Problem Set 3, MIEG, Winter Term, 2016

Osborne: 468.1, 468.2,

1. Consider the following game.

	L	R
U	3,3	0,4
D	6,0	1,1

(i) Suppose that the above game is repeated for infinitely many periods. In a diagram, sketch the set of payoffs that can be sustained as subgame perfect equilibrium of the above infinitely repeated game. Does it include the minmax payoff?

(ii) Let $\delta < 1$ be the common discount factor. A 'tit-for-tat' strategy can be described as follows.
Row player: At period 1 play U. At period t, play U if the column player has played L in period $t-1$ and play D otherwise.
Column player: At period 1 play L. At period t, play L if the row player has played U in period $t-1$ and play R otherwise.
Is 'tit-for-tat' a Nash equilibrium? Explain. [4]

2. Consider an infinitely repeated Bertrand duopoly game with linear demand function and constant marginal cost. Is it possible to sustain the following outcome in SPE?
(i) Both the firms charge the monopoly price (try grim-trigger strategy).
(ii) Both the firms charge the marginal cost.
(iii) One of the firms charges below the marginal cost.

3. Consider an infinitely repeated Cournot duopoly game with linear demand function and constant marginal cost. Is it possible to sustain the monopoly outcome (each producing half of the monopoly quantity) in SPE?

4. Find all pure and mixed strategy NE of the following game.

	L	M	R
U	0,0	3,4	6,0
M	4,3	0,0	0,0
D	0,6	0,0	5,5

If this game is repeated for two periods can (D, R) be sustained as a part of SPE?

5. Country A and Country B have common oil field. In each year, simultaneously, each of these countries decide whether to extract high(H) or low(L) amount of oil from this field. Extracting high amount of oil from the common field hurts the other country. In addition, A has the option of attacking B (action W), which is costly for both countries. The stage game is as follows (IA is row player and B being column player):

	H	L
H	2,2	4,1
L	1,4	3,3
W	-1,-1	-1,-2

Consider the infinitely repeated game with this stage game.
(i) Find a SPNE in which each country extracts low (L) amount of oil every year on the equilibrium path.
(ii) Find a SPNE in which A extracts high (H) amount of oil and B extracts low (L) amount of oil every year on the equilibrium path

6. Consider a infinitely repeated 'offer counteroffer model'. Players have discounted utilities.

(i) Is the following strategy profile Nash equilibrium?
Player 1: (proposer) Offer $(\frac{1}{2}, \frac{1}{2})$ in period 1. Offer $(1, 0)$ in any other period.
Player 1: (responder) Accept any offer $(x, 1-x)$ if and only if $x = 1$
Player 2: (proposer) Offer $(0, 1)$ in every period.
Player 2: (responder) Accept any offer $(x, 1-x)$ if and only if $x \leq \frac{1}{2}$
Is it subgame perfect equilibrium? If not show a profitable deviation.

(ii) Is the following strategy profile Nash equilibrium?
Player 1: (proposer) Offer $(0, 1)$ in every period
Player 1: (responder) Accept any offer $(x, 1-x)$
Player 2: (proposer) Offer $(0, 1)$ in every period.
Player 2: (responder) Accept any offer $(x, 1-x)$ if and only if $x = 0$
Is it subgame perfect equilibrium? If not show a profitable deviation.

7. Without using the 'uniqueness of SPNE proof', find subgame perfect equilibrium of infinitely repeated 'offer counteroffer model' when (i) $\delta_1 = 0$, $\delta_2 > 0$, (ii) $\delta_2 = 0$, $\delta_1 > 0$.

8. Find subgame perfect equilibrium of infinitely repeated 'offer counteroffer model' when player 1 proposes in two consecutive periods (date $1, 2, 4, 5, 7, 8, \ldots$) while player 2 proposes in every third periods (date $3, 6, 9, \ldots$).

9. Find subgame perfect equilibrium of infinitely repeated 'offer counteroffer model' where each player i incurs a cost $c_i > 0$ for every period in which agreement is not reached. Suppose that player 1 makes the first offer. Show that if $c_1 < c_2$ then the game has unique SPNE, where player 1 keeps the entire surplus.