Solution for MIEG (Part A), 2015

1.

(a) It has n pure strategy Nash equilibria, in each of which exactly one person calls. To be more precise the following are n Nash equilibria : $s^{\star i} = \left(s_i^\star = C,\ (s_j^\star = DC)_{j \neq i}\right)\ for\ i = 1, 2, \dots n$ where C and DC means Call and Don't call.

(b) There is only one such equilibrium exists, in which each person calls with probability $p = 1 - (c/v)^{\frac{1}{n-1}}$

(c) $1 - \left(\frac{c}{v}\right)^{\frac{n}{n-1}}$

(d) The probability of reporting the crime falls with an increase of either n (stronger incentive to freeride) or c (increasing cost of reporting).

2.:

(a) At $T = 1$ the discounted values of movies for week 1, 2 and 3 are 22, 21 and 24 respectively. So the optimal choice at $T = 1$ is to wait till week 3 and watch movie at $T = 3$

(b) SPNE as follows (applying backward induction) :
At $T = 3$ Watch movie 3 (else he gets 0)
At $T = 2$ Watch movie 2 (because the immediate payoff is more than the discounted payoff from $T = 3$, $(35 > 30)$.
At $T = 1$ Watch movie 1 (because the immediate payoff is more than the discounted payoff from $T = 2$, $(22 > 21)$.

(c) Yes. He will make movie 2 invalid, which makes him watching movie 3 (the optimal choice at $T = 1$).

(d) Hyperbolic discounting may lead to immediate consumption instead of savings (as above). To solve such commitment problem, one may prefer fixed deposit instead of savings deposit.

3.

(a) Note this game is same as standard Rubinstein bargaining model, with only difference is that players are bargaining over Π instead of 1 i.e. the proposer always keep $\frac{\Pi}{1+\delta}$ for himself and offers $\frac{\delta\Pi}{1+\delta}$ to the respondent. Now, you can write SPNE strategies accordingly.

(b) From part (a), we know in the usual scenario firm will offer a wage equal to $\frac{\delta\Pi}{1+\delta}$. So if outside option (R) is less than this then worker will accept this. But if outside option is more then firm knows if it makes his usual offer worker will reject it and accept outside option and firm will get zero. So firm optimally will make a offer exactly equal to outside option (offering more will only reduce firm's payoff) and makes worker indifferent between the offer made by firm and his outside option which makes worker accepting firm's offer. Note that this will also change the wage offered by worker when its his turn. He will also make a offer such that the firm is indifferent between accepting his offer (which gives payoff of $\Pi - w_W$) or rejecting (which gives present value of next time period payoff i.e. $\delta(\Pi - R)$ because in next period firm makes a offer of equal to R as just argued above)

Therefore SPNE will depend on the value of R.
Case 1 : $R \leqslant \frac{\delta\Pi}{1+\delta}$: Then R has no role to play. And SPNE is same as part (a) above.
Care 2: $R > \frac{\delta\Pi}{1+\delta}$: Then SPNE as follows:
At odd dates: Worker offers $w_W = \Pi(1-\delta) + \delta R$
At even dates: Firm offers $w_F = R$.
Each player will accept\reject any offer accordingly.

(c) Observe that for $t \geqslant 3$ this part is same as part (a) and only for $t = 1, 2$ it is similar to part (b). So SPNE as follows :-
For case 1: $R \leqslant \frac{\delta\Pi}{1+\delta}$: No change in SPNE.
For case 2: $R > \frac{\delta\Pi}{1+\delta}$:
At odd dates : At $t = 1$ worker offers $w_W = \Pi(1-\delta) + \delta R$ and for any $t \geqslant 3$ (*odd dates*) $w_W = \frac{\Pi}{1+\delta}$
At even dates : At $t = 2$ worker offers $w_F = R$ and any $t \geqslant 4$ (*even dates*) $w_F = \frac{\delta\Pi}{1+\delta}$.
Each player will accept\reject any offer accordingly.

Note: In part part (b) and (c), I have just mentioned "Each player will accept\reject any offer accordingly". (Hope by now) you know that its not a proper way of writing a strategy. So I am leaving you this bit of writing strategy *properly*.