A Cost-Effective Tax on Emissions (Cost Minimisation)

References:

1. Hanley, Shogren and White (1997) Chapter 4 – follows Fisher (1981) Chapter 6
2. Baumol and Oates (1988) Chapter 11
3. Baumol and Oates *SJE* (1971)

E_t: Uniformly mixed pollutant defined at any point in time as a flow

e_{kt}: Individual discharges from source *k* at time *t*

$E = \sum_k e_k$ (suppressing t) (1)

y_k: Output of k^{th} firm

r_{ik}: input *i* used by firm *k* (*i* = 1,…..,*n*)

Production function for k^{th} firm: $y_k = f^k(r_{1k} \dots \dots \dots r_{nk})$

v_k: End of pipe technology to reduce polluting emissions

Emissions function for k^{th} firm: $e_k = b^k(y_k, v_k)$ $\frac{\partial b^k}{\partial y_k} > 0$ $\frac{\partial b^k}{\partial v_k} < 0$

p_v: Cost of unit of abatement technology

p_i: Price of inputs

Social planner's problem:

Min $\sum_i \sum_k p_i r_{ik} + \sum_k p_v v_k$ (costs are separable in the two 'outputs' (2)

s.t. $f^k(r_{ik} \dots \dots \dots r_{nk}) = y_k^*$ (3)

and $b^k(y_k^*, v_k) = e_k$ (4)

and $\sum_k e_k \leq \bar{E}$ (5)

and $e_k, y_k \geq 0 \;\; \forall\, k = 1 \dots \dots \dots .. K$ (6)

$L = \sum_i \sum_k p_i r_{ik} + \sum_k p_v v_k + \sum_k \lambda_k [y_k^* - f^k(.)] + \mu[\sum_k b^k(.) - E^*]$ (7)

First order conditions:

$$\frac{\partial L}{\partial r_{ik}} = p_i - \lambda_k \frac{\partial f^k}{\partial r_{ik}} = 0 \quad \forall\, i, k \tag{8a}$$

$$\frac{\partial L}{\partial v_k} = p_v + \mu \frac{\partial b^k}{\partial v_k} = 0 \qquad \forall\, k \tag{8b}$$

Now consider t* : per unit tax on emissions to achieve $\bar{E}$ level of emissions

Firm k facing an emissions tax t* will minimize the cost:

Min $\quad \sum_i p_i r_{ik} + p_v v_k + t_k^* e_k$ (9)

s.t. $\; f^k(r_{1k} \ldots . r_{nk}) = y_k^*$

and $\; b^k(y_k^*, v_k) = e_k$

and $\; e_k,\; y_k \geq 0$

$$L_k = \sum_i p_i r_{ik} + p_v v_k + t^* e_k + \beta^k [y_k^* - f^k(.)] \tag{10}$$

Differentiating L_k with respect to input and abatement use first order conditions for a minimum are:

$$p_i - \beta^k \frac{\partial f^k}{\partial r_i} = 0 \quad \forall\, i \tag{11a}$$

$$p_v + t^* \frac{\partial b^k}{\partial v_k} = 0 \tag{11b}$$

Comparing equation (8) with equation (11), it can be seen that firm's optimum will coincide with social optimum if:

(i) p_i and p_v correspond to their competitive levels, i.e., firm has no price setting power in the input or pollution abatement markets (we also require $\beta^k = \lambda^k$ for all k);

(ii) The tax rate t^* is equal to μ, the shadow price of pollution reduction in the social planner's problem. In other words the least cost tax is equal to the marginal cost of abatement at $\bar{E}$. Rearranging equation (11), $t^* = -p_v / b_v^k$ (where$b_v^k \equiv \partial b^k / \partial v_k$) since $(-p_v / b_v^k)$ is the marginal abatement cost for firm k. This also implies for a given t*, that MACs across all firms must be equal under the cost-minimizing solution.

Non-Uniformly Mixed Pollutants

q_j: weighted function of emissions from all sources at any monitoring point *j*

d_{kj}: transfer coefficients forming $(J \times K)$ matrix
where *k = 1…….K* sources and *j = 1………J* monitoring points

$$q_j = \sum_k d_{kj} e_k \tag{12}$$

The control agency target might be specified as:

$$\sum_k d_{kj} e_k \leq q_j^* \quad \forall j \tag{13}$$

The social planner's problem is to minimize equation (2) subject to equation (3), (4), (6), (13)

$$L = \sum_i \sum_k p_i r_{ik} + \sum_k p_v v_k + \sum_k \lambda_k [y_k^* - f^k(.)] + \sum_j \mu_j \sum_k d_{kj} [b^k(.) - q^*] \tag{14}$$

First order conditions:

$$\frac{\partial L}{\partial r_{ik}} = p_i - \lambda_k \frac{\partial f^k}{\partial r_{ik}} = 0 \; \forall \, i, k \tag{15a}$$

$$\frac{\partial L}{\partial v_k} = p_v + \frac{\partial b^k}{\partial v_k} \sum_j \mu_j \, d_{kj} = 0 \;\; \forall \, k \tag{15b}$$

Firm *k* faced with pollution tax t_k^*

$$L_k = \sum_i p_i r_{ik} + p_v v_k + t_k^* e_k + \beta^k [y_k^* - f^k(.)] \tag{16}$$

Differentiating L_k with respect to input and abatement use first order conditions for a minimum are:

$$p_i - \beta^k \frac{\partial f^k}{\partial r_i} = 0 \quad \forall \, i \tag{17a}$$

$$p_v + t_k^* \frac{\partial b^k}{\partial v_k} = 0 \tag{17b}$$

Comparing equations (15b) and (17b), it can be shown that in order to achieve an efficient solution, <u>each firm faces a different tax rate</u> t_k^* which is determined by that firm's degradation of environmental quality at each monitoring point (given by the transfer coefficients) and the ambient target itself.

Shadow prices of improving ambient quality μ_j, are positive so long as emission reductions are necessary to meet the ambient target, and where the ambient standard is met <u>exactly</u> after the imposition of tax. In other words, shadow prices are 'dual' values and exist only for constraints which are binding in the optimal solution.

Also, a unique shadow price or tax rate μ_j exists at each monitoring point *j*, and each firm *k* pays a tax equal to $[d_{kj}\mu_j]$ for emissions affecting point *j* (if its transfer coefficient is zero for a given j it does not pay tax there).

The total tax paid by a firm t_k^*would be $\left[\sum_j d_{kj}\mu_j\right]$ per unit of emissions.

[Should you have a uniform tax rate at each *j*? Equal to the most degraded monitoring point? Would this be a good idea?]