

EEE-05, 2015

Series 05

Time. 3 hours

Maximum marks. 100

General Instructions: Please read the following instructions carefully

- Check that you have a bubble-sheet and an answer book accompanying this examination booklet. Do **not** break the seal on this booklet until instructed to do so by the invigilator.
- Immediately on receipt of this booklet, fill in your Signature, Name and Roll number in the space provided below.
- Do **not** disturb your neighbours at any time.
- Make sure you do **not** have **mobile, papers, books, etc.**, on your person. The exam does not require use of a calculator. However, you can use non-programmable, non-alpha-numeric memory simple calculator. **Anyone engaged in illegal practices will be immediately evicted and that person's candidature will be canceled.**
- If want to leave the examination hall before the stipulated time for this exam, hand in this **booklet, the bubble-sheet and the answer book** to the invigilator. You are **not allowed to leave** the examination hall during the first 30 minutes and the last 15 minutes of the examination time.
- If you stay till the end of examination, hand in this **the bubble-sheet and the answer book** to the invigilator. You can take this booklet with you.
- **Only after the invigilator announces the start of the examination**, break the seal of the booklet. Check that this booklet has pages 1 through 8. Report any missing pages to the invigilator.

Signature _____

Name _____

Roll number _____

Part I

Instructions for Part I

- This part of the examination will be **checked by a machine**. Therefore, it is very important that you follow the instructions on the bubble-sheet.
- Fill in the required information in Boxes on the bubble-sheet. **Do not write anything in Box 3 - the invigilator will sign in it.**
- This part of the exam consists of 20 multiple-choice questions. Each question is followed by four possible answers, at least one of which is correct. If more than one choice is correct, choose **only the ‘best one’**. The ‘best answer’ is the one that implies (or includes) the other correct answer(s). Indicate your chosen best answer on the **bubble-sheet** by shading the appropriate bubble.
- For each question, you will get: 2 marks if you choose only the best answer; 0 mark if you choose none of the answers. **However, if you choose something other than the best answer or multiple answers, then you will get $-2/3$ mark for that question.**
- To do **‘rough work’** for this part, you can use the last pages of the separate answer book provided to you. Your rough work will be neither read nor checked.

The first Three questions pertain to the following: A simple linear regression of wages on gender, run on a sample of 200 individuals, 150 of whom are men, yields the following

$$W_i = 300 - 50D_i + u_i$$

(20) (10)

where W_i is the wage in Rs per day of the i^{th} individual, $D_i = 1$ if individual i is male, and 0 otherwise, u_i is a classical error term, and the figures in parentheses are standard errors.

Question 1. What is the average wage in the sample?

- (a) Rs. 250 per day
- (b) Rs. 275 per day
- (c) Rs. 262.50 per day
- (d) Rs. 267.50 per day

Question 2. The most precise estimate of the difference in wages between men and women would have been obtained if, among these 200 individuals,

- (a) There were an equal number (100) of men and women in the sample
- (b) The ratio of the number of men and women in the sample was the same as the ratio of their average wages
- (c) There were at least 30 men and 30 women; this is sufficient for estimation: precision does not depend on the distribution of the sample across men and women
- (d) None of the above

Question 3. The explained (regression) sum of squares in this case is:

- (a) 93750
- (b) 1406.25
- (c) 15000
- (d) This cannot be calculated from the information given

Question 4. A researcher estimate the following two models using OLS

$$\text{Model A: } y_i = \beta_0 + \beta_1 S_i + \beta_2 A_i + \varepsilon_i$$

$$\text{Model B: } y_i = \beta_0 + \beta_1 S_i + \varepsilon_i$$

where y_i refers to the marks (out of 100) that a student i gets on an exam, S_i refers to the number of hours spent studying for the exam by the student, and A_i is an index of innate ability (varying continuously from a low ability score of 1 to a high ability score of 10). ε_i the usual classical error term.

The estimated β_1 coefficient is 7.1 for Model A, but 2.1 for Model B; both are statistically significant. The estimated β_2 coefficient is 1.9 and is also significantly different from zero. This suggests that:

- (a) Students with lower ability also spend fewer hours studying
- (b) Students with lower ability spend more time studying
- (c) There is no way that students of even high ability can get more than 40 marks
- (d) None of the above

Question 5. An analyst estimates the model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + u$ using OLS. But the true $\beta_3 = 0$. In this case, by including X_3

- (a) there is no harm done as all the estimates would be unbiased and efficient
- (b) there is a problem because all the estimates would be biased and inconsistent
- (c) the estimates would be unbiased but would have larger standard errors
- (d) the estimates may be biased but they would still be efficient

Question 6. Let $\hat{\beta}$ be the OLS estimator of the slope coefficient in a regression of Y on X_1 . Let $\tilde{\beta}$ be the OLS estimator of the coefficient on X_1 on a regression of Y on X_1 and X_2 . Which of the following is true:

- (a) $\text{Var}(\hat{\beta}) < \text{Var}(\tilde{\beta})$
- (b) $\text{Var}(\hat{\beta}) > \text{Var}(\tilde{\beta})$
- (c) $\text{Var}(\hat{\beta}) < \text{or } > \text{Var}(\tilde{\beta})$
- (d) $\text{Var}(\hat{\beta}) = \text{Var}(\tilde{\beta})$

Question 7. You estimate the multiple regression $Y = a + b_1 X_1 + b_2 X_2 + u$ with a large sample. Let t_1 be the test statistic for testing the null hypothesis $b_1 = 0$ and t_2 be the test statistic for testing the null hypothesis $b_2 = 0$. Suppose you test the joint null hypothesis that $b_1 = b_2 = 0$ using the principle 'reject the null if either t_1 or t_2 exceeds 1.96 in absolute value', taking t_1 and t_2 to be independently distributed.

- (a) The probability of error Type 1 is 5 percent in this case
- (b) The probability of error Type 1 is less than 5 percent in this case
- (c) The probability of error Type 1 is more than 5 percent in this case
- (d) The probability of error Type 1 is either 5 percent or less than 5 percent in this case

The following set of information is relevant for the next Six questions. Consider the following version of the Solow growth model where the aggregate output at time t depends on the aggregate capital stock (K_t) and aggregate labour force (L_t) in the following way:

$$Y_t = (K_t)^\alpha (L_t)^{1-\alpha}; \quad 0 < \alpha < 1.$$

At every point of time there is full employment of both the factors and each factor is paid its marginal product. Total output is distributed equally to all the households in the form of wage earnings and interest earnings. Households' propensity to save from the two types of earnings differ. In particular, they save s_w proportion of their wage earnings and s_r proportion of their interest earnings in every period. All savings are invested which augments the capital stock over time ($\frac{dK}{dt}$). There is no depreciation of capital. The aggregate labour force grows at a constant rate n .

Question 8. Let $s_w = 0$ and $s_r = 1$. An increase in the parameter value α

- (a) unambiguously increases the long run steady state value of the capital-labour ratio
- (b) unambiguously decreases the long run steady state value of the capital-labour ratio
- (c) increases the long run steady state value of the capital-labour ratio if $\alpha > n$
- (d) leaves the long run steady state value of the capital-labour ratio unchanged

Question 9. Now suppose $s_r = 0$ and $0 < s_w < 1$. An increase in the parameter value n

- (a) unambiguously increases the long run steady state value of the capital-labour ratio
- (b) unambiguously decreases the long run steady state value of the capital-labour ratio
- (c) increases the long run steady state value of the capital-labour ratio if $\alpha > n$
- (d) leaves the long run steady state value of the capital-labour ratio unchanged

Question 10. Now let both s_w and s_r be positive fractions such that $s_w < s_r$. In the long run, the capital-labour ratio in this economy

- (a) approaches zero
- (b) approaches infinity
- (c) approaches a constant value given by $\left[\frac{(1-\alpha)s_w + \alpha s_r}{n} \right]^{\frac{1}{1-\alpha}}$
- (d) approaches a constant value given by $\left[\frac{\alpha s_w + (1-\alpha)s_r}{n} \right]^{\frac{1}{\alpha}}$

Question 11. Suppose now the government imposes a proportional tax on wage earnings at the rate τ and redistributes the tax revenue in the form of transfers to the capital-owners. People still save s_w proportion of their net (post-tax) wage earnings and s_r proportion of their net (post-transfer) interest earnings. In the new equilibrium, an increase in the tax rate τ

- (a) unambiguously increases the long run steady state value of the capital-labour ratio
- (b) unambiguously decreases the long run steady state value of the capital-labour ratio
- (c) increases the long run steady state value of the capital-labour ratio if $\alpha > n$
- (d) leaves the long run steady state value of the capital-labour ratio unchanged

Question 12. Let us now go back to case where both s_w and s_r are positive fractions such that $s_w < s_r$ but without the tax-transfer scheme. However, now let the growth rate of

labour force be endogenous such that it depends on the economy's capital-labour ratio in the following way:

$$\frac{1}{L_t} \frac{dL}{dt} = \begin{cases} Ak_t & \text{for } k_t < \bar{k}; \\ 0 & \text{for } k_t \gg \bar{k}, \end{cases}$$

where $\bar{k} > \left[\frac{(1-\alpha)s_w + \alpha s_r}{A} \right]^{\frac{1}{2-\alpha}}$ is a given constant. In the long run, the capital-labour ratio in this economy

- (a) approaches zero
- (b) approaches infinity
- (c) approaches a constant value given by $\left[\frac{\alpha s_w + (1-\alpha)s_r}{A} \right]^{\frac{1}{\alpha}}$
- (d) approaches infinity or a constant value given by $\left[\frac{(1-\alpha)s_w + \alpha s_r}{A} \right]^{\frac{1}{1-\alpha}}$ depending on whether the initial $k_0 >$ or $< \bar{k}$

Question 13. In the above question, an increase in the parameter value A

- (a) unambiguously increases the long run steady state value of the capital-labour ratio
- (b) unambiguously decreases the long run steady state value of the capital-labour ratio
- (c) decreases the long run steady state value of the capital-labour ratio only when the initial $k_0 < \bar{k}$
- (d) leaves the long run steady state value of the capital-labour ratio unchanged

The next Two questions are based on the following. Consider a pure exchange economy with three persons, 1, 2, 3, and two goods, x and y . The utilities are given by $u^1(.) = xy$, $u^2(.) = x^3y$ and $u^3(.) = xy^2$, respectively.

Question 14. If the endowments are (2,0), (0,12) and (12,0), respectively, then

- (a) an equilibrium price ratio does not exist
- (b) $p_X/p_Y = 1$ is an equilibrium price ratio
- (c) $p_X/p_Y > 1$ is an equilibrium price ratio
- (d) $p_X/p_Y < 1$ is an equilibrium price ratio

Question 15. If the endowments are (0,2), (12,0) and (0,12), respectively, then

- (a) an equilibrium price ratio does not exist
- (b) equilibrium price ratio is the same as in the above question
- (c) $p_X/p_Y < 1$ is an equilibrium price ratio
- (d) $p_X/p_Y > 1$ is an equilibrium price ratio

Question 16. A city has a single electricity supplier. Electricity production cost is Rs. c per unit. There are two types of customers. Utility function for type i is given by $u_i(q, t) = \theta_i \ln(1 + q) - t$, where q is electricity consumption and t is electricity tariff. High type customers are more energy efficient, that is, $\theta_H > \theta_L$; moreover $\theta_L > c$. Suppose the supplier cannot observe type of the consumer, i.e., whether $\theta = \theta_H$ or $\theta = \theta_L$. If the supplier decides to sell packages (q_H, t_H) and (q_L, t_L) , meant for for whom $\theta = \theta_H$ and for whom $\theta = \theta_L$ respectively, then profit maximizing tariffs will be

- (a) $t_H = c \ln \left(\frac{\theta_H}{c} \right)$ and $t_L = c \ln \left(\frac{\theta_L}{c} \right)$

- (b) $t_H = \theta_H \ln\left(\frac{\theta_H}{c}\right)$ and $t_L = 0$
- (c) $t_H = \theta_H \ln\left(\frac{\theta_H}{c}\right)$ and $t_L = \theta_L \ln\left(\frac{\theta_L}{c}\right)$
- (d) None of the above

Question 17. Suppose buyers of ice-cream are uniformly distributed on the interval $[0, 1]$. Ice-cream sellers 1 and 2 simultaneously locate on the interval, each locating so to maximize her market share given the location of the rival. Each seller's market share corresponds to the proportion of buyers who are located closer to her location than to the rival's location.

- (a) Both will locate at $1/2$.
- (b) One will locate at $1/4$ and the other at $3/4$.
- (c) One will locate at 0 and the other at 1 .
- (d) One will locate at $1/3$ and the other at $2/3$.

Question 18. In the context of previous question, suppose it is understood by all players that seller 3 will locate on $[0, 1]$ after observing the simultaneous location choices of sellers 1 and 2. Seller 3 aims to maximize market share given the locations of 1 and 2. The locations of sellers 1 and 2 are as follows:

- (a) Both will locate at $1/2$.
- (b) One will locate at $1/4$ and the other at $3/4$.
- (c) One will locate at 0 and the other at 1 .
- (d) One will locate at $1/3$ and the other at $2/3$.

Question 19. Consider an exchange economy with agents 1 and 2 and goods x and y . Agent 1 lexicographically prefers x to y . Agent 2's utility function is $\min\{x, y\}$. Agent 1's endowment is $(0, 10)$ and agent 2's endowment is $(10, 0)$. The competitive equilibrium price ratio, p_x/p_y , for this economy

- (a) can be any positive number
- (b) is greater than 1
- (c) is less than 1
- (d) does not exist

Question 20. A profit maximizing firm owns two production plants with cost functions $c_1(q) = \frac{q^2}{2}$ and $c_2(q) = q^2$, respectively. The firm is free to use either just one or both of the plants to achieve any given level of output. For this firm, the marginal cost curve

- (a) lies above the 45 degree line through the origin, for all positive output levels
- (b) lies below the 45 degree line through the origin, for all positive output levels
- (c) is the 45 degree line through the origin
- (d) none of the above

End of Part I. Proceed to Part II of the examination on the next page.

Part II

Instructions for Part II

- Attempt any **FOUR** questions, in the separate **answer book** provided to you for this part.
- Each question carries 15 marks. The allocation of marks is given in parentheses at the end of each question.
- If you attempt more than four questions, only the first four will be counted.
- Fill in your Name and Roll Number on the detachable slip of the answer book.

1. You are hired by the Labour Ministry to estimate wage elasticity of labour supply among casual labour in India. The Ministry conducted an all-India survey last year and collected individual level data on the days worked per week, weekly wage earnings (in rupees) and standard socio-demographic characteristics namely, age (in years), gender, education (coded in three categories: Less than Higher Secondary, Higher Secondary and Above Higher Secondary), caste (Scheduled Caste or Scheduled Tribe or not) and whether area of residence is urban or not, for a nationally representative sample of casual workers. The researcher at the Labour Ministry ran the following OLS regression:

$$\ln Days_i = Cons + \beta_1 \ln Wage_i + \beta_2 Age_i + \beta_3 Age_squared_i + \beta_4 Male_i + \beta_5 SC_ST_i + \beta_6 Higher_Secondary_i + \beta_7 Above_Higher_Secondary_i + \beta_8 Urban_i + \varepsilon_i$$

The output generated by the software package is given in table 1 provided at the end of the question paper.

Please answer the following questions:

- (a) Fill in the two cells marked BLANK1 and BLANK2 in table 1.
 - (b) Define p-value in the context of hypothesis testing in Econometrics. Further, on the top right of the table above (third row from top) you see $Prob > F$. For this reported p-value, define the Null Hypothesis.
 - (c) The researcher interprets the coefficient attached to $\ln Wage$ as the price elasticity of labour supply. However, this is not the correct interpretation. Using standard labour supply and demand curves, diagrammatically explain why this is an incorrect interpretation.
 - (d) Suggest an Instrument for $\ln Wage$ in order to estimate price elasticity of labour supply. (3,2,5,5)
2. Suppose you wanted to study the effect of years of schooling (a continuous variable), and of gender (a binary variable), on income among the employed population. You have access to a nationally representative dataset that asks employees the following question:

“What amount (in Rs. per month) do you earn?” The responses could be one of the following codes:

less than or equal to Rs.1000 — **1**;

Rs. 1001 to Rs. 1500 — **2**;

Rs. 1501 to Rs. 2000 — **3**;

Rs. 2001 to Rs. 2500 — **4**;

Rs. 2501 to Rs. 3000 — **5**;

more than Rs. 3000 — **6**.

Suppose you decide to model monthly income as follows:

$$y_i = \exp(\mathbf{x}_i\beta + u_i) \quad (1)$$

where y_i is the monthly income of individual i , \mathbf{x}_i are observable traits that include years of education and gender. u_i is the error and we assume:

$$u_i | \mathbf{x}_i, \mathbf{r}_i \sim N(0, \sigma^2) \quad (2)$$

where \mathbf{r}_i are the thresholds applicable to individual i depending on his response to the survey question given above.

(a) Suppose x_j is years of education. Write down the expression for

$$\frac{\partial E[(\ln y | \mathbf{x})]}{\partial x_j}$$

in terms of the parameters of the model given in expression (1) and (2) above.

(b) Consider semi elasticity of income w.r.t. education, given by,

$$\theta = \frac{\partial \ln[E(y | \mathbf{x})]}{\partial x_j}$$

How is this related to

$$\frac{\partial E[(\ln y | \mathbf{x})]}{\partial x_j}?$$

(c) How will you estimate the parameters β and σ^2 using Maximum Likelihood? Your answer should include the log likelihood function.

(d) Derive the expression for the exact percentage change in monthly income in moving from being male (female dummy=0) to being female (female dummy=1)?

(3,3,4,5)

3. Suppose we are interested in estimating the model:

$$(A) : \quad Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + u_i$$

where $E[u_i|X_1, X_2] = 0$ but instead we are forced to estimate

$$(B) : \quad Y_i = \beta_0 + \beta_1 X_{1i} + v_i$$

because of lack of data on variable X_2 .

- (a) Show that $\hat{\beta}_1$, the OLS estimator of β_1 in equation (B), is biased, whereas $\tilde{\beta}_1$, the estimator of β_1 in equation (A), is not.
- (b) Show that $Var(\hat{\beta}_1) \leq Var(\tilde{\beta}_1)$
- (c) Show that if we retain specification (B) on the condition that $MSE(\hat{\beta}_1) \leq MSE(\tilde{\beta}_1)$, that amounts to testing the null hypothesis $H_0 : \beta_2 = 0$ (against the alternative $H_A : \beta_2 \neq 0$) and retaining (B) if the associated t-statistic is ≤ 1 . Note: *MSE* is mean squared error.

(3,4,8)

- 4. (a) What is the difference between ‘neutrality’ and ‘superneutrality’ of money?
 - (b) “When agents have rational expectations, money will necessarily be neutral” - Do you agree? Explain your answer with suitable examples/counter-examples.
- (5,10)

5. Consider the strategic form game below.

| | | |
|---|--------|--------|
| | L | R |
| T | a, b | $c, 2$ |
| M | $1, 1$ | $1, 0$ |
| B | $3, 2$ | $0, 1$ |

where Player 1 has 3 strategies T,M,B and Player 2 has 2 strategies L,R, and a, b, c are numbers.

- (a) For what sets of values of a, b , and c is the outcome (T,L) a strictly dominant strategy equilibrium?
- (b) For what sets of values of a, b , and c is the outcome (T,L) a Nash Equilibrium?
- (c) For what sets of values of a and c does the mixed strategy of playing T and B with probabilities 1/2 each dominate the strategy M?
- (d) Is there any set of a, b, c for which the above is the strategic form induced by an extensive form game of perfect information? Explain your answer.
- (e) Consider an extensive form game in which Player 1 moves first. If he plays T, the game ends. If he plays M or B, Player 2 gets to move, without knowing which of M or B was played. Player 2 plays L or R, and then the game ends. For what values of a, b , and c will this extensive form game induce the strategic form game specified above?

- (f) Draw the extensive form specified in (e). For what sets of values of a, b , and c is the strategy profile (T,L) a sub-game perfect equilibrium?

(3,3,3,1,3,2)

6. Consider an economy with two agents. Agents live for two periods and in each period consume some quantity of food. Agents have identical preferences:

$$u(f_1, f_2) = \ln(1 + f_1) + \beta \ln(1 + f_2)$$

where β ($0 < \beta < 1$) is the discount factor, f_1 and f_2 denote food consumptions in period 1 and period 2 respectively. It is also known that Agent 1 can grow 1 unit of food only in the first period, while agent 2 can grow 1 unit of food only in the second period.

- (a) Identify the underlying resource allocation problem and find all Pareto optimal allocations. (5)
- (b) Now, suppose that food can be borrowed or lent in the first period at an interest rate r . For instance, if one borrows 1 unit of food in period one then she must return $(1 + r)$ units of food in period two. (5)
- (c) Model this economy in general equilibrium framework and find the equilibrium interest rate. How does the equilibrium interest rate change with respect to β ? Provide an economic justification. (5)

End of Part II