Course 603 (Environmental Economics)
Summer semester 2016

PROBLEM SET 1 (August 10, 2016)

1. Consider an economy in which there are two individuals, *A* and *B*, and a single firm. There are two goods in the economy, 1 and 2. There is an initial endowment of good 1, r_1, that is evenly divided between the two individuals. The firm uses good 1 to produce good 2, according to the production relationship,

$$y_2 = f(y_1, s)$$

where y_2 is the total quantity of good 2 produced by the firm, y_1 is the quantity of good 1 used as an input by the firm, and s is the quantity of "trash"generated by the firm. The two consumers' preferences are captured by the utility functions,

$$u^j(x_{1j}, x_{2j}, s) \quad j = A, B$$

where x_{ij} denotes the quantity of the good i consumed by individual j.

(a) Write out the central planner's problem, the solution to which characterises the Pareto optimum of this economy, and then derive the first order conditions to this problem.
(b) Would the regulator be able to achieve the first best optimum by levying a Pigouvian tax on trash generated? Derive an expression for the Pigouvian tax.
(c) Now suppose the externality is depletable. Specifically, the externality is only suffered by one individual, say *A*, because the firm dumps all the trash it generates in *A*'s backyard. Thus, the two individual's utility functions are now,

$$u^A(x_{1A}, x_{2A}, s) \qquad u^B(x_{1B}, x_{2B}, 0)$$

Would your answer to part (b) change, if so how?

2. A smoker and a nonsmoker work together in an office. The smoker gets daily marginal benefits of $(24 - x)$ where x is the number of cigarettes smoked each day. The nonsmoker incurs marginal damages of $2x$ each day from smoke inhalation.

(a) What is the socially optimal level of smoke?
(b) The employer strongly favours incentives rather than regulations. What incentive would she apply to achieve the best possible amount of smoking? Explain.
(c) A local company is advertising a new air filter machine that is guaranteed to remove 50% of cigarette smoke from a room. How would such a machine affect the "socially optimal" smoking rate and tax?
(d) The air filter rents for Rs. 18 per day. If, as a result of the rental, the tax adjusts to the level determined in part c, would it be worthwhile for the smoker to rent the machine? Would it be worthwhile for the non-smoker to rent the machine?

3. Suppose two asthmatic neighbours are deciding how many crackers to set off on Diwali. Neighbour i chooses amount x_i and gets utility $u_i(x_i)$ from this choice; we assume that $u_i'(x_i) > 0$. However, the more crackers they set off, the more likely it is that they **both** fall sick with probability p due to the ensuing smoke pollution. Thus, $p(x_1, x_2)$ is the probability of falling sick and is assumed to be increasing in each argument. Finally, let $c_i > 0$ be the cost being sick imposes on neighbour i. Assume that each neighbour's utility is linear in money.

(a) Show that each neighbour has an incentive to set off too many crackers from the social point of view.
(b) If neighbour i is fined an amount t_i in case sickness happens, how large should t_i be to internalise the externality?
(c) If the optimal fines are being used, what are the total costs, including fines, paid by the neighbours? How does this compare to the total cost of sickness?
(d) Suppose now that neighbour i gets utility $u_i(x_i)$ only if there is no sickness. What is the appropriate fine in this case?

4. A beekeeper (Ms. Madhu) and a farmer (Seb Singh) with an apple orchard are neighbours. Bees pollinate the apple trees: one beehive pollinates one acre of orchard. But there are not enough bees next door to pollinate the whole orchard and pollination costs are Rs 10/acre. The beekeeper has total costs of $TC = H^2 + 10H + 10$ where H is the number of beehives. Each beehive yields Rs. 20 worth of honey.
(a) How many beehives would the beekeeper maintain if operating independently of the farmer?
(b) What is the socially efficient number of hives?
(c) In the absence of transaction costs, what outcome do you expect to arise from bargaining between the beekeeper and the farmer?
(d) How high would transaction costs have to be to erase all gains from bargaining?

5. A lake can be freely accessed by fishermen. The cost of sending a boat out on the lake is $r > 0$. When b boats are sent out onto the lake, $f(b)$ fish are caught in total (so each boat catches $f(b)/b$ fish), where $f'(b) > 0$ and $f''(b) < 0$ at all levels $b \geq 0$. The price of fish is $p > 0$, which is unaffected by the level of the catch from the lake. (This problem is also known as the *problem of the commons*, i.e., open access.)

(a) Characterise the equilibrium number of boats that are sent out on the lake.
(b) Characterize the optimal number of boats that should be sent out on the lake.
(c) What per-boat fishing tax would restore efficiency?
(d) Suppose that the lake is instead owned by a single individual who can choose how many boats to send out. What level would this owner choose?

6. A power station uses labour l_1 (competitively supplied) to produce electricity y which it sells in a perfectly competitive market at a price p per unit.

$$l_1 = y^2 / 2$$

It also produces an equal number of units of smoke, z, that is, $y = z$

The smoke increases labour l_2 required to clean shirts x at a laundry nearby. The laundry receives q rupees per shirt laundered,

$$l_2 = (x^2 / 2) + wxz$$

Thus, the power station generates a negative externality on the laundry, wxz, where w is the intensity of the externality. The only costs for both firms are labour costs and the wage rate is unity. If the two firms take the actions of the other as given and maximise profits independently by setting marginal private cost equal to price, derive expressions for their respective profits π_1 and π_2 *as a function of the amount of smoke produced*, i.e., $\pi_1(z)$ and $\pi_2(z)$. If instead, they were to maximise joint profits how much smoke would be produced? Denote this socially optimal level of smoke z^*. Now suppose that a *Pigouvian* tax on smoke (set equal to marginal damage at the optimal level of smoke z^*) is levied on firm 1 when it is maximising profits independently. Show that the *rate* of tax (i.e., tax / z^*) is given by the expression

$$t = w(q - wp) / (p - wq)$$

7. An old lady lives in a little old house and grows beautiful flowers in her garden. This is the only thing she enjoys in life. Donald Trump buys the plot next door and wants to build a new Trump Tower next to the little old house. The problem is this will shade the garden and will prevent the old lady from raising flowers. Now Trump could put a special sun hole though the tower that would let the flowers grow. This avoidance technology would cost $1,000,000.

If Trump has the right to shade the garden, he would sell the right to the lady for any price above $1,000,000. Unfortunately, she does not have this money and the outcome is that Trump retains the property rights, the garden is shaded and the flowers die. Now suppose the old lady has the right to have sun on her garden. Trump would pay as much as $1,000,000 to acquire the rights since this is the cost of avoidance. However, since the only joy in life for her is growing flowers, she chooses to refuse this offer. The outcome is Trump puts the sun hole in the tower and the old lady retains the property rights and the flowers live.

Thus, different assignments of property rights seem to make a difference to the outcome.
(i) Explain why? (ii) Is this a violation of the Coase theorem?

8. In a world with a uniformly mixed pollutant, and in which each firm has a cost of emissions function $C(e)$, $C'(e) < 0$, $C''(e) > 0$, for $0 < e < e^*$, $C(e^*) = 0$, show that a tax on emissions minimises the cost of achieving a desired level of aggregate emissions (i.e., an emissions standard derived from a desired level of pollution concentration). What properties does the cost-minimising solution have? What determines the level of the tax? Show (and explain) how your answer would change if the pollutant were non-uniformly dispersed, that is, if each firm had a different impact on the standard at the margin?

Short questions:

1. Research by a pharmaceutical company (Pfizer) results in a fall in the price of chemical inputs that increase the profits of its rival (Glaxo). Do you agree that this is a positive externality in the Pigouvian sense? Why or why not?

2. When sulphur dioxide is emitted into the air it is transported over long distances and is converted to sulphuric acid. This gradually falls to the ground, either as rain or snow or simply by settling out of the air. This is called acid deposition (acid rain). In what way could acid deposition be considered a rival bad?

3. In the presence of Coasian bargaining is a Pigouvian tax Pareto optimal? Explain using a diagram.

Extra credit:

1. On Madhu's farm only honey is produced. Suppose there are two ways to make honey: with and without bees. It takes one litre of maple syrup and one unit of labour to make one bucket of artificial honey (indistinguishable from real honey). If honey is made with bees, k units of labour and b bees are required per bucket. Either way, H buckets of honey can be produced on her farm. The neighbouring farm belonging to Seb Singh, produces apples. If bees are present less labour is needed because bees pollinate the blossoms instead of workers. Thus, c bees replace one worker in pollination. Up to A bushels of apples can be grown on Singh's farm. Suppose that the market wage rate is w, bees cost p_b per bee and maple syrup costs p_m per litre.

(a) If each farmer produces her maximum output at the cheapest cost (assume the output prices they face make maximal production efficient), is the resulting outcome efficient?
(b) How does the answer depend on k, b, c, w, p_b, p_m? Give an intuitive explanation of your result.
(c) Up to how much would Singh be willing to bribe Madhu to produce honey with bees?
(d) What would happen to efficiency if both farms belonged to the same owner?
(e) How could the government achieve efficient production through taxes?

2. In addition to pollution externalities there are other types of environmental externalities as well. Consider, for instance, the following case of *ecosystem externalities*. Suppose there are three species that are linked: grain which is consumed by rodents, which in turn are consumed by predators (such as owls, foxes and coyotes). Individual consumers are affected only by grain for making bread and by predators; rodents do not directly affect their allocation of resources (but prove to be the missing link behind the ecosystem externality). Suppose a consumer's utility is increasing in bread consumed b, and leisure l, but decreasing in predators p. Predators can be eliminated by labour, whereas the production of grain g, is a function of labour and predator inputs (the latter is the ecosystem externality which is of indeterminate sign at the moment). Finally, production of bread is a function of inputs of labour and grain. Total labour available is allocated between competing uses including leisure. In this model:

(a) Determine the Pareto optimal allocation of resources (i.e., labour). Assume that an interior solution exists. Interpret the first order conditions.

(b) Now determine the competitive equilibrium where k and w represent the (exogenously determined) price of bread and labour, respectively.

(c) Compare the results in (a) and (b) and show that competitive equilibrium does not lead to Pareto optimal allocation of resources. Explain clearly (and briefly!) why.

(d) What must be the sign on the ecosystem externality ($\partial g/\partial p$) to generate the result that the competitive equilibrium results in too few resources devoted to bread production and too many resources devoted to predator removal?