Prices *vs.* Quantities [1,2]

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I. INTRODUCTION

The setting for the problem under consideration is a large economic organization or system which in some cases is best thought of as the entire economy. Within this large economic organization resources are allocated by some combination of commands and prices (the exact mixture is inessential) or even by some other unspecified mechanism. The following question arises. For one particular isolated economic variable that needs to be regulated,[3] what is the best way to implement control for the benefit of the organization as a whole? Is it better to directly administer the activity under scrutiny or to fix transfer prices and rely on self-interested profit or utility maximization to achieve the same ends in decentralized fashion? This issue is taken as the prototype problem of central control which is studied in the present paper. There are a great many specific examples which fit nicely into such a framework. One of current interest is the question of whether it would be better to control certain forms of pollution by setting emission standards or by charging the appropriate pollution taxes.

When quantities are employed as planning instruments, the basic operating rules from the centre take the form of quotas, targets, or commands to produce a certain level of output. With prices as instruments, the rules specify either explicitly or implicitly that profits are to be maximized at the given parametric prices. Now a basic theme of resource allocation theory emphasizes the close connection between these two modes of control. No matter how one type of planning instrument is fixed, there is always a corresponding way to set the other which achieves the same result when implemented.[4] From a strictly theoretical point of view there is really nothing to recommend one mode of control over the other. This notwithstanding, I think it is a fair generalization to say that the average economist in the Western marginalist tradition has at least a vague preference toward indirect control by prices, just as the typical non-economist leans toward the direct regulation of quantities.

That a person not versed in economics should think primarily in terms of direct controls is probably due to the fact that he does not comprehend the full subtlety and strength of the invisible hand argument. The economist's attitude is somewhat more puzzling. Understanding that prices can be used as a powerful and flexible instrument for rationally allocating resources and that in fact a market economy automatically regulates itself in this manner is very different from being under the impression that such indirect controls are generally preferable for the kind of problem considered in this paper. Certainly a careful reading of economic theory yields little to support such a universal proposition.

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2 Many people have made helpful comments about a previous version of this paper. I would like especially to thank P. A. Diamond and H. E. Scarf for their valuable suggestions. The National Science Foundation helped support my research.
3 Outside the scope of this paper is the issue of *why* it is felt that the given economic activity must be regulated. There may be a variety of reasons, ranging all the way from political considerations to one form or another of market failure.
4 Given the usual convexity assumptions. Without convexity it may not be possible to find a price which will support certain output levels. In this connection it should be mentioned that non-convexities (especially increasing returns) are sometimes responsible for regulation in the first place.

Many economists point with favour to the fact that if prices are the planning instrument then profit maximization automatically guarantees total output will be efficiently produced, as if this result were of any more than secondary interest unless the prices (and hence total output) are optimal to begin with.[1] Sometimes it is maintained that prices are desirable planning instruments because the stimulus to obtain a profit maximizing output is built right in if producers are rewarded in proportion to profits. There is of course just as much motivation, e.g. to minimize costs at specified output levels so long as at least some fraction of production expenditures is borne by producers. With both modes of control there is clearly an incentive for self-interested producers to systematically distort information about hypothetical output and cost possibilities in the pre-implementation planning phase. Conversely, there is no real way to disguise the true facts in the implementation stage so long as actual outputs (in the case of price instruments) and true operating costs (in the case of quantity instruments) can be accurately monitored. For the one case the centre must ascertain *ceteris paribus* output changes as prices are varied, for the other price changes as outputs are altered.

A reason often cited for the theoretical superiority of prices as planning instruments is that their use allegedly economizes on information. The main thing to note here is that generally speaking it is neither easier nor harder to name the right prices than the right quantities because in principle exactly the *same* information is needed to correctly specify either. It is true that in a situation with many independent producers of an identical commodity, only a single uniform price has to be named by the centre, whereas in a command mode separate quantities must be specified for each producer. If such an observation has meaningful implications, it can only be within the artificial milieu of an iterative *tâtonnement* type of " planning game " which is played over and over again approaching an optimal solution in the limit as the number of steps becomes large. Even in this context the fact that there are less " message units " involved in each communication from the centre is a pretty thin reed on which to hang claims for the informational superiority of the price system. It seems to me that a careful examination of the mechanics of successive approximation planning shows that there is no principal informational difference between iteratively finding an optimum by having the centre name prices while the firms respond with quantities, or by having the centre assign quantities while the firm reveals costs or marginal costs.[2]

If there were really some basic intrinsic advantage to a system which employed prices as planning instruments, we would expect to observe many organizations operating with this mode of control, especially among multi-divisional business firms in a competitive

[1] An extreme example may help make this point clear. Suppose that fulfilment of an important emergency rescue operation demands a certain number of airplane flights. It would be inefficient to just order airline companies or branches of the military to supply a certain number of the needed aircraft because marginal (opportunity) costs would almost certainly vary all over the place. Nevertheless, such an approach would undoubtedly be preferable to the efficient procedure of naming a price for plane services. Under profit maximization, overall output would be uncertain, with too few planes spelling disaster and too many being superfluous.

[2] The " message unit " case for the informational superiority of the price system is analogous to the blanket statement that it is better to use dual algorithms for solving a programming problem whenever the number of primal variables exceeds the number of dual multipliers. Certainly for the superior large step decomposition type algorithms which on every iteration go right after what are presently believed to be the best instrument values on the basis of all currently available information, such a general statement has no basis. With myopic gradient methods it is true that on each round the centre infinitesimally and effortlessly adjusts exactly the number of instruments it controls, be they prices or quantities. But who can say *how many* infinitesimally small adjustments will be needed? Gradient algorithms are known to be a bad description of iterative planning procedures, among other reasons because they have inadmissably poor convergence properties. If the step size is chosen too small, convergence takes forever. If it is chosen too large, there is no convergence. As soon as a finite step size is selected on a given iteration to reflect a desire for quick convergence, the " message unit " case for prices evaporates. Calculating the *correct* price change puts the centre right back into the large step decomposition framework where on each round the problem of finding the best iterative prices is formally identical to the problem of finding the best iterative quantities. For discussion of these and various other aspects of iterative planning, see the articles of Heal [4], Malinvaud [5], Marglin [7], Weitzman [9].

environment. Yet the allocation of resources within private companies (not to mention governmental or non-profit organizations) is almost never controlled by setting administered transfer prices on commodities and letting self-interested profit maximization do the rest.[1] The price system as an allocator of internal resources does not itself pass the market test.[2]

Of course, all this is not to deny that in any *particular* setting there may be important *practical* reasons for favouring either prices or quantities as planning instruments. These reasons might involve ideological, political, legal, social, historical, administrative, motivational, informational, monitoring, enforcing, or other considerations.[3] But there is little of what might be called a system-free character.

In studying such a controversial subject, the only fair way to begin must be with the tenet that there is no *basic* or *universal* rationale for having a general predisposition toward one control mode or the other. If this principle is accepted, it becomes an issue of some interest to abstract away all " other " considerations in order to develop strictly " economic " criteria by which the comparative performance of price and quantity planning instruments might be objectively evaluated. Even on an abstract level, it would be useful to know how to identify a situation where employing one mode is relatively advantageous, other things being equal.

II. THE MODEL

We start with a highly simplified prototype planning problem. Amount q of a certain commodity can be produced at cost $C(q)$, yielding benefits $B(q)$.[4] The word " commodity " is used in an abstract sense and really could pertain to just about any kind of good from pure water to military aircraft. Solely for the sake of preserving a unified notation, we follow the standard convention that goods are desirable. This means that rather than talking about air pollution, for example, we instead deal with its negative—clean air.

Later we treat more complicated cases, but for the time being it is assumed that in effect there is just one producer of the commodity and no ambiguity in the notion of a cost curve. Benefits are measured in terms of money equivalents so that the benefit function can be viewed as the reflection of an indifference curve showing the trade-off between amounts of uncommitted extra funds and output levels of the given commodity. It is assumed that $B''(q)<0$, $C''(q)>0$, $B'(0)>C'(0)$, and $B'(q)<C'(q)$ for q sufficiently large.

1 Strictly speaking, this conclusion is not really justified because there may be important externalities or increasing returns within an organization (they may even constitute its *raison d'être*). Nevertheless, the almost universal absence of internal transfer pricing within private firms strikes me as a rather startling contradiction with the often alleged superiority of indirect controls.

2 About a decade ago, Ford and GM performed a few administrative trials of a limited sort with some decentralization schemes based on internal transfer prices. The experiments were subsequently discontinued in favour of a return to more traditional planning methods. See Whinston [10].

3 As one example, if it happens to be the case that it is difficult or expensive to monitor output on a continuous scale but relatively cheap to perform a pass-fail litmus type test on whether a given output level has been attained or not, the price mode may be greatly disadvantaged from the start. The pollution by open-pit mining operations of nearby waterways presents a case in point. It would be difficult or impossible to record how much pollutant is seeping into the ground, whereas it is a comparatively straightforward task to enforce the adoption of one or another level of anti-pollution technology. Another realistic consideration arises when we ask who determines the standards under each mode. For example, if an agency of the executive branch is empowered to regulate prices but the legislature is in charge of setting quantities, that by itself may be important in determining which mode is better for controlling pollution. The price mode would have greater flexibility, but might carry with it more danger of caving in to special interest groups. As yet another realistic consideration, equity arguments are sometimes put forward in favour of price (the supposed " justice " of a uniform price to all) or quantity (equal sharing of a deficit commodity) control modes.

4 It might be thought that an equivalent approach would be to work with demand and supply curves, identifying the consumers' (producers') surplus area under the demand (supply) curve as benefits (costs) or, equivalently, the demand (supply) curve as the marginal benefit (cost) function. The trouble with this approach is that it tends to give the misleading impression that the market left to itself could solve the problem, obscuring the fact that some key element of the standard competitive supply and demand story is felt to be missing in the first place.

The planning problem is to find that value q^* of q which maximizes

$$B(q)-C(q).$$

The solution must satisfy

$$B'(q^*) = C'(q^*).$$

With

$$p^* \equiv B'(q^*) = C'(q^*),$$

it makes no difference whether the planners announce the optimal price p^* and have the producers maximize profits

$$p^*q-C(q)$$

or whether the centre merely orders the production of q^* at least cost. In an environment of complete knowledge and perfect certainty there is a formal identity between the use of prices and quantities as planning instruments.

If there is any advantage to employing price or quantity control modes, therefore, it must be due to inadequate information or uncertainty. Of course it is natural enough for planners to be unsure about the precise specification of cost and benefit functions since even those most likely to know can hardly possess an exact account.

Suppose, then, that the centre perceives the cost function only as an estimate or approximation. The stochastic relation linking q to C is taken to be of the form

$$C(q, \theta),$$

where θ is a disturbance term or random variable, unobserved and unknown at the present time. While the determination of θ could involve elements of genuine randomness,[1] it is probably more appropriate to think primarily in terms of an information gap.

Even the engineers most closely associated with production would be unable to say beforehand precisely what is the cheapest way of generating various hypothetical output levels. How much murkier still must be the centre's *ex ante* conception of costs, especially in a fast moving world where knowledge of particular circumstances of time and place may be required. True, the degree of fuzziness could be reduced by research and experimentation but it could never be truly eliminated because new sources of uncertainty are arising all the time.[2]

Were a particular output level really ordered in all seriousness, a cost-minimizing firm could eventually grope its way toward the cheapest way of producing it by actually testing out the relevant technological alternatives. Or, if an output price were in fact named, a profit maximizing production level could ultimately be found by trial and error. But this is far from having the cost function as a whole knowable *a priori.*

While the planners may be somewhat better acquainted with the benefit function, it too is presumably discernable only tolerably well, say as

$$B(q, \eta)$$

with η a random variable. The connection between q and B is stochastic either because benefits may be imperfectly known at the present time or because authentic randomness may play a role. Since the unknown factors connecting q with B are likely to be quite different from those linking q to C, it is assumed that the random variables θ and η are independently distributed.

As a possible specific example of the present formulation, consider the problem of air pollution. The variable q could be the cleanliness of air being emitted by a certain type of source. Costs as a function of q might not be known beyond doubt because the technology, quantified by θ, is uncertain. At a given level of q the benefits may be unsure since they depend among other things on the weather, measured by η.

1 Like day-to-day fluctuations.
2 For an amplification of some of these points, see Hayek [3].

Now an *ideal* instrument of central control would be a contingency message whose instructions depend on which state of the world is revealed by θ and η. The ideal *ex ante* quantity signal $q^*(\theta, \eta)$ and price signal $p^*(\theta, \eta)$ are in the form of an entire schedule, functions of θ and η satisfying

$$B_1(q^*(\theta, \eta), \eta) = C_1(q^*(\theta, \eta), \theta) = p^*(\theta, \eta).$$

By employing either ideal signal, the *ex ante* uncertainty has in effect been eliminated *ex post* and we are right back to the case where there is no theoretical difference between price and quantity control modes.

It should be readily apparent that it is infeasible for the centre to transmit an entire schedule of ideal prices or quantities. A contingency message is a complicated, specialized contract which is expensive to draw up and hard to understand. The random variables are difficult to quantify. A problem of differentiated information or even of moral hazard may be involved since the exact value of θ will frequently be known only by the producer.[1] Even for the simplest case of just *one* firm, information from different sources must be processed, combined, and evaluated. By the time an ideal schedule was completed, another would be needed because meanwhile changes would have occurred.

In this paper the realistic issue of central control under uncertainty is considered to be the " second best " problem of finding for each producer the single price or quantity message which optimally regulates his actions. This is also the best way to focus sharply and directly on the essential difference between prices and quantities as planning instruments.

The issue of prices *vs.* quantities has to be a " second best " problem by its very nature simply because there is no good *a priori* reason for limiting attention to just these two particular signals. Even if stochastic contingency messages were eliminated on *ad hoc* grounds as being too complicated, there would still be no legitimate justification for not considering, say, an entire expected benefits schedule, or a " kinked " benefit function in the form of a two-tiered price system, or something else. The reason we specialize to price and quantity signals is that these are two *simple* messages, easily comprehended, traditionally employed, and frequently contrasted.[2]

The optimal quantity instrument under uncertainty is that target output $\hat{q}$ which maximizes expected benefits minus expected costs, so that

$$E[B(\hat{q}, \eta) - C(\hat{q}, \theta)] = \max_q E[B(q, \eta) - C(q, \theta)],$$

where $E[.]$ is the expected value operator. The solution $\hat{q}$ must satisfy the first order condition

$$E[B_1(\hat{q}, \eta)] = E[C_1(\hat{q}, \theta]. \qquad \ldots(1)$$

When a price instrument p is announced, production will eventually be adjusted to the output level

$$q = h(p, \theta)$$

which maximizes profits given p and θ. Such a condition is expressed as

$$ph(p, \theta) - C(h(p, \theta), \theta) = \max_q pq - C(q, \theta),$$

implying

$$C_1(h(p, \theta), \theta) = p. \qquad \ldots(2)$$

1 So that it may be inappropriate, for example, to tell him to produce less if costs are high unless a very sophisticated incentive scheme goes along with such a message. For an elaboration of some of these points see Arrow [1], pp. 321-322.

2 There are real costs associated with using more complicated signals. At least implicitly, we are assuming that the magnitude of such costs is sufficiently large to make it uneconomical to consider messages other than prices or quantities. It would be nice to incorporate these costs explicitly into the model, but this is hard to do in any meaningful way.

If the planners are rational, they will choose that price instrument $\tilde{p}$ which maximizes the expected difference between benefits and costs given the reaction function $h(p, \theta)$:

$$E[B(h(\tilde{p}, \theta), \eta) - C(h(\tilde{p}, \theta), \theta)] = \max_{p} E[B(h(p, \theta), \eta) - C(h(p, \theta), \theta)].$$

The solution $\tilde{p}$ must obey the first order equation

$$E[B_1(h(\tilde{p}, \theta), \eta) \, . \, h_1(\tilde{p}, \theta)] = E[C_1(h(\tilde{p}, \theta), \theta) \, . \, h_1(\tilde{p}, \theta)],$$

which can be rewritten as

$$\tilde{p} = \frac{E[B_1(h(\tilde{p}, \theta), \eta) \, . \, h_1(\tilde{p}, \theta)]}{E[h_1(\tilde{p}, \theta)]}. \qquad \ldots(3)$$

Corresponding to the optimal *ex ante* price $\tilde{p}$ is the *ex post* profit maximizing output $\tilde{q}$ expressed as a function of θ,

$$\tilde{q}(\theta) \equiv h(\tilde{p}, \theta). \qquad \ldots(4)$$

In the presence of uncertainty, price and quantity instruments transmit central control in quite different ways. It is important to note that by choosing a specific mode for implementing an intended policy, the planners are at least temporarily locking themselves into certain consequences. The values of η and θ are at first unknown and only gradually, if at all, become recognized through their effects. After the quantity $\hat{q}$ is prescribed, producers will continue to generate that assigned level of output for some time even though in all likelihood

$$B_1(\hat{q}, \eta) \neq C_1(\hat{q}, \theta).$$

In the price mode on the other hand, $\tilde{q}(\theta)$ will be produced where except with negligible probability

$$B_1(\tilde{q}(\theta), \eta) \neq C_1(\tilde{q}(\theta), \theta).$$

Thus neither instrument yields an optimum *ex post*. The relevant question is which one comes closer under what circumstances.[1]

In an infinitely flexible control environment where the planners can continually adjust instruments to reflect current understanding of a fluid situation and producers instantaneously respond, the above considerations are irrelevant and the choice of control mode should be made to depend on other factors. Similar comments apply to a timeless *tâtonnement* milieu where iterations are costless, recontracting takes place after each round, and in effect nothing real is presumed to happen until all the uncertainty has been eliminated and an equilibrium is approached. In any less hypothetical world the consequences of an order given in a particular control mode have to be lived with for at least the time until revisions are made, and real losses will be incurred by selecting the wrong communication medium.

Note that the question usually asked whether it is better to control prices or quantities for *finding* a plan is conceptually distinct from the issue treated in this paper of which mode is superior for *implementing* a plan. The latter way of posing the problem strikes me as more relevant for most actual planning contexts—either because there is no significant informational difference between the two modes in the first place, or because a step in the *tâtonnement* planning game cannot meaningfully occur unless it is really implemented, or because no matter how many iterations have been carried out over time there are always spontaneously arising changes which damp out the significance of knowing past history. In the framework adopted here, the planners are at the decision node where as much information as is feasible to gather has already been obtained by one means or another and an operational plan must be decided on the basis of the available current knowledge.

[1] We remark in passing that the issue of whether it is better to stabilize uncertain demand and supply functions by pegging prices or quantities can also be put in the form of the problem analysed in this paper if benefits are associated with the consumers' surplus area under the demand curve and costs with the producers' surplus area under the supply curve.

III. PRICES *vs.* QUANTITIES

It is natural to define the *comparative advantage of prices over quantities* as

$$\Delta \equiv E[(B(\tilde{q}(\theta), \eta) - C(\tilde{q}(\theta), \theta)) - (B(\hat{q}, \eta) - C(\hat{q}, \theta))]. \qquad \text{...(5)}$$

The loss function implicit in the definition of Δ is the expected difference in gains obtained under the two modes of control. Naturally there is no real distinction between working with Δ or with $-\Delta$ (the comparative advantage of quantities over prices).

The coefficient Δ is intended to be a measure of *comparative* or *relative* advantage only. It goes without saying that making a decision to use price or quantity control modes in a specific instance is more complicated than just consulting Δ. There are also going to be a host of practical considerations formally outside the scope of the present model. Although such external factors render Δ of limited value when isolated by itself, they do not necessarily diminish its conceptual significance. On the contrary, having an objective criterion of the *ceteris paribus* advantage of a control mode is very important because conceptually it can serve as a benchmark against which the cost of " non-economic " ingredients might be measured in reaching a final judgment about whether it would be better to employ prices or quantities as planning instruments in a given situation.

As it stands, the formulation of cost and benefit functions is so general that it hinders us from cleanly dissecting equation (5). To see clearly what Δ depends on we have to put more structure on the problem. It is possible to be somewhat less restrictive than we are going to be, but only at the great expense of clarity.

In what follows, the amount of uncertainty in marginal cost is taken as sufficiently small to justify a second order approximation of cost and benefit functions within the range of $\tilde{q}(\theta)$ as it varies around $\hat{q}$.[1] Let the symbol " $\cong$ " denote an " accurate local approximation " in the sense of deriving from the assumption that cost and benefit functions are of the following quadratic form within an appropriate neighbourhood of $q = \hat{q}$:

$$C(q, \theta) \cong a(\theta) + (C' + \alpha(\theta))(q - \hat{q}) + \frac{C''}{2}(q - \hat{q})^2 \qquad \text{...(6)}$$

$$B(q, \eta) \cong b(\eta) + (B' + \beta(\eta))(q - \hat{q}) + \frac{B''}{2}(q - \hat{q})^2. \qquad \text{...(7)}$$

In the above equations $a(\theta)$, $\alpha(\theta)$, $b(\eta)$, $\beta(\eta)$ are stochastic functions and C', C'', B', B'' are fixed coefficients.

Without loss of generality, $\alpha(\theta)$ and $\beta(\eta)$ are standardized in (6), (7) so that their expected values are zero:

$$E[\alpha(\theta)] = E[\beta(\eta)] = 0. \qquad \text{...(8)}$$

Since θ and η are independently distributed,

$$E[\alpha(\theta) . \beta(\eta)] = 0. \qquad \text{...(9)}$$

Note that the stochastic functions

$$a(\theta) \cong C(\hat{q}, \theta)$$

$$b(\eta) \cong B(\hat{q}, \eta)$$

translate different values of θ and η into pure vertical shifts of the cost and benefit curves.

Differentiating (6) and (7) with respect to q,

$$C_1(q, \theta) \cong (C' + \alpha(\theta)) + C'' . (q - \hat{q}) \qquad \text{...(10)}$$

$$B_1(q, \eta) \cong (B' + \beta(\eta)) + B'' . (q - \hat{q}). \qquad \text{...(11)}$$

[1] Such an approximation can be rigorously defended along the lines developed by Samuelson [8].

Employing the above equations and (8), the following interpretations are available for the fixed coefficients of (6), (7):

$$C' \cong E[C_1(\hat{q}, \theta)]$$

$$B' \cong E[B_1(\hat{q}, \eta)]$$

$$C'' \cong C_{11}(q, \theta)$$

$$B'' \cong B_{11}(q, \eta).$$

From (1),

$$B' = C'. \qquad \text{...(12)}$$

It is apparent from (8) and (10) that stochastic changes in $\alpha(\theta)$ represent pure unbiased shifts of the marginal cost function. The variance of $\alpha(\theta)$ is precisely the mean square error in marginal cost

$$\sigma^2 \equiv E[(C_1(q, \theta)-E[C_1(q, \theta)])^2] \cong E[\alpha(\theta)^2]. \qquad \text{...(13)}$$

Analogous comments hold for the marginal benefit function (11) where we have

$$E[(B_1(q, \eta)-E[B_1(q, \eta)])^2] = E[\beta(\eta)^2].$$

From (10) and (2),

$$h(p, \theta) \cong \hat{q} + \frac{p-C'-\alpha(\theta)}{C''} \qquad \text{...(14)}$$

implying

$$h_1(p, \theta) \cong \frac{1}{C''}. \qquad \text{...(15)}$$

Substituting from (15) into (3) and cancelling out C'' yields

$$\tilde{p} \cong E[B_1(h(\tilde{p}, \theta), \eta)]. \qquad \text{...(16)}$$

Replacing q in (11) by the expression for $h(\tilde{p}, \theta)$ from (14) and plugging into (16), the following equation is obtained after using (8)

$$\tilde{p} \cong B' + \frac{B''}{C''}(\tilde{p}-C'). \qquad \text{...(17)}$$

From (12) and the condition $B''<0<C''$, (17) implies

$$\tilde{p} \cong C'. \qquad \text{...(18)}$$

Combining (4), (14), and (18),

$$\tilde{q}(\theta) \cong \hat{q} - \frac{\alpha(\theta)}{C''}. \qquad \text{...(19)}$$

Now alternately substitute $q = \hat{q}$ and $q = \tilde{q}(\theta)$ from (19) into (6) and (7). Then plugging the resulting values of (6), (7) into (5), using (8), (9), and collecting terms,

$$\Delta \cong \frac{\sigma^2 B''}{2C''^2} + \frac{\sigma^2}{2C''}. \qquad \text{...(20)}$$

Expression (20) is the fundamental result of this paper.[1] The next section is devoted to examining it in detail.

[1] In the supply and demand context B'' is the slope of the (linear) demand curve, C'' is the slope of the (linear) supply curve, and σ^2 is the variance of vertical shifts in the supply curve.

IV. ANALYSING THE COEFFICIENT OF COMPARATIVE ADVANTAGE

Note that the uncertainty in benefits does not appear in (20).[1] To a second-order approximation it affects price and quantity modes *equally* adversely. On the other hand, Δ depends linearly on the mean square error in marginal cost. The *ceteris paribus* effect of increasing σ^2 is to magnify the expected loss from employing the planning instrument with comparative disadvantage. Conversely, as σ^2 shrinks to zero we move closer to the perfect certainty case where in theory the two control modes perform equally satisfactorily.

Clearly Δ depends critically on the curvature of cost and benefit functions around the optimal output level. The first thing to note is that the sign of Δ simply equals the sign of $C''+B''$. When the sum of the " other " considerations nets out to a zero bias toward either control mode, quantities are the preferred planning instrument if and only if benefits have more curvature than costs.

Normally we would want to know the magnitude of Δ and what it depends on, as well as the sign. To strengthen our intuitive feeling for the meaning of formula (20), we turn first to some extreme cases where there is a strong comparative advantage to one control mode over the other. In this connection it is important to bear in mind that when we talk about " large " or " small " values of B'', C'', or σ^2, we are only speaking in a relative sense. The absolute measure of any variable appearing in (20) does not really mean much alone since it is arbitrarily pegged by selecting the units in which output is reckoned.

The coefficient Δ is negative and large as either the benefit function is more sharply curved or the cost function is closer to being linear. Using a price control mode in such situations could have detrimental consequences. When marginal costs are nearly flat, the smallest miscalculation or change results in either much more or much less than the desired quantity. On the other hand, if benefits are almost kinked at the optimum level of output, there is a high degree of risk aversion and the centre cannot afford being even slightly off the mark. In both cases the quantity mode scores a lot of points because a high premium is put on the rigid output controllability which only it can provide under uncertainty.

From (20), the price mode looks relatively more attractive when the benefit function is closer to being linear. In such a situation it would be foolish to name quantities. Since the marginal social benefit is approximately constant in some range, a superior policy is to name it as a price and let the producers find the optimal output level themselves, after eliminating the uncertainty from costs.

At a point where the cost function is highly curved, Δ becomes nearly zero. If marginal costs are very steeply rising around the optimum, as with fixed capacity, there is not much difference between controlling by price or quantity instruments because the resulting output will be almost the same with either mode. In such a situation, as with the case $\sigma^2 = 0$, " non-economic " factors should play the decisive role in determining which system of control to impose.

It is difficult to refrain from noticing that although there are plenty of instances where

[1] This is because the *expected* benefit function (see equation (7)) does not depend on the variance of marginal benefits so long as costs and benefits are independently distributed. If they are *not*, so that

$$\sigma_{bc}^2 \equiv E[\{C_1(q,\theta)-E[C_1(q,\theta)]\}.\{B_1(q,\eta)-E[B_1(q,\eta)]\}] = E[\alpha(\theta).\beta(\eta)] \neq 0,$$

(20) must be replaced by: $\Delta \cong \frac{\sigma^2 B''}{2C''^2} + \frac{1}{2C''}(\sigma^2 - 2\sigma_{bc}^2)$. The sole effect of having costs and benefits correlated with each other is embodied in the term σ_{bc}^2. When marginal costs are positively correlated with marginal benefits, the *ceteris paribus* comparative advantage of the quantity mode is increased. If prices are used as a control mode, the producer will tend to cut back output for high marginal costs. But with σ_{bc}^2 positive, this is the very same time that marginal benefits tend to be high, so that a cutback may not really be in order. In such situations the quantity mode has better properties as a stabilizer, other things being equal. The story is the other way around when σ_{bc}^2 is negative. In that case high marginal costs are associated with low marginal benefits, so that the price mode (which decreases output for high marginal costs) tends to be a better mode of control other things being equal.

the price mode has a good solid comparative advantage (because $-B''$ is small), in some sense it looks as if prices can be a *disastrous* choice of instrument far more often than quantities can. Using (20), $\Delta \to -\infty$ if either $B'' \to -\infty$ or $C'' \to 0$ (or both). The only way $\Delta \to +\infty$ is under the thin set of circumstances where simultaneously $C'' \to 0$, $B'' \to 0$, and $C'' > -B''$. In a world where C'' and B'' are themselves imperfectly known it seems hard to avoid the impression that there will be many circumstances where the more conservative quantity mode will be preferred by planners because it is better for avoiding very bad planning mistakes.[1]

Having seen how C'' and B'' play an essential role in determining Δ, it may be useful to check out a few of the principal situations where we might expect to encounter cost and benefit functions of one curvature or another. We start with costs.

Contemporary economic theory has tended to blur the distinction between the traditional marginalist way of treating production theory with smoothly differentiable production functions and the activity analysis approach with its limited number of alternative production processes. For many theoretical purposes convexity of the underlying technology is really the fundamental property.

However, there are very different implications for the efficacy of price and quantity control modes between a situation described by classically smooth Marshallian cost curves and one characterized by piecewise linear cost functions with a limited number of kinks. In the latter case, the quantity mode tends to have a relative advantage since $\Delta = -\infty$ on the flats and $\Delta = 0$ at the elbows. Of course it is impossible to use a price to control an output at all unless some hidden fixed factors take the flatness out of the average cost curve. Even then, Δ will be positive only if there are enough alternative techniques available to make the cost function have more (finite difference) curvature than the benefit function in the neighbourhood of an optimal policy.

What determines the benefit function for a commodity is contingent in the first place on whether the commodity is a final or intermediate good. The benefit of a final good is essentially the utility which arises out of consuming the good. It could be highly curved at the optimum output level if tastes happen to be kinked at certain critical points. The amount of pollution which makes a river just unfit for swimming could be a point where the marginal benefits of an extra unit of output change very rapidly. Another might be the level of defence which just neutralizes an opponent's offence or the level of offence which just overcomes a given defence. There are many examples which arise in emergencies or natural calamities. Our intuitive feeling, which is confirmed by the formal analysis, is that it doesn't pay to " fool around " with prices in such situations.

For intermediate goods, the shape of the benefit function will depend among other things on the degree of substitutability in use of this commodity with other resources available in the production organization and upon the possibilities for importing this

[1] This idea could be formalized as follows. Consider two generalizations of formulae (6) and (7):

$$C(q, \theta) \cong a(\theta) + (C' + \alpha(\theta))(q - \hat{q}) + \frac{C''}{2f(\theta)}(q - \hat{q})^2$$

$$B(q, \eta) \cong b(\eta) + (B' + \beta(\eta))(q - \hat{q}) + \frac{B'' g(\eta)}{2}(q - \hat{q})^2.$$

The only difference with (6), (7) is that now $1/C_{11}(q, \theta)$ and $B_{11}(q, \eta)$ are allowed to be uncertain. The change in the profit maximizing output response per unit price change is now stochastic, $h_1(p, \theta) = f(\theta)/C''$. Without loss of generality we set

$$E[f(\theta)] = E[g(\eta)] = 1.$$

Note that increasing the variance of f (g) is a mean preserving spread of C_{11} (B_{11}). Suppose for simplicity that f and α are independent of each other. Then we can derive the appropriate generalization of (20) as

$$\Delta \cong \frac{B''\sigma^2(1+\delta^2)}{2C''^2} + \frac{\sigma^2}{2C''},$$

where $\delta^2 \equiv E[\{f(\theta) - E[f(\theta)]\}^2]$ is the variance of $f(\theta)$. The above formula can be interpreted as saying that other things being equal, greater uncertainty in $1/C_{11}(q, \theta)$ increases the comparative advantage of the quantity mode.

commodity from outside the organization. These things in turn are very much dependent on the planning time horizon. In the long run the benefit function probably becomes flatter because more possibilities for substitution are available, including perhaps importing. Take for example the most extreme degree of complete " openness " where any amount of the commodity can be instantaneously and effortlessly bought (and sold) outside the production organization at a fixed price. The relevant benefit function is of course just a straight line whose slope is the outside price.

There is, it seems to me, a rather fundamental reason to believe that quantities are better signals for situations demanding a high degree of coordination. A classical example would be the short run production planning of intermediate industrial materials. Within a large production organization, be it the General Motors Corporation or the Soviet industrial sector as a whole, the need for balancing the output of any intermediate commodity whose production is relatively specialized to this organization and which cannot be effortlessly and instantaneously imported from or exported to a perfectly competitive outside world puts a kink in the benefit function. If it turns out that production of ball bearings of a certain specialized kind (plus reserves) falls short of anticipated internal consumption, far more than the value of the unproduced bearings can be lost. Factors of production and materials that were destined to be combined with the ball bearings and with commodities containing them in higher stages of production must stand idle and are prevented from adding value all along the line. If on the other hand more bearings are produced than were contemplated being consumed, the excess cannot be used immediately and will only go into storage to lose implicit interest over time. Such short run rigidity is essentially due to the limited substitutability, fixed coefficients nature of a technology based on machinery.[1] Other things being equal, the asymmetry between the effects of overproducing and underproducing are more pronounced the further removed from final use is the commodity and the more difficult it is to substitute alternative slack resources or to quickly replenish supplies by emergency imports. The resulting strong curvature in benefits around the planned consumption levels of intermediate materials tends to create a very high comparative advantage for quantity instruments. If this is combined with a cost function that is nearly linear in the relevant range, the advantage of the quantity mode is doubly compounded.[2]

V. MANY PRODUCTION UNITS

Consider the same model previously developed except that now instead of being a single good, $q = (q_1, \ldots, q_n)$ is an n-vector of commodites. The various components of q might represent physically distinct commodities or they could denote amounts of the same commodity produced by different production units. Benefits are $B(q, \eta)$ and the cost of producing the ith good is $c^i(q_i, \theta_i)$. As before, for each i the two random variables η and θ_i are distributed independently of each other.

Suppose the issue of control is phrased as choosing either the quantities $\{\hat{q}_i\}$ which maximize

$$E\left[B(q_i, \eta) - \sum_1^n c^i(q_i, \theta_i)\right],$$

[1] The existence of buffer stocks changes the point at which the kink occurs, but does not remove it. For a more detailed treatment of this entire topic, see Manove [6].

[2] Note that in the context of an autarchic planned economy, such pessimistic conclusions about the feasibility of using Lange-Lerner price signals to control short run output do not carry over to, say, agriculture. The argument just given for a kinked benefit function would not at all pertain to a food crop, which goes more or less directly into final demand. In addition, the cost function for producing a given agricultural commodity ought to be much closer to the classical smooth variety than to the linear programming type with just a few kinks.

or the prices $\{\tilde{p}_i\}$ which maximize

$$E[B(h(p, \theta), \eta)-\Sigma c^i(h_i(p_i, \theta_i), \theta_i)],$$

where $\{h_i(p_i, \theta_i)\}$ are defined analogously to (2).

Naturally the coefficient of comparative advantage is now defined as

$$\Delta_n \equiv E\left[\left\{B\left(\tilde{q}(\theta), \eta\right) - \sum_1^n c^i(\tilde{q}_i(\theta_i), \theta_i)\right\} - \left\{B(\hat{q}, \eta) - \sum_1^n c^i(\hat{q}_i, \theta_i)\right\}\right].$$

Assuming locally quadratic costs and benefits, it is a straightforward generalization of what was done in Section III to derive the analogue of expression (20),

$$\Delta_n \cong \sum_{i=1}^n \sum_{j=1}^n \frac{B_{ij}\sigma_{ij}^2}{2c_{11}^i c_{11}^j} + \sum_{i=1}^n \frac{\sigma_{ii}^2}{2c_{11}^i}, \qquad \ldots(21)$$

where

$$\sigma_{ij}^2 \cong E[\{c_1^i(q_i, \theta_i)-E[c_1^i(q_i, \theta_i)]\}\{c_1^j(q_j, \theta_j)-E[c_1^j(q_j, \theta_j)]\}]. \qquad \ldots(22)$$

To correct for the pure effect of n on Δ_n, it is more suitable to work with the transformed cost functions

$$C^i(x_i, \theta_i) \equiv nc^i(x_i/n, \theta_i). \qquad \ldots(23)$$

The meaning of C^i is most readily interpreted for the situation where n different units are producing the same commodity or a close substitute with similar cost functions. Then C^i is what total costs would be as a function of total output if each production unit were an identical replica of the ith unit. When " other things being equal " n is changed, it is more appropriate to think of C^i being held constant rather than c^i.

With C^i defined by (23), we have

$$C_1^i = c_1^i \qquad \ldots(24)$$

$$C_{11}^i = \frac{c_{11}^i}{n}. \qquad \ldots(25)$$

Relation (24) means that in the quadratic case the coefficients of the marginal cost variance-covariance matrix for the $\{C_1^i\}$ are the same as those given by (22) for the $\{c_1^i\}$. Substituting (25) into (21),

$$\Delta_n \cong \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n \frac{B_{ij}\sigma_{ij}^2}{2C_{11}^i C_{11}^j} + \frac{1}{n} \sum_{i=1}^n \frac{\sigma_{ii}^2}{2C_{11}^i}. \qquad \ldots(26)$$

The above formula shows that in effect the original expression for Δ holds *on the average* for Δ_n when there is more than one producer. Naturally the generalization (26) is more complicated, but the interpretation of it is basically similar to the diagnosis of (20) which was just given in the previous section.

There is, however, a fundamental distinction between having one and many producers which is concealed in formula (26). With some degree of independence among the distributions of individual marginal costs, less weight will be put on the first summation term of (26). Other things being equal, in situations with more rather than fewer independent units producing outputs which substitute for each other in yielding benefits, there is a correspondingly greater relative advantage to the price mode of control. Although this point has general validity, it can be most transparently seen in the special regularized case of one good being produced by many micro-units with symmetrical cost functions. In such a case

$$B_{ij} = B'' \qquad \ldots(27.\text{i})$$

$$C_{11} = C'' \qquad \ldots(27.\text{ii})$$

$$\sigma_{ii}^2 = \sigma^2 \qquad \text{...(27.iii)}$$

$$\sigma_{ij}^2 = \rho\sigma^2, \quad i \neq j, \quad -1 \leqq \rho \leqq 1. \qquad \text{...(27.iv)}$$

The coefficient ρ is a measure of the correlation between marginal costs of separate production units. If all units are pretty much alike and are using a similar technology, ρ is likely to be close to unity. If the cost functions of different units are more or less independent of each other, ρ should be nearly zero. While in theory the correlation coefficient can vary between plus and minus unity, for most situations of practical interest the marginal costs of two different production units will have a non-negative cross correlation.

Using (27), (26) can be rewritten as

$$\Delta_n \cong \rho\left(\frac{B''\sigma^2}{2C''^2} + \frac{\sigma^2}{2C''}\right) + (1-\rho)\left(\frac{1}{n}\frac{B''\sigma^2}{2C''^2} + \frac{\sigma^2}{2C''}\right). \qquad \text{...(28)}$$

If the marginal costs of each identical micro-unit are perfectly correlated with each other so that $\rho = 1$, it is as if there is but a single producer and we are exactly back to the original formula (20). With $n>1$, as ρ decreases, Δ_n goes up. A *ceteris paribus* move from dependent toward independent costs increases the comparative advantage of prices, an effect which is more pronounced as the number of production units is larger. If there are three distinctly different types of sulphur dioxide emitters with independent technologies instead of one large pollution source yielding the same aggregate effect, a relatively stronger case exists for using prices to regulate output.

When it is desired to control different units producing an identical commodity by setting prices, only a single price need be named as an instrument. The price mode therefore possesses the *ceteris paribus* advantage that output is being produced efficiently *ex post*. With prices as instruments

$$c_1^i(\tilde{q}_i, \theta_i) = c_1^j(\tilde{q}_j, \theta_j) = \tilde{p},$$

whereas with quantities

$$c_1^i(\hat{q}_i, \theta_i) \neq c_1^j(\hat{q}_j, \theta_j)$$

except on a set of negligible probability.

Using prices thus enables the centre to automatically screen out the high cost producers, encouraging them to produce less and the low cost units more. This predominance in efficiency makes the comparative advantage of the price mode go up as the number of independent production units becomes larger, other things being equal. The precise statement of such a proposition would depend on exactly what was held equal as n was increased—the variance of *individual* costs or the overall variance of *total* costs. For simplicity consider the case of completely independent marginal costs, $\rho = 0$. Then (28) becomes

$$\Delta_n \cong \frac{1}{n}\frac{B''\sigma^2(n)}{2C''^2} + \frac{\sigma^2(n)}{2C''}, \qquad \text{...(29)}$$

where $\sigma^2(n)$ is implicitly some (given) function of n. If the " other thing " being equal is the constant variance of marginal costs for each individual producing unit, then $\sigma^2(n) \equiv \sigma^2$. If the variance of total costs is held constant as n varies, $\sigma^2(n) \equiv n\sigma^2$. Either way Δ_n in (29) increases monotonically with n and eventually becomes positive.

It is important to note that such *ceteris paribus* efficiency advantages of the price mode as we have been considering for large n are by no means enough to guarantee that Δ_n will be positive in a particular situation for any *given n*. True, what aggregate output is forthcoming under the price mode will be produced at least total cost. But it might be the wrong overall output level to start with. If the $\{-B_{ij}\}$ are sufficiently large or the $\{C_{11}^i\}$ sufficiently small, it may be advantageous to enjoy greater control over total output by setting individual quotas, even after taking account (as our formula for Δ_n does) of the losses incurred by the *ex post* productive inefficiency of such a procedure.[1]

Returning to the general case with which this section began, we note that the basic difference between benefits and costs becomes somewhat more transparent in the n commodity vector formulation. Only the centre knows benefits. Even if it could be done it would not help to transmit $B(.)$ to individual production units because benefits are typically a non-separable function of *all* the units' outputs, whereas a particular unit has control only over its *own* output. In any well formulated mode of decentralized control, the objective function to be maximized by a given unit must depend in some well-defined way on *its* decisions alone. For the purposes of our formulation B need not be a benefit and the $\{c^i\}$ need not be costs in the usual sense, although in many contexts this is the most natural interpretation. The crucial distinction is that B is in principle knowable only by the centre, whereas c^i is best known by firm i.[2]

When uncertainties in individual costs are unrelated so that the random variables θ_i and θ_j are independently distributed, the decision to use a price or quantity instrument to control q_i alone is decentralizable. Suppose it has already been resolved by one means or another whether to use price or quantity instruments to control q_j for each $j \neq i$. To a quadratic approximation, the comparative advantage of prices over quantities for commodity i is

$$\Delta^i \cong \frac{\sigma_{ii}^2 B_{ii}}{2c_{11}^{i2}} + \frac{\sigma_{ii}^2}{2c_{11}^i}, \qquad \ldots(30)$$

which is exactly the formula (20) for this particular case.

In some situations, " mixed " price-quantity modes may give the best results. As a specific example, suppose that q_1 is the catch of a certain fish from a large lake and q_2 from a small but prolific pond. Let q_1 be produced with relatively flat average costs but q_2 have a cost function which is curved at the optimum somewhat more than the benefit function. The optimal policy according to (30) will be to name a quota for q_1 and a price for q_2.

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1 An even better procedure from a theoretical point of view in the case where an identical output is produced by many firms would be to fix *total* output by command and subdivide it by a price mechanism. This kind of solution is proposed by Dales [2] who would set up a market in " pollution rights ", the fixed supply of which is regulated by the government. In effect, such an approach aggregates the individual cost functions, and we are right back to a single cost function. Note that a basic question would still remain: is it better to fix the total amount by a quantity or price control mode?

2 An interesting application of the ideas of this section is provided by the problem of choosing a control mode for best distributing a deficit commodity in fixed supply (say gasoline). In this case what we have been calling an individual cost function, $c^i(q_i, \theta_i)$, would really be the negative of a user's benefit function (as measured by the area under his demand curve). Our B function (of total demand) would just reflect the opportunity loss of having a surplus or shortage of the implied amount when only a fixed supply is available. All the considerations of this section would apply in determining the coefficient of comparative advantage. Accurately characterizing the B function seems especially difficult in the present context. If the commodity can be bought from or sold to the outside world, the B function would just embody the terms of this opportunity (in particular, it would be flat if any amount of the commodity could be bought or sold at some fixed price). Under autonomy, the shape of the B function would depend on what is done in a surplus or deficit situation. With a surplus (from naming too high a price), it would depend on the value to future allocation possibilities of the excess supply, relative to what welfare was lost at the present

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time from having demand less than supply. With a deficit (from naming too low a price), the loss of welfare hinges on how shortages are actually distributed among consumers. If shortages result in some people doing completely without the product, the overall welfare losses may be very great and $|B''|$ could be large. If there is some inherent reason to believe that shortages will automatically be evenly distributed, then $|B''|$ may not be so big. In addition to redistribution losses, there will always be waiting time losses in a shortage. Finally, note that if the amount of the fixed supply is known, a superior policy to naming prices or quantities is to distribute ration tickets (instead of quantities), allowing them to be resold at a competitively determined market price.

Reply to "Prices *vs.* Quantities: A Critical Note on the Use of Approximations" by James M. Malcomson

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The basic point made by Malcomson is, I think, sound. "The fact that an error in the representation of a function is small in relation to the values of that function does not necessarily imply that, after a number of manipulations, the resulting error is small in relation to the results that are derived." But it usually turns out to be true. And in my case, as I think in many cases, conclusions based on a linear approximation can be rigorously defended as accurate in the limit—providing the limiting process is carefully stated. As Malcomson notes, I did not provide a rigorous statement of the appropriate limiting process in my original paper. Let me take the opportunity to do so now.

Following the notation of my footnote on page 436 of "Prices *vs.* Quantities", suppose that marginal costs and benefits are

$$C_1(q, \theta) = (C' + \alpha(\theta)) + (1/f(\theta))C''(q - \hat{q}) \qquad \text{...(3')}$$

$$B_1(q, \eta) = (B' + \beta(\eta)) + g(\eta)B''(q - \hat{q}). \qquad \text{...(4')}$$

This is the same as Malcomson's equations (3), (4) where I use the symbol $1/f(\theta)$ in place of his $\gamma(\theta)$, and $g(\eta)$ in place of his $\delta(\eta)$.

Now, the fact that I work with $f(\theta)$ rather than $\gamma(\theta)$ $(= 1/f(\theta))$ is no accident. I want to introduce the second-order uncertainty in costs as uncertainty in $1/C_{11}(q, \theta)$, the change in profit maximizing output response per unit price change. This turns out to give neater results and facilitates the appropriate way of expressing the relevant approximation theorem.

As a strategic assumption, let $\alpha(\theta)$ and $f(\theta)$ be independently distributed. That is, assume

$$E[f(\theta)\alpha(\theta)] = E[f^2(\theta)\alpha(\theta)] = E[\alpha(\theta)] \equiv 0$$

$$E[f(\theta)] \equiv 1$$

$$E[(f(\theta) - 1)^2] \equiv \delta^2$$

$$E[\alpha^2(\theta)] \equiv \sigma^2.$$

The assumption that the first and second derivatives of the cost function vary independently over all states of the world might conceivably be regarded as a rough approximation of reality in a world of equal ignorance. In any case, it can serve as a point of departure for a more complete analysis.

Then, as in the footnote on page 486, the comparative advantage of prices over quantities is

$$\Delta = \sigma^2 \left(\frac{B''}{2C''^2} + \frac{1}{2C''} + \frac{\delta^2 B''}{2C''^2} \right).$$

Now, as *both* σ^2 and δ^2 go to zero together (for example, $\sigma^2 = k\delta^2$ for some constant k), the first-order approximation on the above expression goes to

$$\sigma^2\left(\frac{B''}{2C''^2}+\frac{1}{2C''}\right),$$

which is just the original coefficient of comparative advantage. So there is no contradiction here.

In " most " distributions of interest, the variance of $\gamma(\theta)$ will go to zero as rapidly as the variance of $f(\theta)$ $(= 1/\gamma(\theta))$. But this need not always be true. Malcomson has constructed a clever example where the variance of $\gamma(\theta)$ goes to zero as the variance of $f(\theta)$ goes to infinity. This occurs because the probability mass of $\gamma(\theta)$ is not all moving compactly toward the mean of one. In fact, some of it gets ever closer to zero, which blows up the variance of $f(\theta)$.

If you take away this sort of behaviour, you take away the driving force behind Malcomson's example. So long as δ^2 and σ^2 are moving toward zero together, the conclusions based on the first-order approximation of " Prices *vs.* Quantities " can be rigorously defended. They certainly cannot be justified if δ^2 goes to infinity as σ^2 goes to zero.

Now when α and f are *not* independently distributed, an altogether different situation arises, and my original formula for the comparative advantage of prices over quantities need not hold as a first-order approximation. That formula was derived on the premise that the second derivative of the cost function, or slope of the supply curve, is constant, and uncertainty merely shifts the supply curve. The formula continues to hold as a first-order approximation if the slope of the supply curve changes independently of shifts in the supply curve. If the slope of the supply curve *systematically* changes with shifts in the supply curve, this will alter the formula for the coefficient of comparative advantage. As an illustrative example of why this occurs, suppose that prices have a comparative advantage by the original formula. Suppose also that α and f are strongly positively correlated. The original calculation in favour of prices was made on the assumption that when α is negative, costs are cheap and output should be expanded. But now $1/f$ is also big, and expanding output could actually *increase* costs. Thus, it might be better to stabilize output by fixing quantities. This kind of an effect won't vanish as the uncertainty diminishes.

I think Malcomson's note is valuable as an illustration of the dangers of using approximations in a non-rigorous fashion. He is right that " in asserting approximate results, it is incumbent upon authors to show that the errors resulting from approximation are small in relation to the results derived, not just in relation to the function which is being approximated ".

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