Solution for MIEG (Part A), 2014

1.
(a) The firm's best response function, $b_f(w)$, is given as below.

$$b_f(w) = \begin{cases} L = 0 & \text{if } w > 100 \\ L = \frac{100-w}{2} & \text{if } 0 < w \leq 100 \\ L \geq 50 & \text{if } w = 0 \end{cases}$$

Given this, the union optimises its payoff.

$$\max_{w} \quad w\left(\frac{100-w}{2}\right)$$

The first order condition yields, $w^* = 50$. The optimal L chosen by the firm at w^* is $L^* = 25$. Corresponding payoffs are $\Pi_u = 1250$ and $\Pi_f = 625$.

(b) At $L = 40$ and $w = 40$, $\Pi_u = 1600$ and $\Pi_f = 800$ is pareto superior to the SPNE payoffs.

(c) If firm rejects any $w > 0$, and hires $L > 50$, $\Pi_u = 0$ and $\Pi_f = 2500$. This is a NE since neither of the players can strictly gain from the deviation. In particular, for any $w > 0$, $L = 0$ and $\Pi_u = 0$. Likewise, for any $L \leq 50$, $\Pi_f < 2500$.

2. Discussed in class

3.
(a) Player 1's dominant strategy is to play D, player 2's dominant strategy is to play R. Therefore, this game has a unique pure Nash equilibrium, (D, R). The minmax payoff is (D, R).

(b) See the diagram below. Note that minmax payoff is included in the set because it is also the Nash equilibrium payoff.

(c) Given if any one player plays 'tit-for-tat' strategy and the other player plays D (R) in any period t, then the former player chooses D (R) in period $t+1$ and continues to choose D (R) in subsequent periods till the latter reverts to U (L).

The present value of payoffs from playing (U, L) in every period is $(1-\delta)\frac{3}{(1-\delta)}$.

Let player 1 choose to deviate in every alternative period, such that her payoffs are $(6 + 0\delta + 6\delta^2 + 0\delta^3 + ...) = (1-\delta)\frac{6}{(1-\delta^2)}$.

For (U, L) to be sustained as Nash equilibrium using the 'tit-for-tat' strategy

$$(1-\delta)\frac{3}{1-\delta} \geq (1-\delta)\frac{6}{(1-\delta^2)}$$

$$3(1+\delta) \geq 6 \Rightarrow \delta \geq 1$$

That is, there are no possible deviations for player 1 for any $\delta < 1$.

Since expected payoff from such deviations is more than that obtained from 'tit-for-tat' strategy which generates outcome associated with (U, L) in each period, 'tit-for-tat' strategy is not a Nash equilibrium as players have an incentive to deviate.

(d) To check if is possible to sustain (U, L) as SPNE, we consider the "Grim Trigger strategy" and check if (U, L) is a Nash equilibrium at every possible history. We check ODP.

For history ending in (U, L):

For player 1,

$$(1-\delta)(3+3\delta+3\delta^2+3\delta^3+...) \geq (1-\delta)(6+\delta+\delta^2+\delta^3+...)$$

The minimum value of δ for player 1 is $\frac{3}{5}$.

For player 2,

$$(1-\delta)(3+3\delta+3\delta^2+3\delta^3+...) \geq (1-\delta)(4+\delta+\delta^2+\delta^3+...)$$

The minimum value of δ for player 2 is $\frac{1}{3}$.

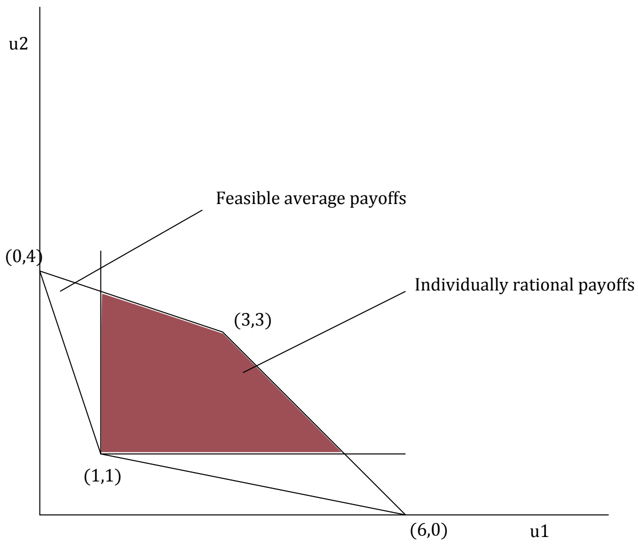
For history ending in (D, R), (U, R) or (D, L):

If (D, R) is played in each subsequent period, then the payoffs is $(1-\delta)\frac{1}{(1-\delta)} = 1$

If player 1 (2) deviates to U (L) for one period, then she gets $(1-\delta)\frac{\delta}{(1-\delta)} = \delta < 1$. There are no profitable deviations in any subgame with this history.

Therefore (U, L) can be sustained as SPNE for $\delta \geq \frac{3}{5}$.

<u>Diagram</u>


u2
Feasible average payoffs
(0,4)
Individually rational payoffs
(3,3)
(1,1)
(6,0)
u1