Theory of Externalities
Partial Equilibrium Analysis

Definition: An externality is present whenever the well–being of a consumer or the production possibilities of a firm are directly affected by the actions of another agent in the economy.

The choice of the word *directly* is important because it distinguishes between pecuniary externalities (external effects that *indirectly* affect actions of other agents — any effects that are mediated by prices) and technological externalities.

For example, a production externality is present if a downstream fishery's productivity is affected by the emissions of a nearby chemical plant, but not simply because the fishery's profitability is affected by the price of chemicals (which, in turn, is affected by the chemical plant's output of chemicals).

Pecuniary or *indirect* externalities are present in any competitive market and do not create any inefficiency.

Technological or *direct* externalities, on the other hand, do affect competitive equilibria.

Examples:

1. Consumption externality: music from neighbour's stereo

x_j^m individual *j*'s consumption of music

$U_j(x_j^m, y_j)$ individual *j*'s utility function

$U_i(x_i, x_j^m, y_i)$ individual *i*'s utility function

If $\frac{\partial U_i}{\partial x_j^m} < 0$, neighbour's music generates a negative consumption externality

If $\frac{\partial U_i}{\partial x_j^m} > 0$, neighbour's music generates a positive consumption externality

2. Production externality: industrial emissions

$y_i = f(x_i)$ firm *i*'s production function

$y_i = z_i$ pollution production function

$y_j = f(x_j, z_i)$ firm *j*'s production function

$\frac{\partial f(x_j, z_i)}{\partial z_i} < 0$ firm *i*'s emissions reduce firm *j*'s output, generating a negative externality

Lecture Plan

1. Simple Bilateral Externality Model — introduce concepts of externality and external costs

2. Derive Pareto Optimum.

3. Derive Competitive Equilibrium

4. Comparative Analysis — discuss suboptimality of competitive equilibrium

5. Centralized (standards, taxes) and decentralized solutions (environmental negotiation and marketable permits)

Simple Bilateral Partial Equilibrium Model

- assume perfect competition — two agents constitute a small part of the economy so their actions do not affect prices

- assume passive pollution victim — victim cannot reduce damages at margin by changing own input (only by shutting down)

- firm *1*: pulp and paper plant dumps effluents into river — effluents cause damages to downstream river users (loss in aquatic life due to reduction in dissolved oxygen, chemical level sufficiently high to be hazardous for swimming and drinking, scum, detergent suds, discolouration which degrades aesthetic beauty of water)

- firm *2*: hotel resort which provides water recreational activities (swimming, kayaking, windsurfing)

- effluents dumped by plant makes water unsafe for swimming and unattractive for other water recreation activities and, as a result, resort's business declines as effluents level rise — as river becomes more polluted, demand for water recreation falls and resort's revenues, hence profits fall

- plant does *not* bear costs it imposes on resort (or any other river users) — these costs are external to plant — do not directly affect plant's profits

Pulp and paper plant

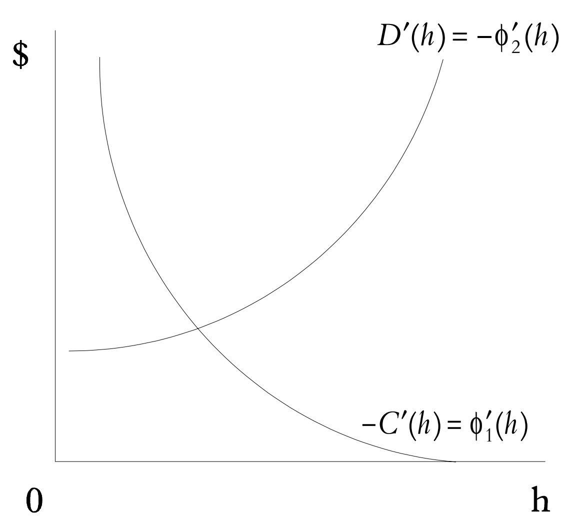
$\pi^P = \pi^p - C(h)$ plant's profits where π^p are zero pollution profits and h is pollution level

$$C(h) < 0 \; \forall \; h > 0 \Rightarrow \pi^P(h) > \pi^P(0) \; \forall \; h > 0$$
$$\Rightarrow C(a) > 0 \; \forall \; a = h_p - h > 0 \Rightarrow \text{abatement costs nonnegative}$$
$$C'(h) < 0, C''(h) > 0 \Rightarrow \pi_h^P > 0, \pi_{hh}^P < 0$$
$$\Rightarrow C'(a) > 0, C''(a) > 0 \Rightarrow \text{abatement costs increasing at increasing rate}$$

Hotel Resort

$\pi^H = \pi^h - D(h)$ hotel's profits where π^h are zero pollution profits

$D(h) \geq 0$ monetary value of damages are nonnegative
$D'(h) > 0$ marginal damages are increasing
$D''(h) > 0$ marginal damages are increasing at increasing rate



Aside: Mas–Colell notation

$$\phi_1(h) = \pi^p - C(h) \geq 0$$
$$\phi_1'(h) = -C'(h) > 0$$
$$\phi_1''(h) = -C''(h) < 0$$

$$\phi_2(h) = \pi^h - D(h) \geq 0$$
$$\phi_2'(h) = -D'(h) < 0$$
$$\phi_2''(h) = -D''(h) < 0$$

Pareto Optimum

Social planner will choose level of externality to maximize social welfare:

$$\max_{h} \quad W = \pi^P + \pi^H = \pi^p - C(h) + \pi^h - D(h)$$

Can rewrite problem as social cost minimization problem:

$$\min_{h} \quad C_s = C(h) + D(h)$$

$$\frac{\partial C_s}{\partial h} = C'(h) + D'(h) = 0$$

$$h^{\star}: \quad -C'(h) = D'(h)$$

Competitive Equilibrium

$$\max_{h} \quad \pi^P = \pi^p - C(h)$$

$$\frac{\partial \pi^P}{\partial h} = -C'(h) = 0$$

$$h_p: \quad -C'(h) = 0$$

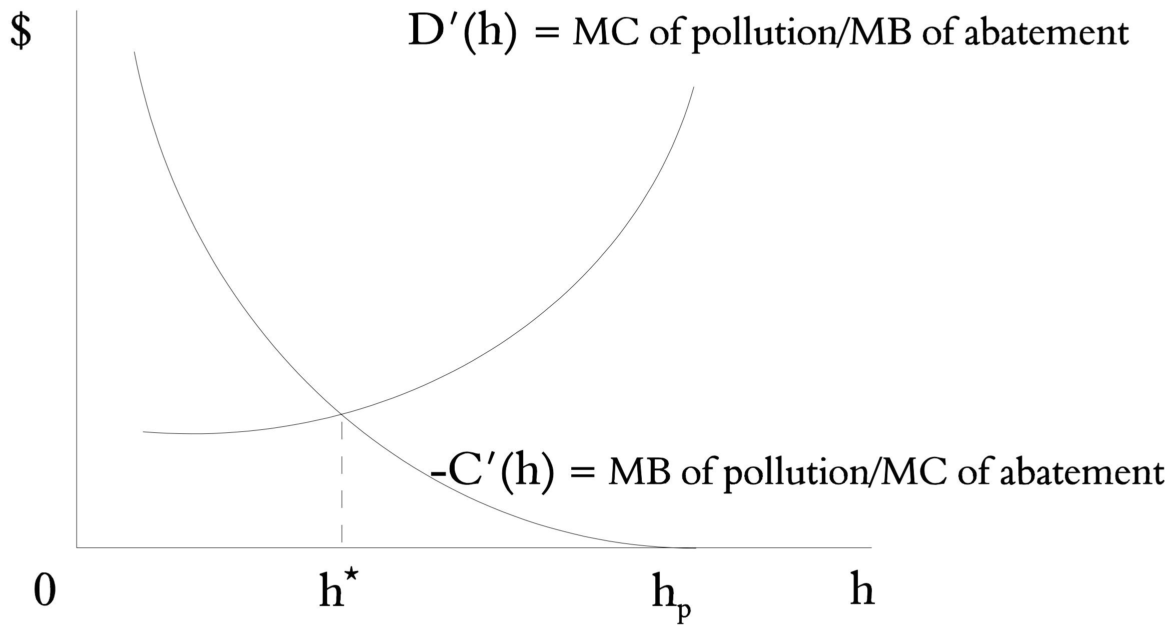
Clearly, $h_p > h^\star$ since environmental damage costs are not internalized at competitive equilibrium.

At $h_p > 0$, the plant imposes imposes excessive damages on the resort:

$$D(h_p) - D(h^\star) = \int_{h^\star}^{h_p} D'(h)dh > 0$$

Social losses will arise:

$$\Delta W = W(h_p) - W(h^\star) = -\int_{h^\star}^{h_p} C'(h) + D'(h)dh < 0$$



- can measure cost of uncontrolled pollution and benefit of (or WTP for) controlling pollution using $D'(h)$

- can measure benefit of uncontrolled pollution and cost of controlling pollution using $-C'(h)$

Key Results

1. $h_p > 0$ in general, optimality does not entail eliminating externality — zero pollution is not optimal — optimality requires balancing costs and benefits of pollution at the margin

2. $h_p > h^\star$ excessive level of pollution generated in a market economy — source of market failure is failure of economic agents to internalize full social costs of actions when making privately optimal decisions — failure to internalize external costs results in a breakdown of "invisible hand"

3. $D(h_p) > D(h^\star)$ excessive level of environmental degradation in market economy

4. $\Delta W = W(h_p) - W(h^\star) = -\int_{h^\star}^{h_p} C'(h) + D'(h)dh < 0$ social welfare loss arises

Centralized Solutions

1. Command–and–Control: Standard

   Set $\bar{h} = h^\star$.

2. Incentive–Based: Pigouvian tax

   - impose tax on externality–generating activity — pulp and paper plant must pay tax t per unit of h — what is optimal tax?

   $$t^\star = D'(h^\star) > 0$$

   - tax is set equal to marginal damages at Pareto optimum
   - tax forces polluter to "internalize" cost of actions imposed on others

$t^\star$ will induce Pareto optimum:

$$\max_{h} \quad \pi^P = \pi^p - C(h) - t^\star h$$

$$\frac{\partial \pi^P}{\partial h} = -C'(h) - t^\star = 0$$

$$h_t: \quad -C'(h) = t^\star \qquad (1)$$

Substituting $t^\star = D'(h^\star)$ into (1) yields

$$h_t: \quad -C'(h_t) = D'(h^\star) \Rightarrow h_t = h^\star$$

hence $t^\star$ will induce the Pareto optimum.

First Welfare Theorem: CE are PO

iff

- complete set of markets exist
- property rights are well–defined
- no externalities
- no strategic behaviour
- perfect information

Three conditions must be met for Pareto optimality:

- Efficient consumption $MRS_{1,2}^{A} = MRS_{1,2}^{B}$
- Efficient production $MRTS_{1,2}^{1} = MRTS_{1,2}^{2}$
- Efficient product mix $MRS_{1,2} = MRT_{1,2}$

Theory of Externalities

General Equilibrium Model

Consider an economy with

x_i two consumption goods indexed by $i = 1,2$ consumed by representative consumer

y_i^k two production goods indexed by $i = 1,2$, produced by two firms indexed by $k = 1,2$, where superscripts denote identity of firm and subscripts denote type of good

There are two externalities affecting firm *2*:

one generated by consumer's consumption of good *1*, x_1

one generated by production of good *1* by firm *1*, y_1^1

Illustrative example: pollution of a river by city inhabitants (municipal sewage) and a firm (industrial effluents) that affect a downstream water–using firm

Consumer

$U(x_1, x_2)$	$U(\bullet)$ is differentiable, increasing and strictly quasi–concave
(ω_1, ω_2)	initial endowment

Producers

$y_1^1 = f^1(y_2^1)$	firm *1*'s production technology, where y_1^1 is firm *1*'s output, f^1 is differentiable and concave and y_2^1 is firm *1*'s input (negative output) of good *2*
$y_2^2 = f^2(y_1^2, y_1^1, x_1)$	firm *2*'s production technology, where y_2^2 is firm *2*'s output, f^2 is differentiable and concave, y_1^2 is firm *2*'s input (negative output) of good *1*, y_1^1 is firm *1*'s output of good *1* and x_1 is consumer's consumption of good *1*

$y_2^1, y_1^2 =$ inputs $=$ negative outputs

Since inputs have negative sign $\Rightarrow \dfrac{\partial f^1}{\partial y_2^1} \leq 0, \dfrac{\partial f^2}{\partial y_1^2} \leq 0$

Hence, firm *1*'s production of good *1* and consumer's consumption of good *1* both enter firm *2*'s production function, generating two negative externalities.

Pareto Optimal Allocation

$$\max_{\{x_1, x_2, y_1^1, y_2^1, y_2^2, y_1^2\}} U(x_1, x_2) \quad \text{s.t.} \quad \left.\begin{array}{l} y_1^1 + y_1^2 + \omega_1 - x_1 \geq 0 \\ y_2^1 + y_2^2 + \omega_2 - x_2 \geq 0 \end{array}\right\} \text{scarcity contraints}$$

$$\left.\begin{array}{l} -y_1^1 + f^1(y_2^1) \geq 0 \\ -y_2^2 + f^2(y_1^2, y_1^1, x_1) \geq 0 \end{array}\right\} \text{technological constraints}$$

We can ignore the inequality constraints because of the convexity and concavity assumptions — these assumptions are sufficient conditions for interior solutions.

The Lagrangian can be written as

$$\max_{\{x_1, x_2, y_1^1, y_2^1, y_2^2, y_1^2\}} L = U(x_1, x_2) + \lambda_1\left(y_1^1 + y_1^2 + \omega_1 - x_1\right) + \lambda_2\left(y_2^1 + y_2^2 + \omega_2 - x_2\right)$$
$$+\mu_1\left(-y_1^1 + f^1(y_2^1)\right) + \mu_2\left(-y_2^2 + f^2(y_1^2, y_1^1, x_1)\right)$$

$$\frac{\partial L}{\partial x_1} = \frac{\partial U}{\partial x_1} - \lambda_1 + \mu_2 \frac{\partial f^2}{\partial x_1} = 0 \tag{1}$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial U}{\partial x_2} - \lambda_2 = 0 \tag{2}$$

$$\frac{\partial L}{\partial y_1^1} = \lambda_1 - \mu_1 + \mu_2 \frac{\partial f^2}{\partial y_1^1} = 0 \tag{3}$$

$$\frac{\partial L}{\partial y_2^1} = \lambda_2 + \mu_1 \frac{\partial f^1}{\partial y_2^1} = 0 \tag{4}$$

$$\frac{\partial L}{\partial y_2^2} = \lambda_2 - \mu_2 = 0 \tag{5}$$

$$\frac{\partial L}{\partial y_1^2} = \lambda_1 + \mu_2 \frac{\partial f^2}{\partial y_1^2} = 0 \tag{6}$$

Solution

Eliminate μ_1 and μ_2. Express efficiency conditions in terms of $\frac{\lambda_1}{\lambda_2}$.

Efficient consumption

Rewrite equations (1) and (2). Substitute $\lambda_2 = \mu_2$ from equation (5):

$$\lambda_1 = \frac{\partial U}{\partial x_1} + \lambda_2 \frac{\partial f^2}{\partial x_1} \qquad (1)$$

$$\lambda_2 = \frac{\partial U}{\partial x_2} \qquad (2)$$

Substituting equation (2) into equation (1) and dividing yields:

$$\frac{\lambda_1}{\lambda_2} = \frac{\frac{\partial U}{\partial x_1} + \frac{\partial U}{\partial x_2}\frac{\partial f^2}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} + \frac{\partial f^2}{\partial x_1} \qquad (7)$$

where the RHS of equation (7) is the social $MRS_{1,2}$. The social $MRS_{1,2}$ takes into account **all** effects of consumption activities (marginal benefits to consumer as well as marginal external costs imposed on others as a result of consumption activity).

The social $MRS_{1,2}$ must take into account that substituting one unit of good *1* for good *2* affects the production of good *2* by $\frac{\partial f^2}{\partial x_1}$ and therefore the individual's utility level is changed by $\frac{\partial U}{\partial x_2}\frac{\partial f^2}{\partial x_1}$.

Efficient production for firm *1*

Rewrite equations (3) and (4). Substitute $\lambda_2 = \mu_2$ from equation (5):

$$\lambda_1 = \mu_1 - \lambda_2 \frac{\partial f^2}{\partial y_1^1} \qquad (3)$$

$$\lambda_2 = -\mu_1 \frac{\partial f^1}{\partial y_2^1} \qquad (4)$$

Substitute equation (4) into equation (3). Dividing the equations yields:

$$\frac{\lambda_1}{\lambda_2} = -\frac{1 + \frac{\partial f^1}{\partial y_2^1}\frac{\partial f^2}{\partial y_1^1}}{\frac{\partial f^1}{\partial y_2^1}} = -\frac{1}{\frac{\partial f^1}{\partial y_2^1}} - \frac{\partial f^2}{\partial y_1^1} \qquad (8)$$

The RHS of equation (8) is the social $MRT_{2,1}$. By using one additional unit of good *2* as an input, firm *1* produces $\frac{\partial f^1}{\partial y_2^1}$ and consequently affects the production of good *2* by $\frac{\partial f^2}{\partial y_1^1}\frac{\partial f^1}{\partial y_2^1}$.

Efficient production for firm *2*

Rewrite equations (5) and (6):

$$\lambda_2 = \mu_2 \qquad (5)$$

$$\lambda_1 = -\mu_2 \frac{\partial f^2}{\partial y_1^2} \qquad (6)$$

Dividing equation (6) by equation (5) yields:

$$\frac{\lambda_1}{\lambda_2} = -\frac{\partial f^2}{\partial y_1^2} \qquad (9)$$

Note that firm *2*'s production activity does not generate an externality, hence the RHS of equation (9), the social $MRT_{1,2}$, equals the private $MRT_{1,2}$.

Pareto Optimal Allocation

$$\frac{\lambda_1}{\lambda_2} \quad = \quad \frac{\dfrac{\partial U}{\partial x_1}}{\dfrac{\partial U}{\partial x_2}} + \frac{\partial f^2}{\partial x_1} \quad = \quad -\frac{1}{\dfrac{\partial f^1}{\partial y_2^1}} - \frac{\partial f^2}{\partial y_1^1} \quad = \quad -\frac{\partial f^2}{\partial y_1^2}$$

$$\begin{array}{c} \textit{relative} \\ \textit{scarcity} \\ \textit{prices} \end{array} \quad = \quad \textit{social}\ MRS_{1,2} \quad = \quad \textit{social}\ MRT_{2,1} \quad = \quad \textit{social}\ MRT_{1,2}$$

Key results

- optimal organization of economy does not necessarily require total elimination of externalities, even when they are negative — consumption and production good *1* affects production of good *2* negatively but optimality does not require eliminating production of good *1*

- in general, zero pollution is not optimal

- external costs (or *direct* effects of one agent's actions upon others) must be internalized in consumption and production decisions

Competitive Equilibrium

Consumer's decision problem

$$\max_{x_1, x_2} \quad U(x_1, x_2) \text{ s.t. } B = p_1\omega_1 + p_2\omega_2 + \pi(p_1, p_2) = p_1 x_1 + p_2 x_2$$

where $p_1\omega_1 + p_2\omega_2$ is the value of the consumer's initial endowment and $\pi(p_1, p_2)$ are industrial profits

$$\max_{x_1, x_2} \quad L = U(x_1, x_2) + \lambda(B - p_1 x_1 - p_2 x_2)$$

$$\frac{\partial L}{\partial x_1} = \frac{\partial U}{\partial x_1} - \lambda p_1 = 0$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial U}{\partial x_2} - \lambda p_2 = 0$$

Eliminating λ yields:

$$\frac{p_1}{p_2} = \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}}$$

$$\text{relative prices} = \text{private } MRS_{1,2}$$

Firm *1*'s decision problem

$$\max_{y_2^1} \quad \pi^1 = p_1 y_1^1 + p_2 y_2^1 \text{ s.t. } y_1^1 = f^1(y_2^1)$$

$$\max_{y_2^1} \quad \pi^1 = p_1 f^1(y_2^1) + p_2 y_2^1$$

$$\frac{\partial \pi^1}{\partial y_2^1} = p_1 \frac{\partial f^1}{\partial y_2^1} + p_2 = 0$$

$$p_1 = -\frac{p_2}{\frac{\partial f^1}{\partial y_2^1}}$$

$$\frac{p_1}{p_2} = -\frac{1}{\frac{\partial f^1}{\partial y_2^1}}$$

Firm *2*'s decision problem

$$\max_{y_1^2} \quad \pi^2 = p_2 f^2(y_1^2, y_1^1, x_1) + p_1 y_1^2$$

$$\frac{\partial \pi^2}{\partial y_1^2} = p_2 \frac{\partial f^2}{\partial y_1^2} + p_1 = 0$$

$$p_2 = -\frac{p_1}{\frac{\partial f^2}{\partial y_1^2}}$$

$$\frac{p_1}{p_2} = -\frac{\partial f^2}{\partial y_1^2}$$

Competitive Equilibrium

A competitive equilibrium is a vector of prices (p_1,p_2) and an allocation $\left(x_1, x_2, y_1^1, y_2^1, y_2^2, y_1^2\right)$ such that

- firms' profits and consumer's utility are maximized (first–order necessary and sufficient conditions are satisfied)

- supply and demand is equalized in both markets

At a competitive equilibrium, self–interest maximization leads each agent to equate his/her private marginal rate of substitution or transformation to the price ratio, resulting in the equalization of private rates:

$$\frac{p_1}{p_2} = \frac{\dfrac{\partial U}{\partial x_1}}{\dfrac{\partial U}{\partial x_2}} = -\frac{1}{\dfrac{\partial f^1}{\partial y_2^1}} = -\frac{\partial f^2}{\partial y_1^2}$$

$$\begin{array}{c}\textit{relative}\\ \textit{prices}\end{array} = \textit{private } MRS_{1,2} = \textit{private } MRT_{2,1} = \textit{private } MRT_{1,2}$$

Comparing Competitive Equilibrium and Pareto Optimum

In contrast, a Pareto optimal allocation requires equalization of social marginal rates of substitution and transformation to relative scarcity or shadow prices.

In general, CE with externalities not PO!

- divergence of private and social valuations
- economic decisions too decentralized at competitive equilibrium
- agents generating negative externality will produce/consume too much
- prices of negative externality–generating production or consumption goods are too low (since full social costs of economic activity not reflected in market prices)

Decentralized Solutions

Optimal Taxation

Is there a tax structure that will sustain a competitive equilibrium as a Pareto optimum? Can we find a set of taxes for polluters and victim damage or compensation payments that will induce consumers and firms to choose Pareto optimal levels of economic activities?

Define: t_c garbage tax per unit of good *1* consumed

t_f emission tax per unit of good *1* produced

τ damage compensation per unit of good *2* produced

Assume taxes collected are redistributed to consumer as lump–sum transfer T.

Consumer's decision problem

$$\max_{x_1, x_2} \quad L = U(x_1, x_2) + \lambda\left(p_1\omega_1 + p_2\omega_2 + T + \pi(p_1, p_2) - (p_1 + t_c)x_1 - p_2 x_2\right)$$

$$\frac{\partial L}{\partial x_1} = \frac{\partial U}{\partial x_1} - \lambda(p_1 + t_c) = 0$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial U}{\partial x_2} - \lambda p_2 = 0$$

Eliminating λ yields:

$$\frac{p_1}{p_2} = \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} - \frac{t_c}{p_2}$$

Firm *1*'s decision problem

$$\max_{y_2^1} \quad \pi^1 = (p_1 - t_f)\, f^1(y_2^1) + p_2 y_2^1$$

$$\frac{\partial \pi^1}{\partial y_2^1} = (p_1 - t_f)\frac{\partial f^1}{\partial y_2^1} + p_2 = 0$$

$$p_1 = -\frac{p_2}{\dfrac{\partial f^1}{\partial y_2^1}} + t_f$$

$$\frac{p_1}{p_2} = -\frac{1}{\dfrac{\partial f^1}{\partial y_2^1}} + \frac{t_f}{p_2}$$

Firm *2*'s decision problem

Damage compensation rates depend on victim's activity level. For example, if exogenous shift in demand leads to an increase in output, damage caused by firm *1* and consumers will necessarily increase and hence compensation payment must increase.

$$\max_{y_1^2} \quad \pi^2 = (p_2 + \tau) f^2(y_1^2, y_1^1, x_1) + p_1 y_1^2$$

$$\frac{\partial \pi^2}{\partial y_1^2} = (p_2 + \tau)\frac{\partial f^2}{\partial y_1^2} + p_1 = 0$$

$$p_2 = -\frac{p_1}{\frac{\partial f^2}{\partial y_1^2}} - \tau$$

$$\frac{p_1}{p_2} = -\frac{\partial f^2}{\partial y_1^2} - \frac{\tau}{p_2}\frac{\partial f^2}{\partial y_1^2}$$

Competitive Equilibrium with Taxes and Compensation

$$\frac{p_1}{p_2} = \frac{\dfrac{\partial U}{\partial x_1}}{\dfrac{\partial U}{\partial x_2}} - \frac{t_c}{p_2} = -\frac{1}{\dfrac{\partial f^1}{\partial y_2^1}} + \frac{t_f}{p_2} = -\left(1 + \frac{\tau}{p_2}\right)\frac{\partial f^2}{\partial y_1^2}$$

$$t_c^* = -p_2 \frac{\partial f^2}{\partial x_1} \qquad t_f^* = -p_2 \frac{\partial f^2}{\partial y_1^1} \qquad \tau^* = 0$$

- $\exists$ set of taxes and compensation payments $\left\{t_c^*, t_f^*, \tau^*\right\}$ which will sustain a competitive equilibrium which is Pareto optimal

- optimal taxes are standard Pigouvian taxes — taxes are set equal to the value of the marginal damage of economic activities evaluated at the Pareto optimal level of these activities — taxes are based on a “polluters’ pay principle”

- Pigouvian taxes are asymmetric — no subsidization or compensation of victims necessary to sustain Pareto optimum provided t_c and t_f set optimally

- market prices are now equal to shadow prices in social planner’s problem

Creation of markets by specifying property rights

Social planner wishes to establish a complete set of competitive markets which incorporates externalities — must assign property rights.

Suppose that social planner assigns firm *2* the right to an unpolluted river. The initial assignment of property rights creates two pollution rights markets:

Consumer must purchase the right to pollute from firm *2*:

p_1^{12} price of right for consumer to pollute one unit

Firm *1* must also purchase the right to pollute from firm *2:*

q_1^{12} price of right for firm *1* to pollute one unit

Consumer's decision problem

$$\max_{x_1, x_2} \quad U(x_1, x_2) \quad \text{s.t. } p_1\omega_1 + p_2\omega_2 + \pi(p_1, p_2) = p_1 x_1 + p_1^{12} x_1^{12} + p_2 x_2$$
$$x_1^{12} = x_1$$

where x_1^{12}, the quantity of rights demanded by consumer, must equal x_1, the number of units of pollution created.

$$\max_{x_1, x_2} \quad L = U(x_1, x_2) + \lambda\left(B - (p_1 + p_1^{12})x_1 - p_2 x_2\right)$$

$$\frac{\partial L}{\partial x_1} = \frac{\partial U}{\partial x_1} - \lambda(p_1 + p_1^{12}) = 0$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial U}{\partial x_2} - \lambda p_2 = 0$$

Eliminating λ yields:

$$\frac{p_1}{p_2} = \frac{\dfrac{\partial U}{\partial x_1}}{\dfrac{\partial U}{\partial x_2}} - \frac{p_1^{12}}{p_2}$$

Firm *1*'s decision problem

$$\max_{y_2^1} \quad \pi^1 = p_1 y_1^1 + p_2 y_2^1 - q_1^{12} y_1^{12} \quad \text{s.t. } y_1^1 = f^1(y_2^1)$$
$$y_1^{12} = y_1^1$$

where y_1^{12} is the quantity of pollution rights demanded by firm *1* and y_1^1 is firm *1*'s level of pollution. Note that, institutionally, firm *1* is constrained to purchase as many rights as it creates units of pollution.

$$\max_{y_2^1} \quad \pi^1 = (p_1 - q_1^{12}) f^1(y_2^1) + p_2 y_2^1$$

$$\frac{\partial \pi^1}{\partial y_2^1} = (p_1 - q_1^{12}) \frac{\partial f^1}{\partial y_2^1} + p_2 = 0$$

$$p_1 = -\frac{p_2}{\frac{\partial f^1}{\partial y_2^1}} + q_1^{12}$$

$$\frac{p_1}{p_2} = -\frac{1}{\frac{\partial f^1}{\partial y_2^1}} + \frac{q_1^{12}}{p_2}$$

Firm *2*'s decision problem

Firm *2* will supply quantities of pollution rights $\hat{y}_1^{12}, \hat{x}_1^{12}$ at prices q_1^{12}, p_1^{12}, respectively.

$$\max_{y_1^2, \hat{y}_1^{12} \hat{x}_1^{12}} \quad \pi^2 = p_2 y_2^2 + p_1 y_1^2 + q_1^{12} \hat{y}_1^{12} + p_1^{12} \hat{x}_1^{12} \text{ s.t. } y_2^2 = f^2(y_1^2, \hat{y}_1^{12}, \hat{x}_1^{12})$$

$$\max_{y_1^2, \hat{y}_1^{12} \hat{x}_1^{12}} \quad \pi^2 = p_2 f^2(y_1^2, \hat{y}_1^{12}, \hat{x}_1^{12}) + p_1 y_1^2 + q_1^{12} \hat{y}_1^{12} + p_1^{12} \hat{x}_1^{12}$$

$$\frac{\partial \pi^2}{\partial y_1^2} = p_2 \frac{\partial f^2}{\partial y_1^2} + p_1 = 0 \Rightarrow p_1 = -p_2 \frac{\partial f^2}{\partial y_1^2} \Rightarrow \frac{p_1}{p_2} = -\frac{\partial f^2}{\partial y_1^2}$$

$$\frac{\partial \pi^2}{\partial \hat{y}_1^{12}} = p_2 \frac{\partial f^2}{\partial \hat{y}_1^{12}} + q_1^{12} = 0 \Rightarrow q_1^{12} = -p_2 \frac{\partial f^2}{\partial \hat{y}_1^{12}} \Rightarrow \frac{q_1^{12}}{p_2} = -\frac{\partial f^2}{\partial \hat{y}_1^{12}}$$

$$\frac{\partial \pi^2}{\partial \hat{x}_1^{12}} = p_2 \frac{\partial f^2}{\partial \hat{x}_1^{12}} + p_1^{12} = 0 \Rightarrow p_1^{12} = -p_2 \frac{\partial f^2}{\partial \hat{x}_1^{12}} \Rightarrow \frac{p_1^{12}}{p_2} = -\frac{\partial f^2}{\partial \hat{x}_1^{12}}$$

Note that firm *2* will sell the right to pollute so as to equate the market value of the property right (*MB* of selling the right) to the value of the damage created by selling the right (*MC* of selling the right).

Competitive equilibrium with marketable pollution permits

$$\frac{p_1}{p_2} = \frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} - \frac{p_1^{12}}{p_2} = -\frac{1}{\frac{\partial f^1}{\partial y_2^1}} + \frac{q_1^{12}}{p_2} = -\frac{\partial f^2}{\partial y_1^2}$$

Definition of competitive equilibrium requires market–clearing in both pollution permit markets:

$$\hat{x}_1^{12} = x_1^{12} = x_1$$
$$\hat{y}_1^{12} = y_1^{12} = y_1^1$$

From firm *2*'s profit–maximization problem and market–clearing conditions we have that

$$p_1^{12} \quad = \quad -p_2 \frac{\partial f^2}{\partial \hat{x}_1^{12}} = -p_2 \frac{\partial f^2}{\partial x_1^{12}} \quad = \quad t_c^*$$

$$q_1^{12} \quad = \quad -p_2 \frac{\partial f^2}{\partial \hat{y}_1^{12}} = -p_2 \frac{\partial f^2}{\partial y_1^{12}} \quad = \quad t_f^*$$

permit prices	=	marginal damage at Pareto optimum	=	Pigouvian taxes

Substituting market–clearing permit prices into competitive equilibrium conditions will yield conditions for Pareto optimal allocation.

- by defining property rights, social planner can create missing markets
- can sustain a competitive equilibrium as Pareto optimum with a complete set of markets!

Marketable Permits versus Taxes

- two decentralized solutions to externality problem
  - price set by central authority in tax regime
  - price determined by market forces in marketable permit system — economic agents find optimal price of pollution in decentralized, atomistic economy — "invisible hand" works provided there is a complete set of markets
- both solutions force economic agents to fully internalize costs of actions
- solutions have identical allocative consequences but different distributional consequences

Caveats

- models assume externalities the only source of institutional failure
- other possible sources of institutional failure
  - market failure
    - transaction costs — administrative costs may differ
    - strategic behaviour
    - trading restrictions
  - regulatory failure
    - other distortionary policies may be in place such as subsidization of externality–generating activity
    - incomplete information — regulator must know MB and MD curves exactly to set policies efficiently
    - incomplete enforcement — models assumes full compliance — highly unlikely regulator has the resources and knowledge to design perfect *ex–post* governance structure
    - regulatory capture
  - global failure