The second weakness occurs even if we confine ourselves to the partial equilibrium nature of the preceding analysis. This difficulty is due to the specification that the demand for output depends only on the price of a particular period, that is, $Y_t = D(p_t, t)$. Once the firm announces its pricing policy, $p_1, p_2, \ldots, p_T$, then consumers will naturally adjust their demand for the commodity accordingly. Then the demand function should more properly be specified as $Y_t = D(p_1, p_2, \ldots, p_T, t)$, $t = 1, 2, \ldots, T$ (e.g., Mohring 1970). Since the firm will decide its optimum pricing policy based on such consumer behavior, this would create a game-theoretic situation. Such an extension, though very important, is again assumed away here. We leave the task of pursuing these problems to the interested reader.

4.7 On the Coase Theorem[35]

4.7.1 Introduction

One of the fundamental theorems of welfare economics is that a competitive equilibrium achieves a Pareto optimum, and that, as a corollary to that proposition, a competitive equilibrium achieves productive efficiency. This constitutes a modern formulation of Adam Smith's well-known insight into the "invisible hand."[36] Although such propositions hold under a weak set of assumptions, these will break down in the presence of "externalities," external economies and diseconomies in production and consumption in which "relevant effects of production or welfare go wholly or partially unpaid" (Mishan 1964, p. 103). The presence of such externalities then constitutes a fundamental challenge to the well-accepted doctrine with the welfare implications of competitive

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[35]This section is adapted from Takayama (1979a). Ronald Coase was the 1991 Nobel laureate in Economic Science.

[36]There are two fundamental classical propositions in welfare economics: (*a*) every competitive equilibrium realizes a Pareto optimum, and (*b*) every Pareto optimum can be supported by a competitive equilibrium with a suitable allocation of resources among individuals. The seminal work on welfare economics was by Pigou (1932). This has been criticized on the grounds that it allowed interpersonal comparisons of utility by then young economists such as Lerner, Kaldor, Hicks, and others under the apparent influence of Lionel Robbins of the London School of Economics. The Pigouvian procedure of welfare has been replaced by the concept that is now called "Pareto optimum." The classical propositions of welfare economics under such a new approach were analyzed in terms of calculus by Bergson (1938), Lange (1942), and others. In the early 1950s these propositions were reformulated in terms of a modern set theoretic framework by K. J. Arrow, G. Debreu, L. W. McKenzie, and others. See Takayama (1985, chap. 2, sec. C).

equilibrium.[37] A well-known example of external economies is that of an apple grower and a bee keeper in an adjacent field, wherein the latter obtains a benefit from the production of the former (see Meade 1952).[38] Although examples of this kind may have been taken to be rather unimportant, the question of externalities has attracted a great deal of attention more recently due to the problem of external diseconomies and the liability rules associated with it. Smoke, noise, and many forms of air and water pollution have intensified the interest in this problem.

The well-known solution to "market failure" due to externalities was proposed by Pigou (1932) and has been widely accepted. In such a case, market failure can be corrected by the governmental intervention of devising a proper tax-subsidy scheme. An important article by Coase (1960) challenged the Pigouvian solution in a fundamental way: he eloquently argues that the Pareto optimum resource allocation can be achieved via private negotiations *without* invoking governmental intervention.

The externality considered by Coase is a production externality in

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[37]Various "deficiencies" of competitive equilibrium are known as *market failures.* For good summaries of the problem of market failures, see Imai et al. (1971), Homma (1980), Boadway and Bruce (1984, pp. 103–36), Ledyard (1987), and Stiglitz (1988, pp. 71–81); for example. For an early but excellent exposition of this problem, see Bator (1957 and 1958). For the market failures associated with incomplete markets and imperfect information, see Greenwald and Stiglitz (1986). Market failures are often used as the *prima facie* economic justification for the existence of government and government interventions. The Coase theorem is an important contribution to the antithesis of such an argument.

[38]Meade (1952) represents the first modern discussion on externalities, or non-market interdependence of economic agents, which is responsible for divergence between social and private costs, as well as causing an important type of market failure. (James Meade, jointly with Bertil Ohlin, was awarded the 1977 Nobel Prize in Economic Science for contributions to international economics.) A simplified variant of a production model by Meade in terms of one factor ("labor") was presented by Bator (1958). (It may be worthwhile to note the distinction with regard to different sources of market failure, one due to non-market interdependence or externalities, and the other due to economies of scale. This distinction is pointed out by Scitovsky (1954). The pioneering and classical work on the problem of divergence between private cost and social cost (and related issues) is due to Pigou (1932). In this regard, Mishan (1964, pp. 14–15) writes,

> *The Economics of Welfare* is frequently associated with the controversies of the thirties over interpersonal comparison of utility. But its enduring contribution is to be found in the continued emphasis on the vital distinction between social and private valuation of economic activities, a distinction evoked nowadays more by reference to "external effects" or "external economies and diseconomies of production and consumption."

which the production of one good becomes a negative input in the production of some other goods. A celebrated example used by Coase is the case of straying cattle; a rancher-producer raises cattle that trample a neighboring farm's crop. Another example used by Coase is the case of a confectioner and a doctor, in which the doctor, in his consulting room, is disturbed by the noise and vibrations caused by the machinery in the confectioner's kitchen.

The proposition known as the "Coase theorem" states that the Pareto optimum resource allocation (under some ideal conditions) can be achieved via private negotiation without invoking the Pigouvian tax-subsidy scheme, and that this private solution is independent of liability rules. Coase presented his proposition verbally by using interesting and eye-opening examples drawn from actual legal cases. Although this is justifiably and undoubtedly an important reason for the popularity of his proposition, here we shall present it in terms of a mathematical model. This approach should more clearly reveal the basic logical structure and assumptions involved in his proposition.[39]

4.7.2 Externality and the Pigouvian Scheme

To understand Coase's contribution properly, it is useful to first discuss the basic argument involved in the classical Pigouvian scheme. For simplicity, following Coase (1960) and others, we consider an economy consisting of two producers (1 and 2). Assume that Firm 1's production function for its output Y_1 is given by

$$Y_1 = F_1(L_1),\ F_1(0) = 0,\ F_1' > 0,\ F_1'' < 0, \tag{79}$$

where L_1 is its input (labor), while Firm 2's production function for its output Y_2 is given this,

$$Y_2 = \tilde{F}_2(Y_1,\ L_2),\ \tilde{F}_2(Y_1,\ 0) = 0, \tag{80}$$

where $\partial \tilde{F}_2/\partial L_2 > 0$, $\partial_1^2 \tilde{F}_2/\partial L_2^2 < 0$, $\partial \tilde{F}_2/\partial Y_1 < 0$, and where L_2 is its direct input (labor). Notice also that here Firm 1's output (Y_1) enters as a negative input to Firm 2's production, that is, $\partial \tilde{F}_2/\partial Y_1 < 0$.

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[39]In many ways, Coase (1960) constitutes another significant step forward from Pigou (1932). Coase has inspired a number of works. It provides illuminating reading with many interesting legal examples, and a simple mathematical exposition as presented here may not do full justice to this important work. For a recent succinct discussion of the problem involved with this "theorem," see Cootner (1987).

Substituting (79) into $Y_2 = \tilde{F}_2(Y_1, L_2)$, we may obtain

$$Y_2 = \tilde{F}_2[F_1(L_1), L_2] \equiv F_2(L_1, L_2), \tag{81}$$

where $F_2(L_1, 0) = 0$, $\partial F_2/\partial L_1 < 0$, $\partial F_2/\partial L_2 > 0$, $\partial^2 F_2/\partial L_2^2 < 0$. Note that $\partial F_2/\partial L_1 < 0$ if and only if $\partial \tilde{F}_2/\partial Y_1 < 0$. It is assumed that Firm 2 knows function F_1.[40] The model, in terms of (79) and (81), should capture the essence of the externality considered by the Coase theorem.

A **Pareto optimum** point in production, or the efficient configuration of production, is given as a solution of the constrained maximization problem in which L_1 and L_2 are chosen so as to

$$\text{Maximize } \alpha_1 F_1(L_1) + \alpha_2 F_2(L_1, L_2),$$

$$\text{subject to } L_1 + L_2 \leqq L,\ L_1 \geqq 0, \text{ and } L_2 \geqq 0,$$

where L is the fixed amount of the resource (labor) in this economy, and α_1 and α_2 are fixed nonnegative constants with $(\alpha_1, \alpha_2) \neq 0$. Assume that $L_1 > 0$ and $L_2 > 0$ at the optimum (an interior solution), where we omit $(^*)$ to denote the optimum for simplicity. Then the first-order (necessary) conditions for optimality are given by[41]

$$\alpha_1 F_1' + \alpha_2 \partial F_2/\partial L_1 = \alpha_2 \partial F_2/\partial L_2, \tag{82}$$

$$L_1 + L_2 = L. \tag{83}$$

These two equations determine the Pareto optimum values of L_1 and L_2 for given values of the parameters, α_1 and α_2 (or, for a given value of α_1/α_2 where we assume $\alpha_2 > 0$).[42] Varying the value of α_1/α_2, we

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[40] This presupposes that Firm 2 knows Firm 1's production function, F_1, as well as its own, i.e., $\tilde{F}_2$. Alternatively, we may suppose that the externality enters into Firm 2's production function directly in the form of L_1.

[41] Write the Lagrangian as $\Phi \equiv \alpha_1 F_1(L_1) + \alpha_2 F_2(L_1, L_2) + \lambda(L - L_1 - L_2)$. By setting $\partial\Phi/\partial L_1 = 0$ and $\partial\Phi/\partial L_2 = 0$, we at once obtain

$$\alpha_1 F_1' + \alpha_2 \partial F_2/\partial L_1 = \lambda,\ \alpha_2 \partial F_2/\partial L_2 = \lambda, \tag{82'}$$

which yields (82). The second equation of (82′) ensures $\lambda > 0$, which in turn yields (83).

[42] When $\partial F_2/\partial L_1 \equiv 0$ (no externality), the assumptions stated in (79) and (80) are sufficient to furnish a unique (globally) Pareto optimum point for a given value of α_1/α_2. Although this is not necessarily the case when the externality is present, we shall assume that such is also the case when this externality is present. The reader should be able to check the appropriate second-order condition.

obtain different Pareto optimum points.[43] Condition (82) signifies the productive efficiency condition. We may call (82) the **Pareto optimum condition** (in production).

Suppose that these two firms are immersed in a "much bigger" economy in which the prices of the two outputs (p_1 and p_2) and the factor price (w) are determined more or less independently of the two firms' levels of production, and that these two firms are "competitive" in the sense that they take these prices (p_1, p_2, and w) as exogenously given constants.[44] Under such circumstances, it is well known (and can easily be shown) that the joint profit maximization yields a Pareto optimum situation in which α_1/α_2 is set equal to p_1/p_2. To see this, consider the problem of choosing L_1 and L_2 to maximize the *joint profit* of the two firms,

$$\pi(L_1,\ L_2) \equiv p_1F_1(L_1) + p_2F_2(L_1,\ L_2) - w(L_1 + L_2),$$

subject to $L_1 \geqq 0$ and $L_2 \geqq 0$ Assuming $L_1 > 0$ and $L_2 > 0$ at the optimum, the first-order condition is given by $\partial\pi/\partial L_1 = \partial\pi/\partial L_2 = 0$, or

$$p_1F_1' + p_2\partial F_2/\partial L_1 = p_2\partial F_2/\partial L_2 (= w). \tag{84}$$

This in turn ensures the Pareto optimum (82), for $\alpha_1/\alpha_2 = p_1/p_2$. Under the given partial equilibrium situation in which each firm is able to employ any amount of labor at a fixed wage rate w, (83) is trivially satisfied.[45] Hence we may conclude that *the joint profit maximization point defined by (84) achieves a Pareto optimum* in which α_1/α_2 is taken to be equal to p_1/p_2.[46]

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[43] Mathematically speaking, the Pareto optimum problem is formulated as the one of *vector maximization*, which can be reformulated in terms of the problem of maximizing a weighted average of the objective functions of the economic agents (producers and/or consumers) involved. For the discussion on vector maximization, see Takayama (1985, chap. 1, sec. E). Needless to say, the problem stated in the text can equivalently be reformulated as the one of choosing L_1 and L_2 so as to maximize $F_1(L_1)$ subject to $F_2(L_1,\ L_2) \geqq Y_2$, $L_1 + L_2 \geqq 0$, $L_1 \geqq 0$, and $L_2 \geqq 0$, where Y_2 is taken to be a parameter.

[44] Such a "partial equilibrium" set of circumstances corresponds to the situation considered by Coase (1960), as well as Meade (1952) and others.

[45] Note that (84) determines the optimum values of L_1 and L_2 as functions of p_1, p_2, and w, that is, $L_1 = L_1(q)$ and $L_2 = L_2(q)$ where $q = (p_1,\ p_2,\ w)$. Then under the partial equilibrium circumstances described here, we may define L by $L \equiv L_1(q) + L_2(q)$.

[46] This is because (82) is satisfied by (84) when $p_1 = \alpha_1$ and $p_2 = \alpha_2$. Also,

In the competitive situation, however, each firm is *not* typically interested in maximizing the joint profit or the "social profit," $\pi(L_1, L_2)$. Rather, each is interested in maximizing its own private profit. Namely, Firm 1 chooses L_1 to maximize its own profit,

$$\pi_1(L_1) \equiv p_1F_1(L_1) - wL_1,$$

subject to $L_1 \geqq 0$, while Firm 2 chooses L_2 to maximize its own profit,

$$\pi_2(L_2, L_2) \equiv p_2F_2(L_1, L_2) - wL_2,$$

subject to $L_2 \geqq 0$, for a given value of L_1. Assuming $L_1 > 0$ and $L_2 > 0$ at the optimum, the first-order conditions for Firms 1 and 2 are, respectively, given by

$$p_1F_1' = w, \text{ and} \tag{85}$$

$$p_2\partial F_2/\partial L_2 = w. \tag{86}$$

This in turn implies

$$p_1F_1' = p_2\partial F_2/\partial L_2. \tag{87}$$

Under such circumstances, the Pareto optimum condition (82) cannot be satisfied in general. In the presence of an externality ($\partial F_2/\partial L_1 \neq 0$), the "market solution," each firm's maximization of its own private profit, will not achieve a Pareto optimum (or a "social optimum"). This result is at the heart of the problem of market failures due to externalities.

Note that if there is no externality in this case ($\partial F_2/\partial L_1 \equiv 0$), then condition (82) is ensured by the private profit maximization (87). This is clearly an aspect of the well-known proposition by Adam Smith that "free competition" realizes a "social optimum."

A. C. Pigou proposed to remove market failure in the presence of an externality by a "tax-cum-subsidy" scheme. Suppose that Firm 1, which inflicts "damage" on Firm 2, is subject to $100 \cdot t\%$ revenue *tax*. Then Firm 1 chooses L_1 to maximize its after tax profit,

$$(1 - t)p_1F_1(L_1) - wL_1,$$

subject to $L_1 \geqq 0$, and the given tax rate t.

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letting λ signify the shadow price of the resource constraint, $L_1 + L_2 \leqq L$, λ is equal to the market wage rate, where we may recall (82′).

Assuming $L_1 > 0$ at the optimum, the first-order condition is given by

$$(1 - t)p_1 F_1' = w. \tag{88}$$

Suppose that Firm 2 is subject to no tax (nor subsidies). Then its profit maximization condition is given by (86). Hence, combining (86) and (88), we obtain

$$p_1 F_1' - t p_1 F_1' = p_2 \partial F_2 / \partial L_2 = w. \tag{89}$$

If the government should choose the tax rate t so that

$$-t p_1 F_1' = p_2 \partial F_2 / \partial L_1, \quad \text{that is,} \quad t = -\frac{p_2 \partial F_2 / \partial L_1}{p_1 F_1'} > 0, \tag{90}$$

then condition (89) ensures (82) with $\alpha_1/\alpha_2 = p_1/p_2$.[47] The Pareto optimum with $\alpha_1/\alpha_2 = p_1/p_2$ can be achieved via private profit maximization when the proper tax is levied on Firm 1.

The same solution can be achieved by providing *subsidies* to Firm 2. Suppose that Firm 2 receives a subsidy in the amount of $100 \cdot s\%$ of its revenue. Then Firm 2 chooses L_2 to maximize its (after subsidy) profit,

$$(1 + s)p_2 F_2(L_1, L_2) - wL_2,$$

subject to $L_2 \geqq 0$, for given values of s and L_1. Assuming $L_2 > 0$ at the optimum, the first-order condition gives

$$(1 + s)p_2 \partial F_2 / \partial L_2 = W. \tag{91}$$

Assuming that Firm 1 is subject to no taxes or subsidies, its profit maximization condition is given by (85). Combining (85) and (91), we obtain

$$p_1 F_1' - s p_2 \partial F_2 / \partial L_2 = p_2 \partial F_2 / \partial L_2. \tag{92}$$

Hence, if the government chooses s such that

$$-s p_2 \partial F_2 / \partial L_2 = p_2 \partial F_2 / \partial L_1, \quad \text{that is} \quad s = -\frac{\partial F_2 / \partial L_1}{\partial F_2 / \partial L_2} > 0, \tag{93}$$

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[47]Note that $t > 0$ since $\partial F_2 / \partial L_1 < 0$. To compute the appropriate tax rate t, obtain the optimum values of L_1 and L_2 from (89) as $L_1 = L_1(q;\, t)$ and $L_2 = L_2(q;\, t)$, where $q = (p_1,\, p_2,\, w)$, and substitute them into (90):

$$-t p_1 F_1'[L_1(q;\, t)] = p_2 \partial F_2 [L_1(q;\, t),\; L_2(q;\, t)] / \partial L_1,$$

which gives the tax rate t as $t = t(q)$, i.e., $t = t(p_1,\, p_2,\, w)$.

then the Pareto optimum (82) is ensured.

An obvious difficulty in this solution is that it is not clear how the government spends the tax revenue in the tax scheme case, or how the government obtains income to subsidize Firm 2 in the subsidy scheme case. One (possible satisfactory) answer is that since these firms are presumably very "small" in the economy, the amount (and hence the effects) of the tax revenue or the subsidy is negligible, and hence can be ignored. On the other hand, if such is the case, the significance of the problem of externalities considered here may also be small. Also, these can add up to a non-trivial amount.

4.7.3 The Coase Theorem

A remarkable feature of the Coase theorem is that a Pareto optimum can be achieved by private negotiations of the two firms without any intervention of the government. Thus, bureaucratic red tape and other (explicit or implicit) costs associated with government intervention can be dispensed with. Furthermore, the Pareto optimum can be achieved whether or not Firm 1 is liable for the damage that Firm 2 suffers. In connection with this, we may note that it is not necessarily clear why Firm 1 should always be liable to Firm 2, even if Firm 1's output is a negative input of Firm 2's production function. As an example, consider Firm 2 coming into existence after Firm 1 is already established. In such an event, it is not clear why Firm 1 should pay compensation to a newcomer, Firm 2. If Firm 1 has to pay compensation, it is possible that Firm 1 may have to cease its operation when Firm 2 is established. This may be unjustifiable.

It is often the case that the damage suffered by Firm 2 is due to the fact that Firm 1 destroys a certain "environment" (as in the case in which noise made by the confectioner inflicts damage on the doctor in the confectioner vs. doctor case). The question is then who owns the property right with regard to the environment. As in the confectioner-doctor example by Coase, if the confectioner is in operation before the doctor moves nearby, it is possible to argue that the confectioner owns the "environmental right," and hence can make noise without any compensation to the doctor. The Coase theorem asserts that the Pareto optimum can be achieved by private negotiations *regardless of* who owns the environmental right.

To begin the discussion, recall that, under private profit maximization, Firm 2's output depends on the level of Firm 1's operation. To see this, recall Firm 2's profit maximization condition,

$$p_2 \partial F_2(L_1, L_2)/\partial L_2 = w. \tag{86}$$

This in turn gives the optimum value of L_2 for a *given* level of L_1. Namely, (86) can be solved for L_2 (for a given value of w/p_2) as

$$L_2 = L_2(L_1). \tag{94}$$

The maximum profit of Firm 2, π_2^*, thus depends on L_1. In other words,

$$\pi_2^*(L_1) \equiv p_2 F_2[L_1, L_2(L_1)] - wL_2(L_1). \tag{95}$$

We now examine the two cases that arise due to two different liability rules.

Case A (Firm 1 Is Liable). When Firm 1 ceases its operation, Firm 2's maximum profit is given by $\pi_2^*(0)$. This profit is reduced to $\pi_2^*(L_1)$, when Firm 1's level of operation is given by $L_1 > 0$, where $\pi_2^*(0) > \pi_2^*(L_1)$. Hence, damage Firm 2 suffers from Firm 1 via externality is equal to $\pi_2^*(0) - \pi_2^*(L_1)$. In Case A, Firm 1 is liable for compensating this entire amount to Firm 2. Under such circumstances, Firm 1 chooses L_1 to maximize its profit after compensation,

$$p_1 F_1(L_1) - wL_1 - [\pi_2^*(0) - \pi_2^*(L_1)]. \tag{96}$$

The optimality condition is given by

$$p_1 F_1' - w + d\pi_2^*/dL_1 = 0. \tag{97}$$

Firm 2 chooses L_2 to maximize profit inclusive of the compensation from Firm 1,

$$p_2 F_2(L_1, L_2) - wL_2 + [\pi_2^*(0) - \pi_2^*(L_1)]. \tag{98}$$

Then the optimality condition is given by

$$p_2 \partial F_2/\partial L_2 - w = 0. \tag{99}$$

From (97) and (99), we obtain

$$p_1 F_1' + d\pi_2^*/dL_1 = p_2 \partial F_2/\partial L_2. \tag{100}$$

The expression for $d\pi_2^*/dL_1$ can be computed, by recalling (95), as:

$$d\pi_2^*/dL_1 = p_2[\partial F_2/\partial L_1 + (\partial F_2/\partial L_2)L_2'] - wL_2' = p_2 \partial F_2/\partial L_1 \tag{101}$$

where $L_2' \equiv dL_2/dL_1$, and where the second equality is obtained by using (99). In view of (100), (101) reduces to the Pareto optimum condition (82). Thus, we may conclude that the Pareto optimum can be achieved when Firm 1 compensates Firm 2 for the entire amount of the damage.

Case B (Firm 2 Is Liable). In this case, Firm 2 (say, a "newcomer") is responsible for compensating Firm 1. Firm 2 wishes Firm 1 reduces its production level, and is willing pay compensation for that. When Firm 1's level of production is reduced from L_1° to L_1, Firm 2's profit will increase by the amount of $\pi_2^*(L_1) - \pi_2^*(L_1^\circ)$, where $L_1 < L_1^\circ$. Let L_1° be the level of Firm 1's operation when Firm 1 receives no compensation. Since Firm 1 receives the compensation from Firm 2 by the amount of $\pi_2^*(L_1) - \pi_2^*(L_1^\circ)$, Firm 1's profit inclusive of the compensation is given by

$$p_1 F_1(L_1) - wL_1 + [\pi_2^*(L_1) - \pi_2^*(L_1^\circ)]. \tag{102}$$

Firm 1 chooses L_1 to maximize such a profit, and the optimality condition is given by

$$p_1 F_1' - w + d\pi_2^*/dL_1 = 0. \tag{103}$$

Firm 2 chooses L_2 to maximize its profit after the compensation,

$$p_2 F_2(L_1, L_2) - wL_2 - [\pi_2^*(L_1) - \pi_1^*(L_1^\circ)]. \tag{104}$$

The optimality condition for this is given by

$$p_2 \partial F_2/\partial L_2 - w = 0. \tag{105}$$

Combining (103) and (105), we get

$$p_1 F_1' + d\pi_2^*/dL_1 = p_2 \partial F_2/\partial L_2. \tag{106}$$

Then recalling $d\pi_2^*/dL_1 = p_2\partial F_2/\partial L_1$ from (101), we can readily observe that (106) reduces to the Pareto optimum, condition (82). Thus, we may also conclude that the Pareto optimum can also be achieved when Firm 2 compensates Firm 1 for the entire amount of the damage. In summary, we obtain the following result (the **Coase theorem**). The above argument provides a formal proof of the theorem.

Proposition 4.8. In the above specification of the model, the Pareto optimum in production can be achieved by private negotiation between firms, whether Firm 1 is liable for compensation (case A) or Firm 2 is liable for compensation (case B).

Some important questions arise once the theorem is formulated. For example, what are the incentives for such negotiations and how would private negotiations result in one of the two cases considered above? Setting aside such questions, we illustrate the Coase theorem diagrammatically in figure 4.13.[48] The BB'-curve signifies the $[-d\pi_2^*(L_1)/dL_1]$-locus, where $OA = L_1^\circ$. Since $d\pi_2^*(L_1)/dL_1 = p_2\partial F_2(L_1, L_2)/\partial L_1 < 0$ by (101), the BB'-curve is above the OL_1-axis. Assume, for simplicity, $d^2\pi_2^*(L_1)/dL_1^2 < 0$.[49] Then the BB'-curve is upward sloping. In figure 4.13, the AE-curve signifies the locus of $[p_1F_1'(L_1) - w]$. Since $F_1'' < 0$, the AE-curve is negatively sloped. L_1° is the amount of labor that Firm 1 employs when the firm neither pays nor receives any compensation, and it is determined by Firm 1's profit maximization under no restrictions, that is, by $p_1F_1'(L^\circ) = w$. Denote the level of L_1 determined by the intersection of the BB'-curve and the AE-curve by L_1^*. Then, when $L_1 = L_1^*$, we have

$$p_1F_1' - w = -p_2\partial F_2/\partial L_1, \quad \text{so that} \quad p_1F_1' + p_2\partial F_2/\partial L_1 = w.$$

Recalling (84), we may at once conclude from this that L_1^* signifies the Pareto optimum level of L_1. When the level of L_1 decreases from L_1° to L_1^*, Firm 1's profit decreases by the amount of triangle ACD, while Firm 2's profit increases by the amount of trapezoid $ABCD$ in figure 4.13. There is a net increase in the "joint profit" by the amount of triangle ABC. The Coase theorem asserts that such a net increase can be achieved regardless of which firm is liable for compensation.

In Case A, Firm 1 is liable for compensation by the amount of damage that it inflicts upon Firm 2. The amount of damage $[\pi_2^*(L_1^*) - \pi_2^*(0)]$ is measured by the trapezoid area $OB'CD$. Firm 1's profit before the compensation, when its level of operation is equal to L_1^* is measured by trapezoid $OECD$, while its profit after the compensation, when $L_1 = L_1^*$, is measured by triangle $B'CE$. In Case B, Firm 2 is liable for compensating Firm 1 for restricting its production. The amount of compensation is equal to the amount of increase in profit due to Firm 1's restriction of

[48] We are indebted to Kudoh and Yabushita (1974) for a similar diagram and some of the subsequent discussion.

[49] Recalling (96) and (102), we may observe that condition $d^2\pi_2^*/dL^2 < 0$ ensures the second-order (sufficiency) condition for Firm 1's maximization problems for both cases A and B.

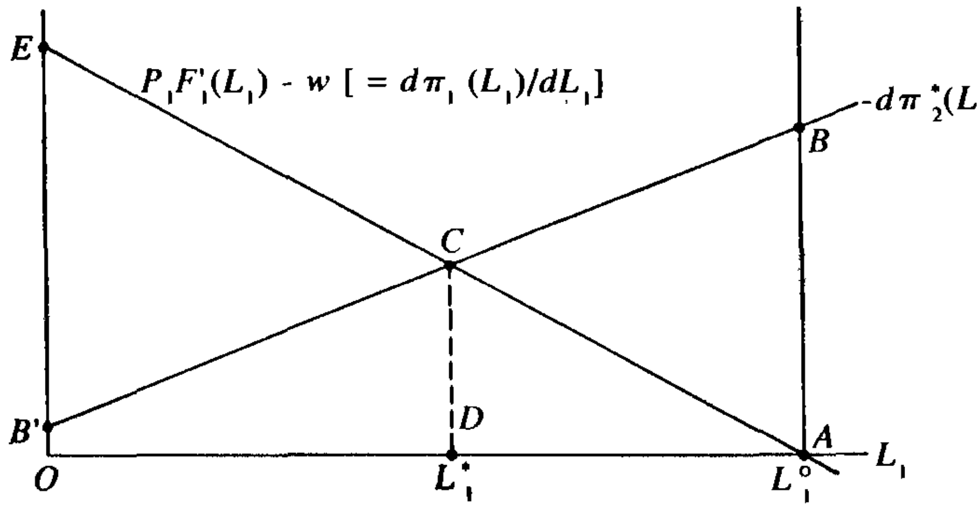

Figure 4.13: Illustration of the Coase theorem

its production, $[\pi_2^*(L_1^*) - \pi_2^*(L_1^0)]$, when L_1 is restricted from L_1^0 to L_1^*. In figure 4.13, this is measured by the trapezoid area $ABCD$. Thus in Case B, an increase in the joint profit due to Firm 1's reduction of its production level all goes to Firm 1.

Having formulated the Coase theorem mathematically, we can easily extend the analysis to more general circumstances, such as the cases (1) when the firm's production involves factors other than labor, (2) when both firms are subject to taxation (e.g., corporate profit tax), (3) when one firm is subject to some sort of governmental regulations, and (4) when one party is a consumer instead of producer, etc. Such extensions should be easy and are left to the interested reader.

Setting such generalizations aside, we may note that some important assumptions are made implicitly in the preceding formulation of the Coase theorem. One is the absence of negotiation and transaction costs. The court procedure that determines the amount of damage is very often quite costly in terms of both nominal and real costs including "time." Such costs may become inhibitively high for many typical environmental problems in which thousands of people are involved (sometimes in both parties) such as in the pollution of air and water, nuclear reactor accidents, etc. Another assumption concerns the fact that the income distribution between parties is largely influenced by the liability rule, although the final level of production after negotiation remains the same.

But the question of determining which party is liable depends on the question of which party owns the property right to the "environment," or which party has the right to harm the other. The determination of such property rights may not be easy. The third problem is that the damage may not occur with certainty, but rather occurs under a certain probability distribution (as in the case of an "accident"), and that the probability of the accident occurence depends on the cost that each party absorbs. Note that the probability of accidents depends on the position taken by both parties, which would make the concept of the "perpetrator of injury" and the "victim" even more ambiguous. On top of these economic problems, there is a well-known mathematical problem involved in externalities. The production possibility set, under even simple circumstances described by (79) and (81), is not necessarily convex.[50] This implies that the Pareto optimum is only a "local" optimum and a more careful examination of the Pareto optimum condition (82) is necessary.

A number of interesting studies have been done on all of these aspects of the problem. However, the message of the Coase theorem is clear. Using a simple framework, it clarifies a certain basic structure involved with externalities and shows that it is possible, under certain situations, to achieve the Pareto optimum by private negotiations; that is, by avoiding governmental interventions. This is a remarkable result.

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[50] Under the present "partial equilibrium" circumstances, the concept of the "production possibility set" may not be appropriate. It may be more proper to use the concept of the "feasible set" in the bargaining solution in the theory of cooperative games with side payments.

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