P+r} [V(r)-P-r]dF(C)+\alpha[\int_{P+r}^{V(r)} [V(r)-C]dF(C)].$$

Note when when $C < P + r$, we have $V(r) - P - r \leq V(r) - C$. And, when $P + r < C < V(r)$ we have $V(r) - P - r \geq V(r) - C$. So, rewriting the last two terms, we get

$$(1-\alpha)\int_0^{P+r)}[V(r)-P-r]dF(C)+\alpha\int_0^{V(r)} min\left\{\begin{array}{l} V(r)-P-r; \\ V(r)-C \end{array}\right\} dF(C). \tag{10}$$

When $\alpha = 0$ the r opted by the Buyer, solves

$$V'(r)-1=-\frac{f(P+r)[V(r)-P-r]}{F(P+r)}, i.e., \tag{11}$$

r'_{RD} opted by the Buyer is the same as in the case of no renegotiations.

Now, from (10) note that for all $r \leq r_{ED}$ the following holds:
(i) $V(r)-r \leq V(r_{ED})-r_{ED}$ - recall r_{ED} solve $\max_r\{V(r)-r\}$. And
(ii) the area of integral increases with r.

Therefore, both the terms in (10) and hence their sum will attain a maxima at $r \geq r_{ED}$, i.e., for all $\alpha \in [0,1]$, $r_{RD}(\alpha) \geq r_{ED}$. It can be shown that for all $\alpha \in [0,1]$, $r_{RD} \geq r_{RD}(\alpha) \geq r_{ED}$.

Proposition 5 *In the presence of ex-post negotiations, Expectation damages are K-H superior to Reliance damages*

Proof: Consider an arbitrary contract under Reliance damages, say $(D(r),P) = (r,P)$; where P is the agreed price. Suppose, under this contract the outcome is (BS_{RD}, r_{RD}). We know that for all $\alpha \in [0,1]$, $r \geq r_{ED}$.

Also, for any given r, $Z(r, BS^*(r)) \geq Z(r, BS(r))$. Moreover, $Z(r, BS^*(r)) = \int max\{V(r)-r-C, -r\}dF(C)$. Since r_{ED} solves $max\{V(r)-r\}$, $V(r_{ED}) - r_{ED} > V(r_{RD}) - r_{RD}$, i.e., $Z(r_{ED}, BS^*_{ED}(r_{ED})) \geq Z(r_{RD}, BS^*(r_{RD})) \geq Z(r_{RD}, BS(r_{RD}))$. *Q.E.D.*

4.3 Specific Performance:

Under this remedy, if there is no renegotiation S will always deliver and therefore B's payoff will be $V(r)-P-r$. This means that the Buyer can make sure that he always gets at least $V(r)-P-r$. When $V(r) \geq C$ it is easy to see that there is no scope for mutually beneficial re-negotiations. When $C > V(r)$ the social surplus from non-performance is $C - V(r)$. So, if there is renegotiation, B would want to make sure that he ends up with at least $V(r)-P-r$ plus a fraction of surplus from renegotiations. Assume that the Buyer gets $\alpha \in$

$[0,1]$ of this surplus. Specifically, in the event of renegotiation, payoff will be $V(r) - P - r + \alpha[C - V(r)]$.

As before, when ex-post negotiations are possible, under Specific Performance the breach set $BS(r) = \{C | C > V(r)\} = BS^*(r)$.

The Buyer chooses r that solves

$$\max_{RD} \{V(r) - P - r + \alpha E[C - V(r) | C > V(r)]\}, i.e.,$$

$$\max_{RD} \{V(r) - P + \alpha[\int_{V(r)}^{\infty} (C - V(r)) dF(C)] - r\}, i.e.,$$

Using Leibniz Rule, the optimal r opted by the Buyer satisfies the following FOC

$$V'(r) + \alpha[-(1 - F(V(r))V'(r))] - 1 = 0, i.e.,$$

$$V'(r) - 1 = \alpha(1 - F(V(r))V'(r)), i.e.,$$

$\forall \alpha \in [0,1] (r_{SP} \in [r^*, r_{ED}])$. When $\alpha = 0$, $r_{SP} = r_{ED}$, and when $\alpha = 1$, $r_{SP} = r^*$.

Lemma 4 *When ex-post negotiations are possible, under Specific Performance choice of r by B is a function of α. Moreover,*

$$\frac{dr_{SP}}{d\alpha} < 0.$$

Proof: Under Specific Performance, the Buyer's payoff is

$$V(r) - P + \alpha[\int_{V(r)}^{\infty} (C - V(r)) dF(C)] - r, i.e.,$$

$$V(r) - P - r + \int_{V(r)}^{\infty} (C - V(r)) dF(C) - (1 - \alpha) \int_{V(r)}^{\infty} (C - V(r)) dF(C), i.e.,$$

$$V(r) - \int_{V(r)}^{\infty} V(r) dF(C) - P - r + \int_{V(r)}^{\infty} C dF(C) - (1 - \alpha) \int_{V(r)}^{\infty} (C - V(r)) dF(C), i.e.,$$

Since $\int_{V(r)}^{\infty} V(r) dF(C) = [1 - F(V(r))]V(r)$, therefore the above expression reduces to

$$F(V(r))V(r) - P - r + \int_{V(r)}^{\infty} C dF(C) - (1 - \alpha) \int_{V(r)}^{\infty} (C - V(r)) dF(C), i.e.,$$

In view of definition of $Z(r, BS^*(r))$, and that $E(C) = \int_{V(r)}^{\infty} CdF(C)$, the above expression reduces to

$$F(V(r))V(r) - P - r - \int_0^{V(r)} CdF(C) + \int_0^{\infty} CdF(C) - (1-\alpha)\int_{V(r)}^{\infty} (C - V(r))dF(C), i.e.,$$

$$Z(r, BS^*(r)) - P + E(C) - (1-\alpha)\int_{V(r)}^{\infty} (C - V(r))dF(c). \tag{12}$$

We assume that (12) has a unique solution for every $\alpha \in [0, 1]$. Let $1 - \alpha = \beta$ and $g(r) = -\int_{V(r)}^{\infty}(C - V(r))dF(c)$. Note that $g'(r) \geq 0$, in fact $g'(r) > 0$. Now consider

$$Z(r, BS^*(r)) + \beta g(r)). \tag{13}$$

Note that, since P and $E(C)$ are constants, (12) and (13) have the same solution. Let r solve

$$\max_r \{Z(r, BS^*(r)) + \beta g(r))\}. \tag{14}$$

Clearly r is a function of β. First we show that $r'(\beta) > 0$.

Let r_i solve

$$\max_r \{Z(r, BS^*(r)) + \beta_i g(r))\}. \tag{15}$$

Suppose $\beta_1 < \beta_2$ and $r_1 > r_2$ holds. Since $g'(.) > 0$, $r_1 > r_2$ and $\beta_1 < \beta_2$ imply that

$$[\beta_2 - \beta_1]g(r_1) > [\beta_2 - \beta_1]g(r_2). \tag{16}$$

Since r_i solves (15),

$$Z(r_1, BS^*(r_1)) + \beta_1 g(r_1) > Z(r_2, BS^*(r_2)) + \beta_1 g(r_2). \tag{17}$$

(16)+(17) gives us $Z(r_1, BS^*(r_1)) + \beta_2 g(r_1) > Z(r_2, BS^*(r_2)) + \beta_2 g(r_2)$, which is a contradiction.

Therefore, $\beta_1 < \beta_2 \Rightarrow r_1 < r_2$ i.e., $\alpha_1 < \alpha_2 \Rightarrow r_1 > r_2$.

Hence, $r'_{SP}(\beta) > 0$ and $r'_{SP}(\alpha) < 0$.

Proposition 6 *In the presence of ex-post negotiations, Specific Performance is K-H superior to Expectation damages.*

Proof: Let r_1 solve $\max_r\{Z(r, BS^*(r))+\beta_1 g(r))\}$ and r_2 solve $\max_r\{Z(r, BS^*(r))+\beta_2 g(r))\}$. First of all, we show that $\beta_1 < \beta_2 \Rightarrow Z(r_1, BS^*(r_1)) > Z(r_2, BS^*(r_2))$ i.e., $\alpha_1 > \alpha_2 \Rightarrow Z(r_1, BS^*(r_1)) \geq Z(r_2, BS^*(r_2))$.

Suppose $\beta_1 < \beta_2$ and $Z(r_1, BS^*(r_1)) \leq Z(r_2, BS^*(r_2))$ holds.

Since, $\beta_1 < \beta_2$, we have $r_1 < r_2$, i.e., $g(r_1) < g(r_2)$, i.e., $\beta_1 g(r_1) < \beta_1 g(r_2)$. Therefore, we get $Z(r_1, BS^*(r_1)) + \beta_1 g(r_1) < Z(r_2, BS^*(r_2)) + \beta_1 g(r_2)$, which is a contradiction. Therefore, $\beta_1 < \beta_2 \Rightarrow Z(r_1, BS^*(r_1)) > Z(r_2, BS^*(r_2))$,[2] i.e., $\alpha_1 > \alpha_2 \Rightarrow Z(r_1, BS^*(r_1)) > Z(r_2, BS^*(r_2))$. That is, the total social surplus is an increasing function of α.

Since, the case of Expectation Damages corresponds to the case $\alpha = 0$, whereas under Specific Performance $0 \leq \alpha \leq 1$. Therefore, $Z_{SP}(r_{SP}, BS_{SP}(r_{SP})) \geq Z(r_{ED}, BS_{ED}(r_{ED}))$.

Proposition 7 *In the presence of ex-post negotiations, Specific Performance is K-H superior to Reliance damages*

From Proposition 5 we know that Expectation Damages are K-H superior to the Reliance Damages. $Q.E.D.$

4.4 Restitution Damages:

Under our assumption restitution damages means no damages, i.e., $D_N(r, P) = 0$. If S refuses to deliver, B has no remedies available. Alternatively, if B refuses to accept deliver S can do nothing. Following the literature, we assume that depending on C and $V(r)$, either S or B will have temptation disrespect the contract. Anticipating this, both parties would know that the scenario is as in the case of *no contract* at all. But, the parties will renegotiate at time 1, in that case B gets α fraction of surplus. So, the Buyer chooses r that maximizes

$$\alpha \int_0^{V(r)} [V(r) - C] dF(C) - r.$$

You can easily check that $r_N(\alpha)$ opted by the Buyer will solve

$$\alpha F(V(r))V'(r) - 1 = 0$$

that is, will be such that $r_N(\alpha) \leq r^* < r_{ED}$. Moreover, $\alpha = 1 \Rightarrow r_N = r^*$.

[2] $Z(r_1, BS^*(r_1)) = Z(r_2, BS^*(r_2))$ cannot hold since by assumption $\max_r\{Z(r, BS^*(r)) + \beta_i g(r))\}$ has unique solution.

5 Damages with fixed r and no ex-post negotiations

(Shavell 1984)

OPTIONAL Section

Suppose, the level of reliance investment by the Buyer, r, is given. Also, to start with, we assume away the ex-post negotiations.

Since r is given, $V(r)$ is also a given constant. It is easy to see that regardless of the damages measure supplied by the law, the contract price $P \leq V(r) - r$.

Proposition 8 *When r is given and ex-post negotiations are not possible, Reliance Damages is Pareto superior to the Restitution (No) Damages*

Proof: Consider an arbitrary contract under Restitution Damages, i.e., consider a contract $(D_N, P_N) = (0, P_N)$. Clearly, $P_N \leq V(r) - r$, and $BS_N = \{C | C > P_N\}$. Assume $P_N < V(r) - r$. The social surplus under this contract is

$$Z_N(P_N) = \int_0^{P_N} [V(r) - C] dF(C) - r. \tag{18}$$

Let E_N^S be the Seller's expected payoff under Restitution (No) Damages

$$E_N^S(P_N) = \int_0^{P_N} [P_N - C] dF(C).$$

Clearly,

$$E_N^S(P_N) \leq Z_N(P_N).$$

Now consider a contract under Reliance Damages $(D_{RD}, P_{RD}) = (r, P_{RD})$. The social surplus under this contract is

$$Z_{RD}(P_{RD}) = \int_0^{P_{RD}+r} [V(r) - C] dF(C) - r. \tag{19}$$

Let E_{RD}^S be the Seller's expected payoff under Reliance Damages.

$$E_{RD}^S(P_{RD}) = \int_0^{P_{RD}+r} [P_{RD} - C] dF(C) - \int_{P_{RD}+r}^{\infty} r dF(C).$$

Therefore,

$$E_{RD}^B(P_{RD}) = \int_0^{P_{RD}+r} [V(r) - r - P_{RD}] dF(C). \tag{20}$$

From (17) and (18), for a given price, P, $Z_{RD}(P) > Z_N(P)$, i.e., the social surplus is strictly greater under Reliance Damages. Therefore, Reliance Damages is K-H superior to the Restitution (No) Damages. Also, from (18), $\frac{\partial}{\partial P} Z_{RD}(P) > 0$.

For a given price, P, $E^S_{RD} < E^S_N$. However, $\frac{\partial}{\partial P} E^S_{RD} > 0$. Moreover, from (19),

$$\lim_{P_r \to (V(r)-r)} [E^B_{RD}(.) = 0,\ i.e.,\ E^B_{RD}(.) = Z_{RD}]. \tag{21}$$

Now, under Reliance Damages consider an increase in price from P_N to $V(r) - r$. $\frac{\partial}{\partial P} E^S_{RD} > 0$ and (20) imply that $\exists P'_{RD} \in (P_N, V(r) - r)$ such that $E^S_{RD}(P'_{RD}) = E^S_N(P_N)$ and $Z_{RD}(P'_{RD}) > Z_N(P_N)$. Hence, $E^B_{RD}(P'_{RD}) > E^B_N(P_N)$. Now at price $P'_{RD} + \epsilon$ both parties are better off. Q.E.D.

Proposition 9 *When r is given and ex-post negotiations are not possible, Expectation damages is Pareto superior to other damages measures*

Proof: Consider a contract under Reliance Damages $(D_{RD}, P_{RD}) = (r, P_{RD})$. Clearly, $P_{RD} < V(r) - r$, and $BS_{RD} = \{C | C > P_{RD} + r\}$. The social surplus under this contract is

$$Z_{RD}(P_{RD}) = \int_0^{P_{RD}+r} [V(r) - C] dF(C) - r. \tag{22}$$

Since r is given, $Z_{RD}(.)$ is function of P only.

As before, let $E^S_{RD}(P_{RD})$ and $E^B_{RD}(P_{RD})$ be the Seller's and the Buyer's expected payoff under Reliance Damages.

Under Expectation Damages, for a contract $(D_{ED}, P_{ED}) = (V(r) - P, P_{ED})$, the social surplus under this contract is

$$Z_{ED} = \int_0^{V(r)} [V(r) - C] dF(C) - r. \tag{23}$$

Therefore, irrespective of P_{RD}, $Z_{ED}(.) > Z_{RD}(P)$, i.e., Expectation Damages is always K-H superior to the Reliance Damages. Since $Z_{ED}(.) > Z_{RD}(.)$, it is always possible to have P_{ED} such that $E^S_{ED} > E^S_{RD}$ and $E^B_{ED} > E^B_{RD}$, i.e., Expectation Damages is Pareto superior to the Reliance Damages.

Likewise, it is easy to show that Expectation Damages is Pareto superior to the Restitution Damages.

6 Damages with fixed r and costless ex-post negotiations

OPTIONAL SECTION

In the presence of ex-post negotiations, under every damage measure the breach set is always $BS = \{C|C > V(r)\}$. This implies that in terms of K-H criterion all measures are equally good.

Since initial contract price as well as the surplus a party gets in ex-post negotiations will depend on the parties bargaining strengths, in general it is not possible to rank the damage measures in terms of Pareto criterion.

Damages with fixed r and costless ex-post negotiations:

In the presence of costly ex-post negotiations, as long as these cost are small, the breach set is always $BS = \{C|C > V(r)\}$, regardless of the damage measure.

However, the negotiations costs are highest under the Restitution Damages, moderate under Reliance Damages and the least under the Expectation Damages. Therefore, it is easy to show the following.

Proposition 10 *When r is given and ex-post negotiations are costly, Expectation damages is K-H superior to all other damages measures*