Public Goods

Private versus Public Goods

A private good (bread) exhibits the following two properties:

exclusive: A good is *exclusive* if once you have purchased a good, then you can exclude others from consuming it.

rival: A good is *rival* in consumption, in the sense that once someone buys a loaf of bread and consumes it, then that precludes you from consuming that same loaf of bread.

- A rival good is *depletable*. A technical consequence of depletability is that consumption of additional amounts of rival goods involve some marginal costs of production.

A public good (air quality) may exhibit the following two properties:

nonexclusive: A good is *nonexclusive* if no one can be excluded from benefiting from or consuming the good once it is produced. An implication of nonexclusivity is that goods can be enjoyed without direct payment.

nonrivalrous: One person's consumption of a good does not diminish the amount or quality available for others.

- A nonrival good is *nondepletable*. A technical consequence of nondepletability is that the marginal cost of providing a nonrival good to an additional consumer is zero.

- All public goods exhibit the nonexcludability property but they do not necessarily exhibit the nonrivalrous property.

<table>
<tr><td></td><td>nonrival</td><td>rival</td></tr>
<tr><td>excludable</td><td>• water pollution in small body of water, indoor air pollution</td><td>private good</td></tr>
<tr><td>nonexcludable</td><td>pure public good/bad
• users neither interfere with each other nor increase good's usefulness to each other (free–rider problem)
• biodiversity, greenhouse gases
• noise, defence, radio signal</td><td>congestible public good/bad
• users affect good's usefulness to others — mutual interference of users creates negative externality (free–access problem)
• ocean fishery, parks
• bridge, highway</td></tr>
</table>

Aggregate Demand Curves for Private and Public Goods

1. Private Goods

- To construct aggregate demand curve for a private good we must find total quantity demanded by all individuals at different price levels. To do so, we aggregate or sum the quantity demanded by each individual demand at a given price.

    - Thus, aggregate demand curves for private goods are derived by **horizontally summing** individual demand curves.

2. Public Goods

- To construct aggregate demand curve for a public good we must find total willingness to pay by all individuals for all possible quantity levels. To do so, we aggregate or sum the willingness to pay by each individual demand at a given quantity level.

    - Thus, aggregate demand curves for public goods are derived by **vertically summing** individual demand curves. Vertical summation is necessary because of *nonexcludability* — everyone simultaneously consumes the same amount of the public good.

Notation

I consumers, $i = 1,\ldots,I$

q public good (nonexclusive, nonrival)

y_i consumer i's consumption of numeraire good (exclusive, rival)

$\phi_i(q, y_i) = u_i(q) + y_i$ where $u_i' > 0, u_i'' < 0 \ \forall \ q \geq 0$

- assuming quasilinear preferences implies quantity of public goods doesn't affect prices of traded private goods

- marginal utility derived from consumption of public good positive but decreasing

c marginal cost of public good

$\omega = cq + y$ resource constraint, where $\omega = \sum_{i=1}^{I} \omega_i, y = \sum_{i=1}^{I} y_i$

Pareto–efficient allocation

$$\max_{q} \quad W = \sum_{i=1}^{I} \phi_i(q, y_i) \text{ s.t. } \omega = cq + y$$

$$\max_{q} \quad W = \sum_{i=1}^{I} u_i(q) + \omega - cq$$

$$\frac{\partial W}{\partial q} = \sum_{i=1}^{I} \frac{\partial u_i(q)}{\partial q} - c = 0$$

$$q^*: \quad \sum_{i=1}^{I} \frac{\partial u_i(q)}{\partial q} = c \qquad (1)$$

Equation (1) is the Samuelson condition for optimal public good provision. It states that the public good should be provided up to the point where the social marginal utility or benefit from provision equals the social marginal cost:

$$q^*: \quad MB_s = MC_s \text{ where } MB_s = \sum_{i=1}^{I} \frac{\partial u_i(q)}{\partial q} = \sum_{i=1}^{I} MB_i \,.$$

If preferences are not quasilinear, then Samuelson condition is written as:

$$q^*: \quad \frac{\sum_{i=1}^{I} \frac{\partial \phi_i(q, y_i)}{\partial q}}{\sum_{i=1}^{I} \frac{\partial \phi_i(q, y_i)}{\partial y_i}} = c$$

Or, alternatively, $q^*: \quad \sum_{i=1}^{I} MRS_{q,y}^{i} = MRT_{q,y}$

Private Provision of Public Good

- public good provided through private purchases by consumers — private purchases are voluntary contributions of the public good
- each consumer chooses how much of the public good to provide, $q_i \geq 0$
- $q = \sum_{i=1}^{I} q_i$ level of public good provided by voluntary contributions

Consumer *i* will choose to provide the level of the public good which maximizes their utility subject to their budget constraint:

$$\max_{q_i} \quad \phi_i\left(q_i + \sum_{\substack{j=1 \\ j\neq i}}^{I} q_j, y_i \right) \text{ s.t. } \omega_i = cq_i + y_i$$

- note that consumer *i*'s utility depends upon their own contribution as well as contributions by others — since public good is nonexclusive, consumer *i* cannot be excluded from enjoying the utility or benefits they derive from voluntary contributions by others
- assume Nash behaviour — consumer *i* takes as given the amount of good purchased by other consumers — consumer *i* assumes that other consumers will each contribute q_j

Voluntary Contribution Decision

Substitute budget constraint into quasilinear preferences and solve:

$$\max_{q_i} \quad u_i\left(q_i + \sum_{\substack{j=1 \\ j\neq i}}^{I} q_j\right) + \omega_i - cq_i$$

$$\frac{\partial \phi_i(q, y_i)}{\partial q_i} = \frac{\partial u_i(q)}{\partial q_i} - c \leq 0 \text{ with equality if } q_i > 0$$

$$\hat{q}_i: \quad \frac{\partial u_i}{\partial q_i} = c \qquad (2)$$

Equation (2) states that each consumer only considers their private rather than the social benefits of public good provision when choosing their own contribution — $\hat{q}_i: \quad MB_i = MC$. Hence, individuals do not internalize external benefits accruing to others from their contributions.

If preferences are not quasilinear, then voluntary contribution condition is written as:

$$\hat{q}_i: \quad \frac{\dfrac{\partial u_i(q, y_i)}{\partial q_i}}{\dfrac{\partial u_i(q, y_i)}{\partial y_i}} = c$$

Or, alternatively, $\hat{q}_i: \quad MRS^i_{q,y} = MRT_{q,y}$

Comparing Pareto Optimum and Voluntary Contribution Equilibrium

Let $\hat{q} = \sum_{i=1}^{I} \hat{q}_i$ be equilibrium level of public good provided by voluntary contributions.

Will the public good be provided optimally in a market economy?

Comparing equations (1) and (2),

$$q^*: \quad \sum_{i=1}^{I} \frac{\partial u_i(q)}{\partial q} = c \qquad (1)$$

$$\hat{q}_i: \quad \frac{\partial u_i}{\partial q_i} = c \qquad (2)$$

To determine whether $\hat{q} \overset{>}{\underset{<}{=}} q^*$, we must determine if there is a divergence between social marginal benefits and costs at the voluntary contribution equilibrium $\hat{q}$.

Define the indicator function:

$$\delta_i = \begin{cases} 1 & \text{if } \hat{q}_i > 0 \\ 0 & \text{if } \hat{q}_i = 0 \end{cases}$$

From equation (2), it must be true that the following condition holds at a competitive equilibrium with $\hat{q} > 0$:

$$\sum_{i=1}^{I} \delta_i \left[\frac{\partial u_i(\hat{q})}{\partial q} - c \right] = 0 \qquad (3)$$

Expanding and rearranging equation (3) yields

$$\sum_{i=1}^{I} \delta_i \frac{\partial u_i(\hat{q})}{\partial q} = c \sum_{i=1}^{I} \delta_i$$

Hence,

$$\frac{\sum_{i=1}^{I} \delta_i \frac{\partial u_i(\hat{q})}{\partial q}}{\sum_{i=1}^{I} \delta_i} = c \qquad (4)$$

The LHS of equation (4) is the *average* MB_i or $\overline{MB}$. Equation (4) simply states that, at a competitive equilibrium, $\overline{MB} = MC$.

Since $\frac{\partial u_i}{\partial q} > 0$ and $c > 0$, equation (4) implies that whenever $I > 1$ and $\hat{q} > 0$ (so that $\delta_i = 1$ for some *i*) we have that

$$\hat{q}: \quad \sum_{i=1}^{I} \frac{\partial u_i(\hat{q})}{\partial q} > c \qquad (5)$$

To determine whether $\hat{q} \gtreqless q^*$, compare equations (1) and (5):

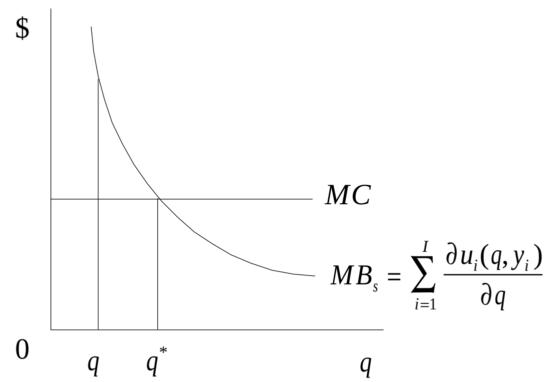
$$q^*: \quad \sum_{i=1}^{I} \frac{\partial u_i(q)}{\partial q} = c \qquad (1)$$

$$\hat{q}: \quad \sum_{i=1}^{I} \frac{\partial u_i(\hat{q})}{\partial q} > c \qquad (5)$$

Clearly, whenever
$q^* > 0$ and $I > 1$,

$$\hat{q} < q^*$$

since $MB_s(\hat{q}) > MC(\hat{q})$



- underprovision of public good in market economy

- positive externalities generated from private provision — failure of each consumer to consider benefits accruing to others from his/her public good provision

Why must equation (5) hold? From equation (4), we know that $\overline{MB} = MC$ at the voluntary contribution equilibrium:

$$\frac{\sum_{i=1}^{I} \delta_i \frac{\partial u_i(\hat{q})}{\partial q}}{\sum_{i=1}^{I} \delta_i} = c \qquad (4)$$

Clearly, when $I > 1$, the sum of the individual MB_i will always exceed the average:

$$\sum_{i=1}^{I} \frac{\partial u_i(\hat{q})}{\partial q} > \frac{\sum_{i=1}^{I} \delta_i \frac{\partial u_i(\hat{q})}{\partial q}}{\sum_{i=1}^{I} \delta_i}$$

$$\sum_{i=1}^{I} MB_i > \overline{MB} \text{ or, alternatively, } MB_s > \overline{MB}$$

For example, if $\delta_i = 1 \; \forall \; i$, then

$$\sum_{i=1}^{I} \frac{\partial u_i(\hat{q})}{\partial q} > \frac{\sum_{i=1}^{I} \frac{\partial u_i(\hat{q})}{\partial q}}{I}$$

Thus, equation (5) must hold at a competitive equilibrium and hence, the public good will be underprovided.

Free–rider Problem

Order consumers according to their individual marginal utility or benefit functions:

$$\frac{\partial u_1}{\partial q} < \ldots < \frac{\partial u_I}{\partial q} \quad \forall \ q > 0$$

It must be true that $\frac{\partial u_i(\hat{q})}{\partial q} = c$ holds for only 1 consumer — consumer *I*.

(Note that if $\frac{\partial u_1(\hat{q})}{\partial q} = c \Rightarrow \frac{\partial u_I(\hat{q})}{\partial q} > c$.)

Thus, only the consumer who derives the highest marginal utility from the public good will provide it — consumers *1* to *I - 1* will set purchases equal to zero in equilibrium, that is, they will free–ride on consumer *I*'s contribution. Hence, at the voluntary contribution equilibrium,

$$\hat{q}: \quad \frac{\partial u_I(\hat{q})}{\partial q} = c$$

Free–rider problem

Voluntary contribution equilibrium is socially inefficient because private valuations of the public good diverge from society's valuation and because of the nonexclusive nature of public goods:

1. individuals do not internalize the benefits that accrue to others as a result of their contributions when making private decisions

   - individuals only consider the private net benefits of their contributions — positive externalities from voluntary contributions are not internalized in private decisions

2. individuals can free–ride on others provision of a public good

   - individuals behave strategically — since individuals cannot be excluded from the benefits accruing from others contributions, individuals incentives to contribute are diminished

   - individuals act as *free–riders*, understating their true value of the good so that they can enjoy its benefit without paying for it — as a result, voluntary contributions will not be sufficient to provide the optimal level of the public good — public good will be undersupplied

   - free–riding occurs because of nonexcludability and nonrivalrous properties of the public good

   - presence of free–riders makes it virtually impossible for market economy to provide public goods efficiently

Problem

Suppose there are I symmetric consumers with the following preferences and budget constraint:

$$\phi_i(q, y_i) = u_i(q) + y_i \text{ where } u_i(q) = Aq - \frac{a}{2}q^2 \text{ and } q = \sum_{i=1}^{I} q_i$$

$$\omega = cq_i + y_i$$

i. Find the Pareto optimal allocation of public goods and the voluntary contribution equilibrium in this economy.

ii. Prove that $\hat{q} < q^* \;\forall\; I > 1$.

iii. Propose a corrective mechanism (either a subsidy or a tax). Derive solution.