Two Lectures on Information Design

10th DSE Winter School

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December 2015

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- We can instead ask what can happen for all information structures...
- .... and pick a favorite information structure...
- .... call this "information design"

Two Lectures

1. Price Discrimination: An Application
2. Information Design: A General Approach

Price Discrimination

- Fix a demand curve
- Interpret the demand curve as representing single unit demand of a continuum of consumers
- If a monopolist producer is selling the good (say, with zero cost), what is producer surplus (monopoly profits) and consumer surplus (area under demand curve = sum of surplus of buyers)?

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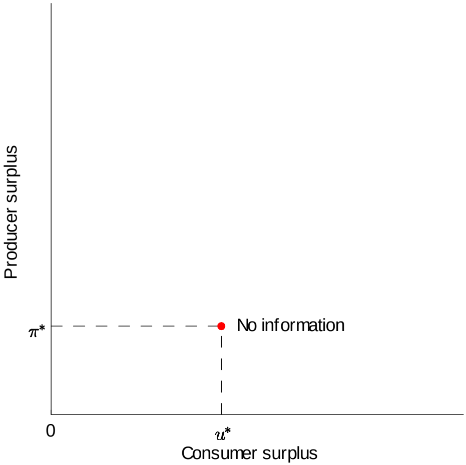
- If a monopolist producer is selling the good (say, with zero cost), what is producer surplus (monopoly profits) and consumer surplus (area under demand curve = sum of surplus of buyers)?
- If the seller cannot discriminate between consumers, he must charge uniform monopoly price

The Uniform Price Monopoly

- Write u^* for the resulting consumer surplus and π^* for the producer surplus ("uniform monopoly profits")



Perfect Price Discrimination

- But what if the producer could observe each consumer's valuation perfectly?
- Pigou (1920) called this "first degree price discrimination"
- In this case, consumer gets zero surplus and producer fully extracts efficient surplus $w^* > \pi^* + u^*$

First Degree Price Discrimination

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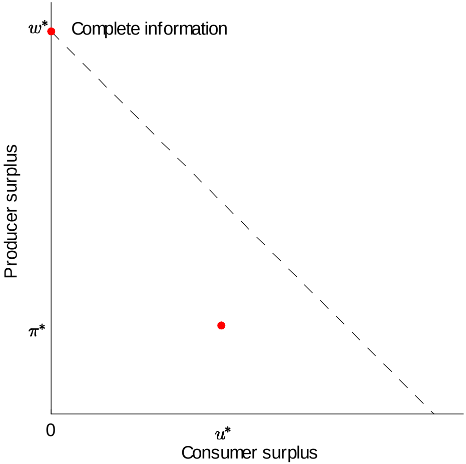

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- What can happen?

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- Pigou (1920) called this "third degree price discrimination"
- What can happen?
- A large literature (starting with Pigou (1920)) asks what happens to consumer surplus, producer surplus and thus total surplus if we segment the market in *particular* ways...

The Limits of Price Discrimination

- Different question:
  - What could happen to consumer surplus, producer surplus and thus total surplus for all possible ways of segmenting the market?

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- Different question:
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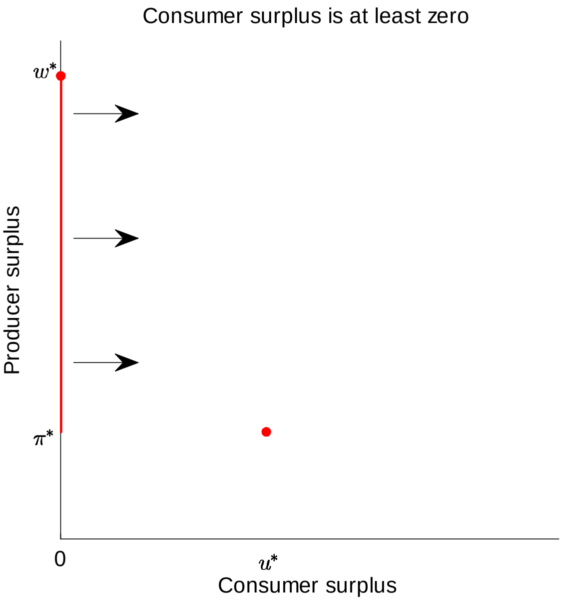
  - Equivalently, what could happen to consumer surplus, producer surplus and thus total surplus for all possible information that the producer might receive about consumer valuations?
- We can provide
  - A complete characterization of all (consumer surplus, producer surplus) pairs that can arise, and thus total surplus...

Three Welfare Bounds

1. Voluntary Participation: Consumer Surplus is at least zero

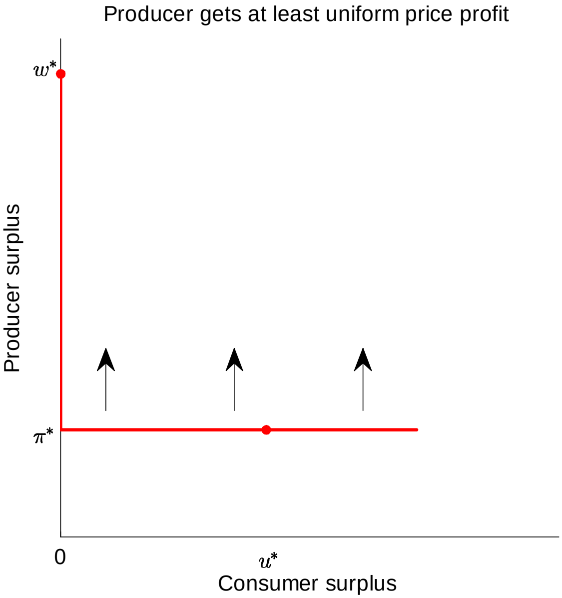
Welfare Bounds: Voluntary Participation



Three Welfare Bounds

1. Voluntary Participation: Consumer Surplus is at least zero
2. Non-negative Value of Information: Producer Surplus bounded below by uniform monopoly profits π^*

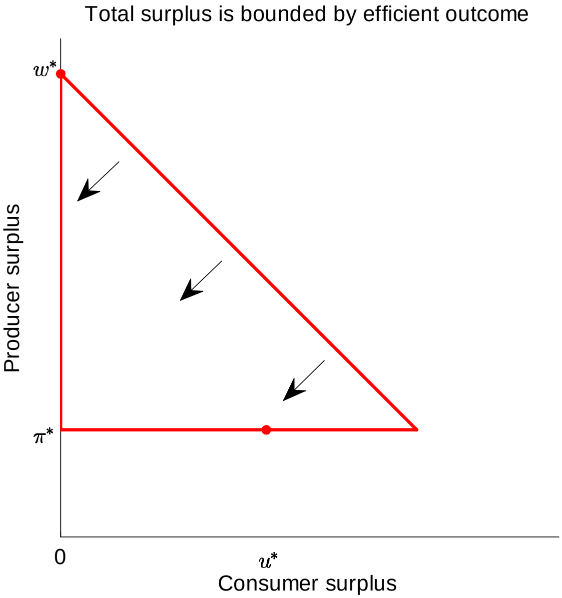
Welfare Bounds: Nonnegative Value of Information



Three Welfare Bounds

1. Voluntary Participation: Consumer Surplus is at least zero
2. Non-negative Value of Information: Producer Surplus bounded below by uniform monopoly profits π^*
3. Social Surplus: The sum of Consumer Surplus and Producer Surplus cannot exceed the total gains from trade

Welfare Bounds: Social Surplus



Beyond Welfare Bounds

1. Includes points corresponding uniform price monopoly, (u^*, π^*), and perfect price discrimination, $(0, w^*)$
2. Convex

Welfare Bounds and Convexity

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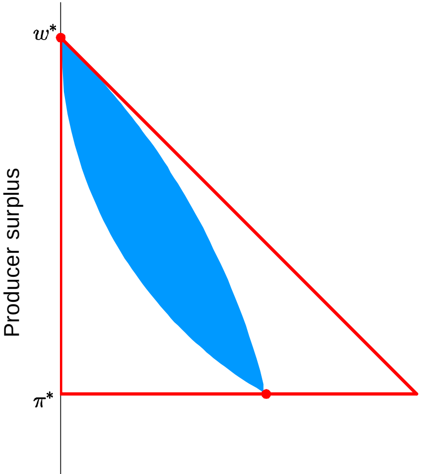
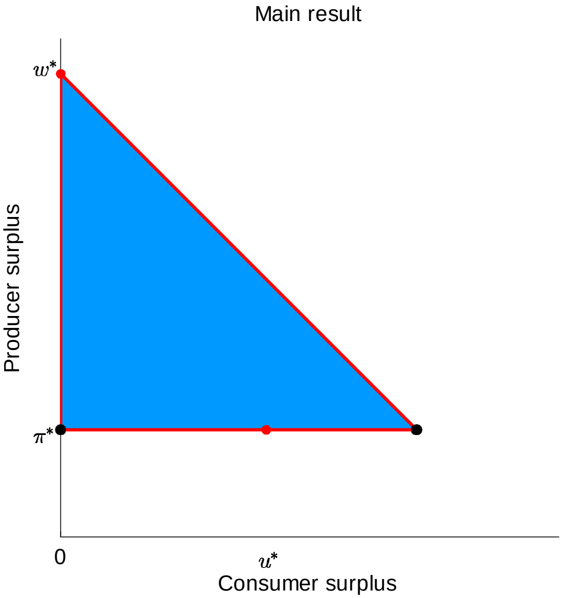
2. Convex

What is the feasible surplus set?



Main Result: Welfare Bounds are Sharp



Main Result

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   1.1 the producer earns uniform monopoly profits,
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1. a *consumer surplus maximizing segmentation* where
   1.1 the producer earns uniform monopoly profits,
   1.2 the allocation is efficient,
   1.3 and the consumers attain the difference between efficient surplus and uniform monopoly profit.
2. a *social surplus minimizing segmentation* where
   2.1 the producer earns uniform monopoly profits,
   2.2 the consumers get zero surplus,
   2.3 and so the allocation is very inefficient.

A Simple "Direct" Construction of Consumer Surplus Maximizing Segmentation (bottom right hand corner)

- We first report a simple direct construction of a consumer surplus maximizing segmentation (bottom right hand corner):
- Assume a finite number of valuations $v_1 < ... < v_K$
- The optimal uniform monopoly price will be one of those values, say v^*

A Simple "Direct" Construction of Consumer Surplus Maximizing Segmentation (bottom right hand corner)

1. first split:
   1.1 We first create a market which contains all consumers with the lowest valuation v_1 and a constant proportion q_1 of valuations greater than or equal to v_2

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2. Iterate this process

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4. continue until we hit the monopoly price

Proof in Three Value Example

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- Argument then extends to continuum of valuations by continuity

Markets and Prices

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- Price 3 gives profits $3x_3$

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$$1 \geq 2\left(x_2 + x_3\right) \text{ and } 1 \geq 3x_3$$

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- Price 3 is optimal if

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Aggregate Market

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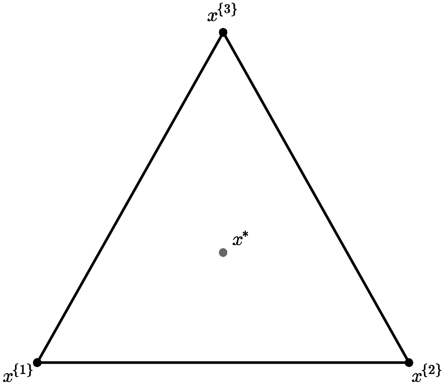
- Price 3 gives profits

$$3\frac{1}{3} = 1$$

- Optimal price is 2

A Visual Representation

- Can represent markets in a diagram:



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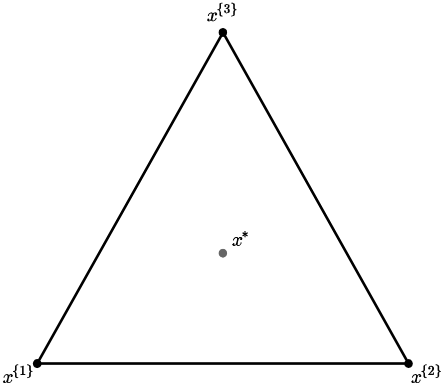

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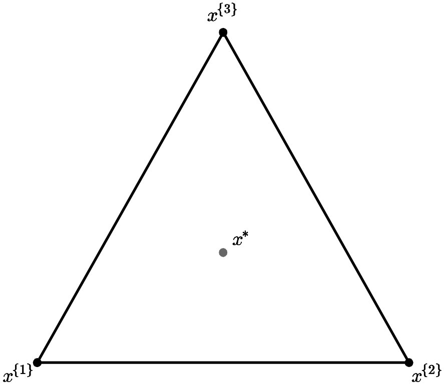

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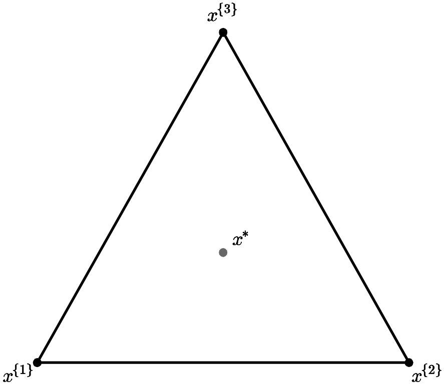

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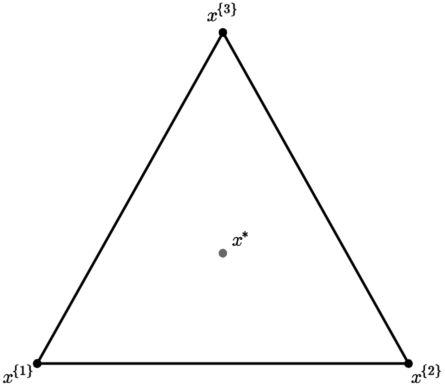

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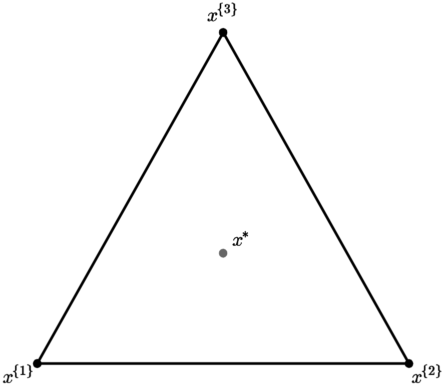

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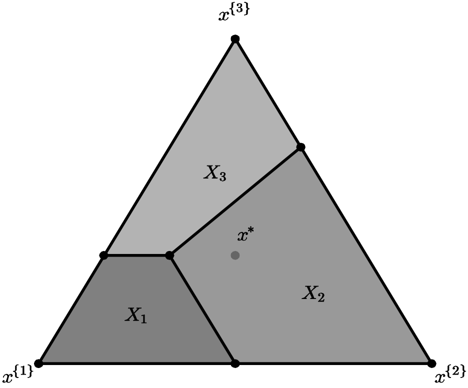
- Aggregate market x^* is at the center of the triangle

A Visual Representation: Segments and (Optimal) Prices

- The optimal pricing inequalities generate regions where each price is optimal:



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- Suppose that we split consumers into three markets

$$
\begin{aligned}
a &= (a_1, a_2, a_3) \\
b &= (b_1, b_2, b_3) \\
c &= (c_1, c_2, c_3)
\end{aligned}
$$

with weights w_a, w_b and w_c respectively

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  with weights w_a, w_b and w_c respectively
- This is a segmentation of our aggregate market if

$$w_a a + w_b b + w_c c = x^*$$

Segmentation Example

	$v = 1$	$v = 2$	$v = 3$	weight
market a	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$
market b	0	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$
market c	0	1	0	$\frac{1}{6}$
total market	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	

Complete Distribution in Example

	$v = 1$	$v = 2$	$v = 3$
market a	$\frac{2}{3}$	$\frac{1}{9}$	$\frac{2}{9}$
market b	0	$\frac{1}{18}$	$\frac{1}{9}$
market c	0	$\frac{1}{6}$	0

Signal Interpretation in Example

	$v = 1$	$v = 2$	$v = 3$
signal A	1	$\frac{1}{3}$	$\frac{2}{3}$
signal B	0	$\frac{1}{6}$	$\frac{1}{3}$
signal C	0	$\frac{1}{2}$	0

"Extremal Segmentation"

- the example is special

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$\{1,2,3\}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{2}{3}$
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- price 2 is optimal in all markets!

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- in fact, seller is always indifferent between all prices in the support of the market

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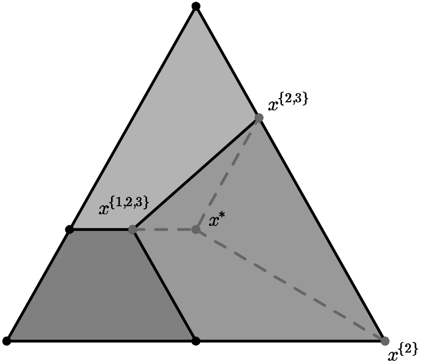
- price 2 is optimal in all markets!
- in fact, seller is always indifferent between all prices in the support of the market
- call these "extremal markets"

Geometry of Extremal Markets

- extremal segment x^S:
  - seller is indifferent between all values in the support of S and puts zero weight on values outside the support



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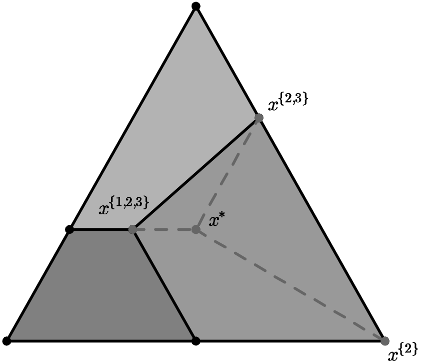

  - can segment the market so that the aggregete market is a segmented into extremal segments only

Consumer Surplus Maximizing Segmentation

- *an* optimal policy: always charge lowest price in the support of every segment:

	$v = 1$	$v = 2$	$v = 3$	price	weight
$\{1, 2, 3\}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	1	$\frac{2}{3}$
$\{2, 3\}$	0	$\frac{1}{3}$	$\frac{2}{3}$	2	$\frac{1}{6}$
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  - the good is sold to every consumer, so the allocation is efficient
  - so consumers get efficient surplus minus uniform monopoly profits
  - we reach the botton right hand corner of the triangle

Social Surplus Minimizing Segmentation

- another optimal policy: always charge highest price in each segment:

	$v=1$	$v=2$	$v=3$	price	weight
$\{1,2,3\}$	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{3}$	3	$\frac{2}{3}$
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  - the good is only ever sold to a consumer when the price equals his valuation

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- In this segmentation....
  - the seller is indifferent between all prices in the segment, so his profits are the same as if he had no information and charged the uniform monopoly price of 2
  - the good is only ever sold to a consumer when the price equals his valuation
  - so consumer surplus is zero

Social Surplus Minimizing Segmentation

- *an* optimal policy: always charge highest price in the support of every segment
- In this segmentation....
  - the seller is indifferent between all prices in the segment, so his profits are the same as if he had no information and charged the uniform monopoly price of 2
  - the good is only ever sold to a consumer when the price equals his valuation
  - so consumer surplus is zero
  - we reach the botton left hand corner of the triangle

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Following the steps for the three values case

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What Do We Learn from Price Discrimination Exercise

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- (Relatively) easy to find out what can happen for all information structures
- Elegant characterization of what can happen for all information structures
- Many different things can happen for some information structure

Information Design: Picking an Information Structure

How gets to choose the information structure and what would they choose?

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   - allow producers to offer discounts (i.e., prices lower the uniform monopoly price)
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3. Malevolent outsider?
   - bottom left hand corner

Context: Third Degree Price Discrimination

- classic topic:
  - Pigou (1920) *Economics of Welfare*
  - Robinson (1933) *The Economics of Imperfect Competition*
- middle period: e.g.,
  - Schmalensee (1981)
  - Varian (1985)
  - Nahata et al (1990)
- latest word:
  - Aguirre, Cowan and Vickers (AER 2010)
  - Cowan (2012)

Existing Results: Welfare, Output and Prices

- examine welfare, output and prices
- focus on two segments
- price rises in one segment and drops in the other if segment profits are strictly concave and continuous: see Nahata et al (1990))
- Pigou:
  - welfare effect = output effect + misallocation effect
  - two linear demand curves, output stays the same, producer surplus strictly increases, total surplus declines (through misallocation), and so consumer surplus must strictly decrease
- Robinson: less curvature of demand $(-\frac{p \cdot q''}{q'})$ in "strong" market means smaller output loss in strong market and higher welfare

These Results (across all segmentations)

- Welfare:
  - Main result: consistent with bounds, anything goes
  - Non first order sufficient conditions for increasing and decreasing total surplus (and can map entirely into consumer surplus)
- Output:
  - Maximum output is efficient output
  - Minimum output is given by *conditionally efficient* allocation generating uniform monopoly profits as total surplus (note: different argument)
- Prices:
  - all prices fall in consumer surplus maximizing segmentation
  - all prices rise in total surplus minimizing segmentation
  - prices might always rise or always fall *whatever* the initial demand function (this is sometimes - as in example - consistent with weakly concave profits, but not always)

An Alternative Perspective: One Player Information Design and Concavification

- Suppose that there is one decision maker and two states, 1 and 2

An Alternative Perspective: One Player Information Design and Concavification

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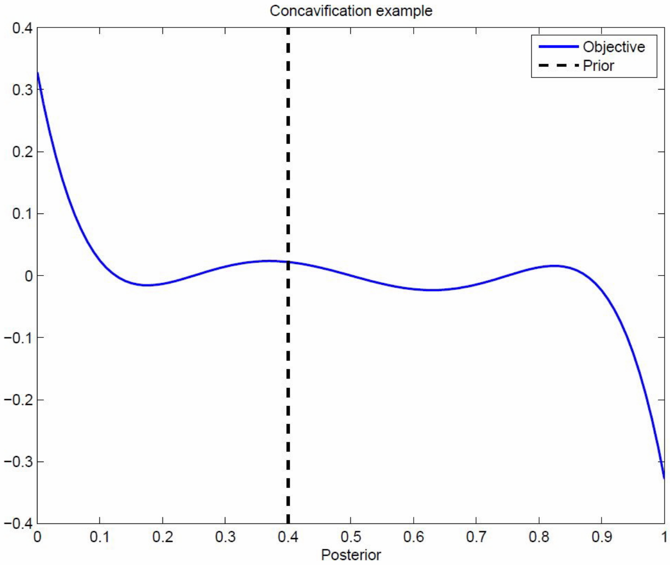
- If the decision maker thinks that the probability of state 1 is x, he will choose an action that will give the information designer utility $u(x)$

Information Designer's Utility



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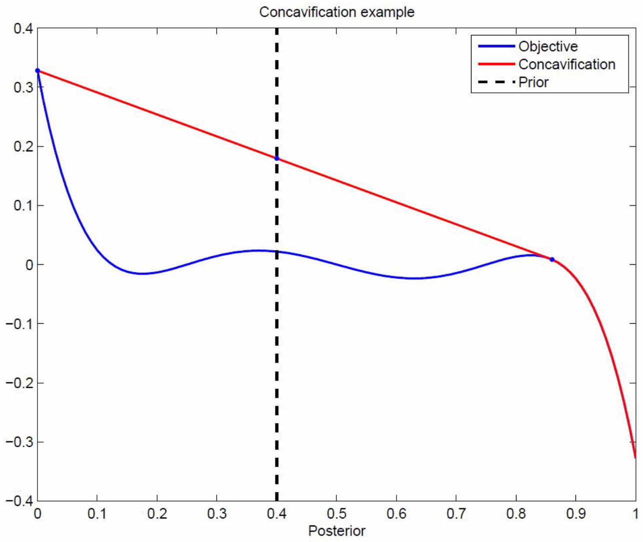
- What information will the designer give and what will his utility be?
- In this simpler two state case, it remains the case that an information structure corresponds to a probability distribution over beliefs that average to the true belief
- Thus the set of utilities that are attainable by the information designer from choosing the information structure is given by the concavification of u

Information Designer's Maximum Utility



Optimal Information Structure

The information designer's utility is maximized if, with equal probabilities,

- the decision maker is told that the state is 2
- the decision maker is told that the state is 1 with probability 0.8

Many States

- Concavification argument works with an arbitrary number of states
- But less easy to use in practise with strictly more than two states

- Suppose that the good has two possible values: 1 and 2

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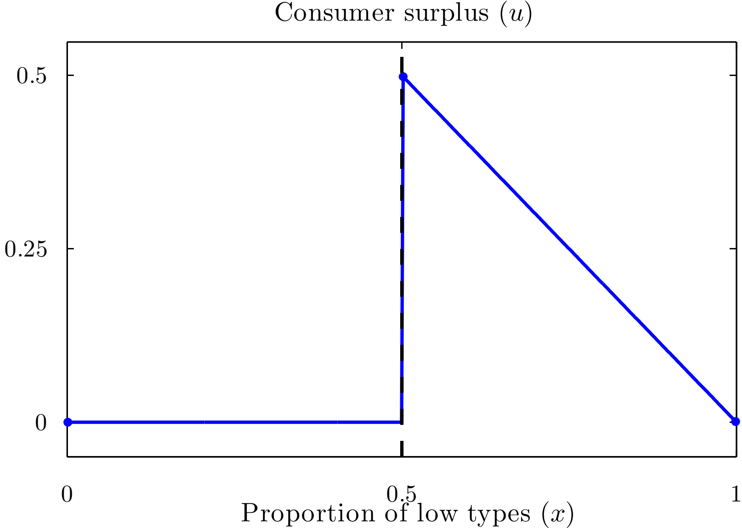
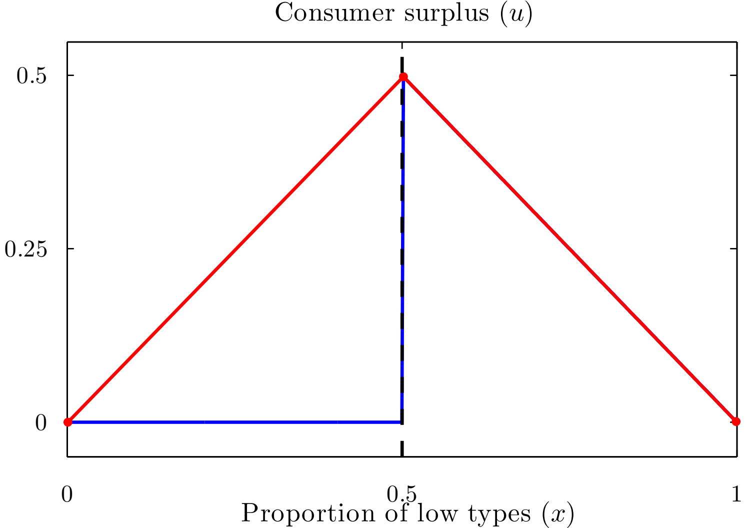
  - $1 - x$ if $x \geq \frac{1}{2}$, since proportion $1 - x$ of consumers with value 2 will get a surplus of 1


Consumer surplus (u)
0.5
0.25
0
0
0.5
1
Proportion of low types (x)



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- Consumer surplus is maximized if the market is segmented as follows:
  - proportion 0.2 of consumers are in a market with only high valuation consumers
  - proportion 0.8 of consumers are in a market with equal numbers of high and low valuation consumers

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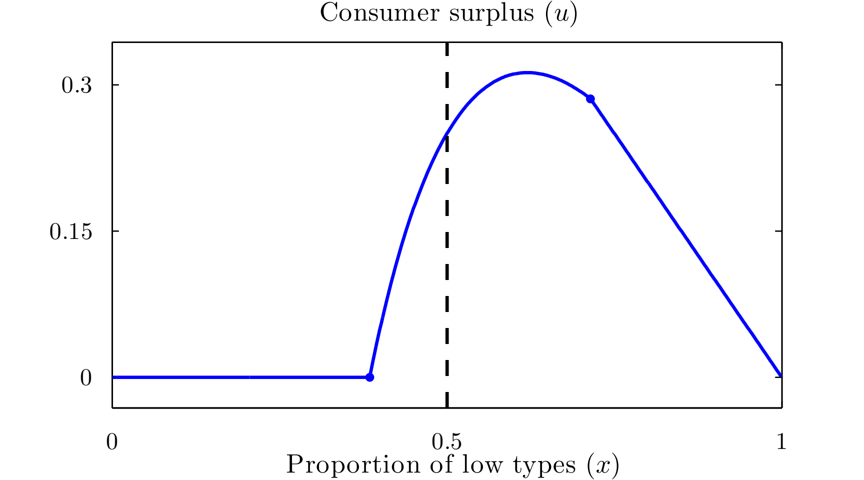
- Equivalent to a concavification argument
- This argument works with one objective: consumer surplus
- Could apply same methodology with arbitrary other objectives to map out surplus triangle

Price Discrimination with Screening

- Now suppose that the consumer's utility from consuming q units is $v\sqrt{q} - t$ where
  - q is quantity consumed
  - t is payment
  - v is "value"
- Cost of production is 1
- Efficient output is $\frac{1}{4}v^2$

Price Discrimination with Screening

- Now suppose that there are two types: low value $v = 1$ and high value $v = 2$
- Suppose that the proportion of consumers with a low value is x
- For small x, the optimal contract is
  - "exclude" low valuation consumers
  - sell efficient quantity $\frac{1}{4}v^2$ to high valuation consumers and charge them their willingness to pay ($\frac{1}{2}v^2$)
- For large x, the optimal contract is
  - sell less than efficient quantity to low valuation consumers and charge them their willingness to pay
  - sell efficient quantity $\frac{1}{4}v^2$ to high valuation consumers and give them some rent to stop them mimicing low valuation types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0
0.5
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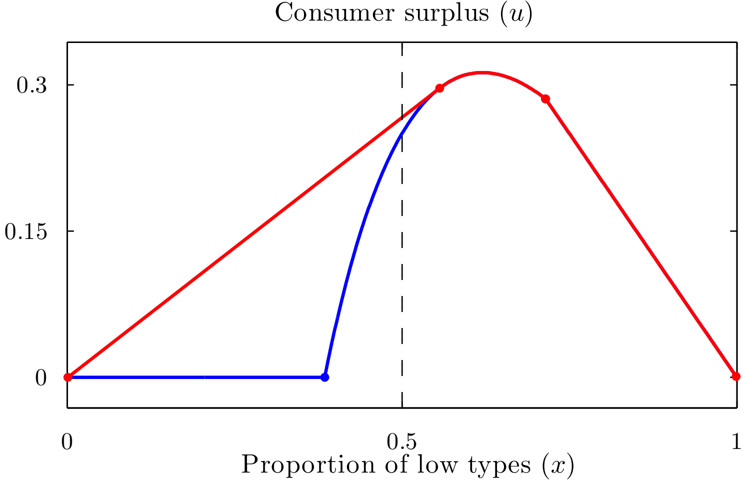
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References

- Bergemann, Brooks and Morris (2015), **The Limits of Price Discrimination**, *American Economic Review.*
- Kamenica and Genzkow (2001), **Bayesian Persuasion**, *American Economic Review*.

After the Break...

A more general perspective on Information Design

Mechanism Design and Information Design

- **Mechanism Design**:
  - Fix an economic environment and information structure
  - Design the rules of the game to get a desirable outcome

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- **Mechanism Design**:
  - Can compare particular mechanisms
    - e.g., first price auctions versus second price auctions

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    - "Information Design": Taneva (2015)

This Lecture

1. Leading Examples
2. General Framework (in words)
3. Examples

Bank Run: one depositor and no initial information

- A bank depositor is deciding whether to run from the bank if he assigns probability greater than $\frac{1}{2}$ to a bad state

Payoff	θ_G	θ_B
Stay	1	-1
Run	0	0

- The depositor knows nothing about the state
- The probability of the bad state is $\frac{2}{3}$

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- Outcome distribution with no information:

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Run	$\frac{1}{3}$	$\frac{2}{3}$

- Probability of run is 1

Optimal Information Design with one depositor and no initial information

- The regulator cannot stop the depositor withdrawing....
  - ... but can choose what information is made available to prevent withdrawals

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Lessons

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1. Without loss of generality, can restrict attention to information structures where each player's signal space is equal to his action space
   - compare with the revelation principle of mechanism design:
     - without loss of generality, we can restrict attention to mechanisms where each player's message space is equal to his type space

Bayesian Persuasion

- This is the leading example in Kamenica-Gentzkow 2011
- We are not exploiting "concavification" logic discussed earlier...

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Is initially more informed depositor good or bad?

- With no information design....
  - ...and no initial information, probability of run is 1
  - ...and initial information, probability of run is $\frac{5}{6}$

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- With information design....
  - ...more initial information is always bad for the regulator

Lessons

1. Without loss of generality, can restrict attention to information structures where each player's signal space is equal to his action space
2. Prior information limits the scope for information design

Bank Runs: two depositors and no initial information (and strategic complements)

- A bank depositor would like to run from the bank if he assigns probability greater than $\frac{1}{2}$ to a bad state OR the other depositor running

state θ_G	*Stay*	*Run*
Stay	1	-1
Run	0	0

state θ_B	*Stay*	*Run*
Stay	-1	-1
Run	0	0

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  - ...with public signals optimal

Bank Runs: two depositors, no initial information and strategic substitutes

- Previous example had strategic complements
- Strategic substitute example: a bank depositor would like to run from the bank if he assigns probability greater than $\frac{1}{2}$ to a bad state AND the other depositor staying

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Run	0	0

outcome θ_B	*Stay*	*Run*
Stay	$\frac{4}{9}$	$\frac{1}{9}$
Run	$\frac{1}{9}$	0

Bank Runs: two depositors and no initial information

- Outcome distribution with no information: mixed strategy equilibrium
- Best information structure:
  - tell the depositors that the state is bad exactly often enough so that they will stay if they don't get the signal.....

outcome θ_G	*Stay*	*Run*
Stay	$\frac{1}{3}$	0
Run	0	0

outcome θ_B	*Stay*	*Run*
Stay	$\frac{4}{9}$	$\frac{1}{9}$
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  - ....with private signals optimal

Lessons

1. Without loss of generality, can restrict attention to information structures where each player's signal space is equal to his action space
2. Prior information limits the scope for information design
3. Public signals optimal if strategic complementarities; private signals optimal if strategic substitutes

Bank Run: two depositors with initial information

have also analyzed elsewhere....

General Formulation (in words!)

- Fix a game with incomplete information about payoff states
- Ask what could happen in equilibrium for any additional information that players could be given....

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  - ....Forges (1993) Bayesian solution if no distributed uncertainty

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- One information structure is "individually sufficient" for another if you can embed both information structures in a combined information structure where a player's signal in the former information structure is sufficient for his signal in the latter...
- This ordering characterizes which information structure imposes more incentive constraints

Applications

1. Price Discrimination

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2. Auctions

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1. Price Discrimination
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3. Linear Normal Applications
    3.1 Oligopoly
    3.2 Volatility
    3.3 Market Power
    3.4 Networks

Application 2: First Price Auctions

- Four Cases:
  1. Symmetric / Complete Information (Bertrand Competition)
  2. Independent Private Values
  3. a few more special cases, e.g., Affiliated Values
  4. (this paper) All Information Structures

A Leading Example

- 2 bidders with private values uniformly distributed on the interval $[0, 1]$; bidders know their private values

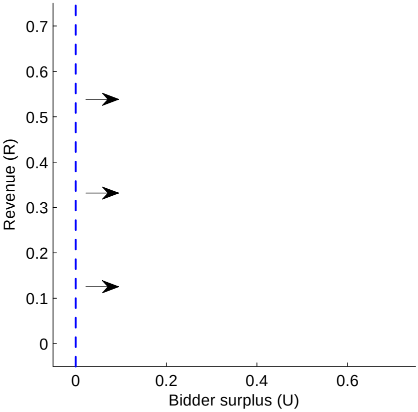
1. Symmetric Information (Bertrand Competition):
    - each bidder bids lower value
    - revenue is expectation of lower value $= \frac{1}{3}$
    - total efficient surplus is expectation of higher value $= \frac{2}{3}$
    - bidder surplus is $\frac{1}{3}$ ($\frac{1}{6}$ each)
2. Independent Private Values

A Leading Example

- 2 bidders with valuations uniformly distributed on the interval $[0, 1]$

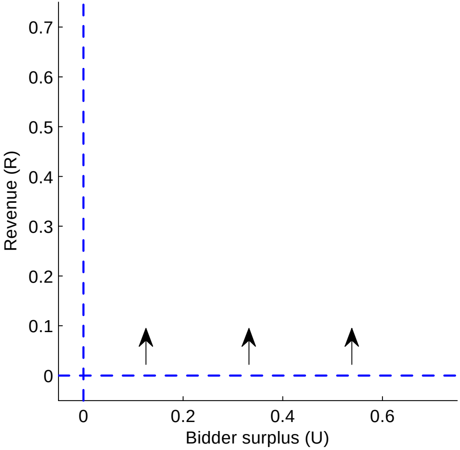
1. Symmetric Information (Bertrand Competition)
2. Independent Private Values
   - each bidder bids half his value
   - revenue equivalence holds....as under complete information or second price auction...
     - revenue is expectation of low value $= \frac{1}{3}$
     - total efficient surplus is expectation of high value $= \frac{2}{3}$
     - bidder surplus is $\frac{1}{3}$

Graphical Summary: Bounds 1



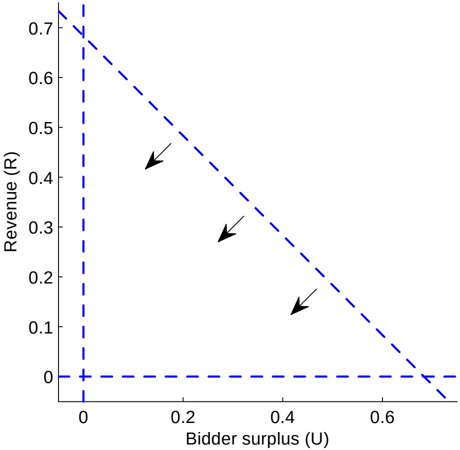
- nonnegative bidder surplus

Graphical Summary: Bounds 2



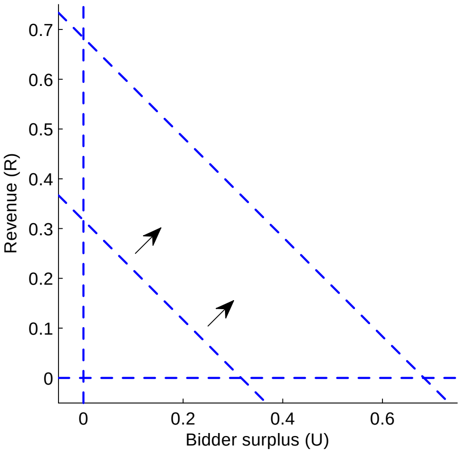
- nonnegative revenues

Graphical Summary: Bounds 3



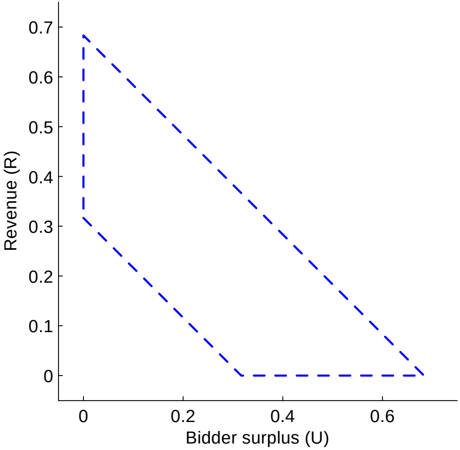
- efficient social surplus: always give the object to the bidder with the highest valuation

Graphical Summary: Bounds 4



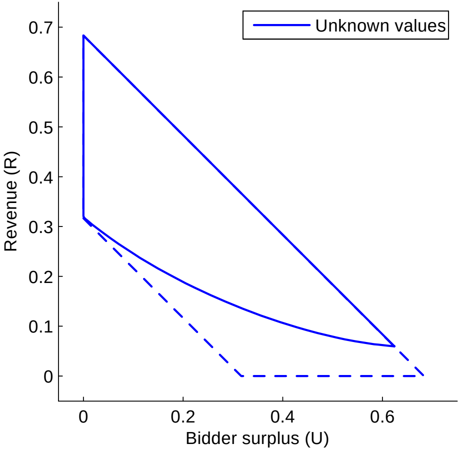
- least efficient allocation: always give the object to the bidder with the lowest valuation

Surplus Trapezoid



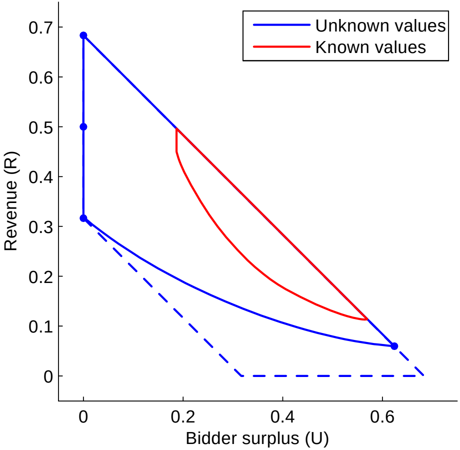
- so far: feasibility and participation constraints

Incentives Imposes Restrictions: Unknown Values



- incentive constraints (optimal bidding) adds new constraints (even if you dont know your own value!)

Information Generates Incentives: Known Values



- each bidder i knows his own value v_i

Applications 3: Linear Normal Model

- continuum of agents: $i \in [0, 1]$
- utility of agent i depends on own action $a_i \in \mathbb{R}$, average action $A \in \mathbb{R}$ and state of the world $\theta \in \mathbb{R}$,

$$u(a, A, \theta) = -(1-r)(a-\theta)^2 - r(a-A)^2$$

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- the state of the world θ is normally distributed

$$\theta \sim N\left(\mu_\theta, \sigma_\theta^2\right)$$

Application 3a: (in words....) Oligopoly

- Lesson 3:
  - with strategic complementaries, public information is best
  - with strategic substitutes, private (conditionally independent) information is best
- In oligopoly...
  - strategic substitutes
  - if uncertainty about demand, firms would like to have
    - good information about the state of demand
    - BUT would like signals to be as uncorrelated as possible with others' signals
  - in general, intermediate conditionally independent private signals about demand are optimal for cartel problem

Application 3b: Aggregate Volatility

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  - Without aggregate uncertainty, intermediate information with common shock

Application 3c: Market Power

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  - e.g., if my information overweights common component, low price suggests high ideosyncratic value
- Any market power is consistent with any number of players....

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  - Information: common noisy components of signals (even with symmetric interaction and independent shocks)
- Equivalence between these three perturbations
- Subtle interactions

References

- General Approach:
  - Bergemann and Morris (2013), **Robust Predictions in Incomplete Information Games**, *Econometrica*.
  - Bergemann and Morris (2015), **Bayes Correlated Equilibrium and the Comparison of Information Structures**, forthcoming in *Theoretical Economics*.
- More Applications:
  - Oligopoly, Ecta paper
  - Auctions: Bergemann, Brooks and Morris (2015), **First Price Auctions with General Information Structures: Implications for Bidding and Revenue**
  - Volatility: Bergemann, Heumann and Morris (2015), **Information and Volatility**, forthcoming in *JET*
  - Market Power: Bergemann, Heumann and Morris (2015), **Market Power**
  - Networks: Bergemann, Heumann and Morris (2015), **Networks and Volatility**

Information Design Recap

- **Mechanism Design**:
  - Incentive constraint: truth-telling
  - Other constraint: participation
- **Information Design**
  - Incentive constraint: obedience
  - Other constraint: prior information