M.A. Economics: Winter Semester, 2016
005: Markets, Institutions and Economic Growth
Mid term1: Answer Key

1. (a) For the row player, each payoff from strategy C is strictly lower than the respective payoffs from strategy A. Therefore C is the strictly dominated strategy for the row player.

(b) If the column player randomizes between strategies A and B with probabilities q and $1-q$ then
$U_2(A, mixed\ strategy) > U_2(A, C) \Rightarrow 2q + 0(1-q) > 1 \Rightarrow q > 0.5$
$U_2(B, mixed\ strategy) > U_2(B, C) \Rightarrow 1q + 3(1-q) > 1 \Rightarrow q < 0.75$
$U_2(C, mixed\ strategy) > U_2(C, C) \Rightarrow 1q + 0.(1-q) > 0.7 \Rightarrow q > 0.7$
Therefore, for $q \in (0.70, 0.75)$, C is strictly dominated by a mixed strategy involving strategies A and B.

(c) From above, strategy C is strictly dominated for both players and is eliminated. The strategies that survive iterated elimination of strictly dominated strategies are A and B for each player.

(d) Given that strategy C is strictly dominated for both players and are eliminated, players randomize between strategies A and B. Let the row player randomize between A and B with probabilities p and $1-p$, respectively, and the column player randomize between A and B with probabilities q and $1-q$.
For the row player, $\mathcal{E}_1[A] = \mathcal{E}_1[B] \Rightarrow 3q + 1 - q = 2(1-q) \Rightarrow q = \frac{1}{4}$
For the column player, $\mathcal{E}_2[A] = \mathcal{E}_2[B] \Rightarrow 2p + 1 - p = 3(1-p) \Rightarrow p = \frac{1}{2}$
Therefore, mixed strategy Nash equilibrium is the strategy profile $\{(\frac{1}{2}, \frac{1}{2}, 0), (\frac{1}{4}, \frac{3}{4}, 0)\}$.

2. (a) Player 1 has four actions: AA, AB, BA, BB
Player 2 has two actions: A, B
Strategic form of the game:

	A	B
AA	1,1	0,1
AB	1,0	0,1
BA	1,0	1,0
BB	1,0	1,0

(b) Five pure strategy Nash Equilibria: 1. AA,A, 2. BA,A, 3. BA,B, 4. BB,A, 5. BB,B

(c) Four pure strategy SPNE: 1. AA,A, 2. BA,A, 3. BA,B, 4. BB,B

(d) BB,A is a Nash Equilibrium which is not a SPNE. Player 2 is better off deviating to B at history following Player 1 playing A at the initial node.

3. (a) After the incumbent chooses a price (p), firm B chooses whether to enter and set a price in case of entry. At a history p, Firm B's optimal action is as follows.
Firm B chooses $\hat{p}$ to maximize profit, which is

$$\pi^B = \begin{cases} -0.09 & \text{if entry and } \hat{p} > p \\ \frac{p(1-p)}{2} - 0.09 & \text{if entry and } \hat{p} = p \\ \hat{p}(1-\hat{p}) - 0.09 & \text{if entry and } \hat{p} < p \\ 0 & \text{if stays out} \end{cases}$$

(i) If $p > p^m$ ($p^m = \frac{1}{2}$ is the monopoly price), then Firm B enters and chooses p^m. It earns a positive profit.
(ii) If $p \leq p^m$ then the maximum profit Firm B can earn by entering is $[(p-\epsilon)(1-p+\epsilon) - 0.09]$, under the assumption that the minimum denomination is ϵ. If ϵ is small enough this expression can be approximated by $p(1-p) - 0.09$, which is positive if and only if $p > 0.1$. Hence B chooses to stay out if $p \leq 0.1$ and chooses $p - \epsilon$ if $p > 0.1$. This is B's SPNE strategy.

Hence profit of Firm A is $\pi^A = \begin{cases} 0 & \text{if } p > 0.1 \\ p(1-p) - 0.09 & \text{if } p \leq 0.1 \end{cases}$

Thus optimal choice of Firm A is 0.1 (lets denote it by p^c). B stays out and profit of A is 0.09.

(b)Consider the following strategy profile. Let p_t^i be the price chosen by Firm i at period t.
Firm A:
• $p_t^A = p^m$ as long as both firms are charging the monopoly price till period t, that is $p_r^A = p_r^B = p^m$, for all period $r < t$.
• (Punishment) For all other history, A chooses p^c perpetually.
Firm B:
• $p_t^B = p^m$ as long as both firms are charging the monopoly price till period t, that is $p_r^A = p_r^B = p^m$, for all period $r < t$ and $p_t^A = p^m$.

• (Punishment) For all other history, B plays its one-period SPNE strategy perpetually.

If these strategies are played then both firms choose p^m in every period and earn half of the monopoly profit, which is 0.125. Let us now check that this strategy pair is SPNE.

Firm A wont deviate from p^m because any deviation will lead to price competition in all future period (including the current period) and hence the maximum profit it can earn per period is 0.09 (one period SPNE profit) instead of 0.125.

Firm B can earn the monopoly profit for one period by deviating but subsequently will earn zero profit. So B will not deviate, if $(0.125 - 0.09) > (1 - \delta)(0.25 - 0.09)$ that is $\delta \geq 0.8$ (approx.).

In the punishment phase both players play the one-period SPNE repeatedly and hence there won't be any deviation.