The Pigouvian Tax Rule under Monopoly

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Over the last decade, significant strides have been made in clarifying the issue of taxation for control of external effects. William Baumol and Wallace Oates have shown that, contrary to the position taken by Ronald Coase, James Buchanan and W. Craig Stubblebine, among others, unilateral taxes are appropriate for the control of public externalities. They have further shown that the majority of policy-relevant externalities are of the public variety.[1] Yet these efforts fail to deal adequately with an analytic complication which arises with the Pigouvian approach. In particular, they fail to deal adequately with the problem of taxing externalities when those being taxed are imperfectly competitive firms. Buchanan has described the nature of the problem which arises when externality-generating firms are monopolistic, and Baumol and Oates have restated and extended Buchanan's work. But these analyses are incorrect on some points and incomplete on others. The purpose of this note is to further extend Buchanan's work and to correct an error made by Baumol and Oates.

I. The Model

Two sources of misallocation can occur with imperfectly competitive polluters. One is the distortion due to the externality, and the other is the underproduction of final products generally associated with the exercise of monopoly power. A tax on pollutants will reduce the generation of external damages, but it may also cause firms to reduce further their production of final products. Thus there may be tradeoffs between the two distortions, one due to monopolistic underproduction and the other due to external diseconomies. A tax based only on marginal external damages ignores the social cost of further output contraction by a producer whose output already is below an optimal level.

An ideal solution to this problem would incorporate two policy actions: a device to increase production of final products together with a tax to control the external diseconomy. It is assumed, however, that the product market distortion cannot be directly corrected, and so the pollution tax must achieve an optimal second best tradeoff of distortions.

To see more clearly the nature of this optimal second best tradeoff, let us assume that the externality in question is air pollution. Consider a polluter who produces a single product output q with the (inverse) demand function $f(q)$, and who discharges smoke s generating $e(s, X)$ in external damages: where X is a vector of activity levels x_i for parties damaged by smoke. Assuming that all parties affected adversely by smoke are utility-maximizing individuals who view any tax imposed on pollution as a parameter, and further assuming that all prices relevant to purchasing decisions by these individuals remain unchanged as the rate of smoke production, s, varies, then

$$x_i = g(s) \text{ for all } x_i \text{ in } X$$

Thus, $e(s, X)$ can be written as a function of s alone, i.e.,

$$e(s, X) = E(s)$$

Let total resource costs for the smoking-generating firm be represented by $c(q, w)$:

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[1]In general, private externalities will persist only when (i) legal or institutional restrictions prevent bargaining, or (ii) the externality is either insignificant or the cost of price rationing is very high. For a discussion of these points, see Baumol and Oates, pp. 20-21.

where w represents resources devoted to smoke treatment. Assume initially that the firm has two ways of reducing its effluent s. It may either reduce output q, or it may devote more resources w to the treatment of smoke once it is produced.

Now let us consider a tax on effluent. Assuming that all conditions for optimal production are met elsewhere in the economy and that society is indifferent to purely redistributional effects, we can use producer's and consumer's surplus as welfare measures.[2] Social welfare will be given by the difference between the sum of producer's and consumer's surplus and any technological external costs which are not accounted for in producer's surplus. To maximize welfare by taxation, we must find some tax rate per unit of smoke discharged t, which will maximize

$$(1)\qquad u=\int_0^q f(q)dq-c(q,w)-E(s)$$

where q, w, and hence s can be written as functions of t. Differentiating equation (1) with respect to t yields the following first-order condition for welfare maximization:

$$(2)\quad f(q)\frac{dq}{dt}-\frac{\partial c(q,w)}{\partial q}\frac{dq}{dt}-\frac{\partial c(q,w)}{\partial w}\frac{dw}{dt}-\frac{dE(s)}{ds}\left[\frac{\partial s}{\partial q}\frac{dq}{dt}+\frac{\partial s}{\partial w}\frac{dw}{dt}\right]=0$$

Profits for the firm are given by

$$(3)\qquad \Pi=f(q)q-c(q,w)-s\cdot t$$

Assuming the polluter views t as a parameter, first-order conditions for profit maximization are

$$(4)\qquad \frac{\partial\Pi}{\partial q}=f(q)+\frac{df(q)}{dq}q-\frac{\partial c(q,w)}{\partial q}-\frac{\partial s}{\partial q}t=0$$

and

$$(5)\qquad \frac{\partial\Pi}{\partial w}=0-\frac{\partial c(q,w)}{\partial w}-\frac{\partial s}{\partial w}t=0$$

Rewriting (4) and (5), we have

$$(6)\qquad \frac{\partial c(q,w)}{\partial q}=f(q)+\frac{df(q)}{dq}q-\frac{\partial s}{\partial q}t$$

$$(7)\qquad \frac{\partial c(q,w)}{\partial w}=-\frac{\partial s}{\partial w}t$$

Substituting (6) and (7) into equation (2) and simplifying yields a first-order condition for welfare maximization combined with profit-maximizing behavior:

$$(8)\quad 0=-\frac{df(q)}{dq}\frac{dq}{dt}q+\frac{\partial s}{\partial q}\frac{dq}{dt}t+\frac{\partial s}{\partial w}\frac{dw}{dt}t-\frac{dE(s)}{ds}\left[\frac{\partial s}{\partial q}\frac{dq}{dt}+\frac{\partial s}{\partial w}\frac{dw}{dt}\right]$$

and the welfare-maximizing tax is

$$(9)\qquad t^*=\frac{\frac{df(q)}{dq}\frac{dq}{dt}q}{\frac{\partial s}{\partial q}\frac{dq}{dt}+\frac{\partial s}{\partial w}\frac{dw}{dt}}+\frac{dE(s)}{ds}$$

Obviously, (9) is not an explicit solution for t because t is on both sides of the equation. But (6) and (7) allow us to write q and w as functions of t. Substituting into (2) then gives one equation in one unknown, t.

II. An Illustration of Two Special Cases

It is now possible to illustrate with (9) two special cases found in the literature concerned with taxation for pollution control. The first case is one in which the only means of smoke abatement is reducing output q and the second case is one in which waste treatment is the only means of smoke abatement. The first of these special cases is often analyzed in the theoretical literature concerned with taxation.[3] The second has

[2]To be strictly correct, I should use a real income constant measure of consumer's surplus, such as equivalent or compensating variation. The area under a money-income constant demand curve is only approximately correct as a welfare measure.

[3]See, for example, Baumol, Baumol and Oates, Earl Thompson and Ronald Batchelder, and James Marchand and Keith Russell.

been used in several empirical studies.[4] If it is assumed that the only way for a polluter to reduce emissions is by reducing output, then terms involving w in (9) disappear and an optimal tax is given by

$$(10)\qquad t^*=\frac{\dfrac{df(q)}{dq}\dfrac{dq}{dt}q}{\dfrac{\partial s}{\partial q}\dfrac{dq}{dt}}+\frac{dE(s)}{ds}$$

On the other hand, if we assume that waste treatment is the only means of smoke abatement, then dq/dt is zero and an optimal tax is given by

$$(11)\qquad t^*=\frac{\dfrac{dE(s)}{ds}\left[\dfrac{\partial s}{\partial w}\dfrac{dw}{dt}\right]}{\dfrac{\partial s}{\partial w}\dfrac{dw}{dt}}=\frac{dE(s)}{ds}$$

The term $(df(q)/dq)(dq/dt)$ reflects in part the market structure within which the polluter operates, and, therefore, the significance of the welfare cost associated with reducing the firm's output by imposing emission taxes. If it is assumed that the polluter responds to a tax on emissions by reducing q only, then dq/dt is negative and demand elasticity is relevant for determining an optimal second best tax. Of course, if the polluter responds to a tax by changing w only, then dq/dt is zero and market structure is not a relevant consideration. I shall now interpret condition (9) for the more general case where both q and w vary with t.

III. A More General Case

The importance of market structure in setting optimal pollution taxes may be seen more clearly by rewriting equation (9) to show explicitly the role of price elasticity of demand. The price elasticity of demand for q, η, is

$$\eta=\frac{dq}{df(q)}\frac{f(q)}{q}$$

thus $\dfrac{df(q)}{dq}q=\dfrac{f(q)}{\eta}$

and, of course, $-\dfrac{df(q)}{dq}q=\dfrac{f(q)}{|\eta|}$

Substituting this identity into equation (9) yields the following expression for an optimal second best tax:[5]

$$(12)\qquad t^*=\frac{-\dfrac{f(q)}{|\eta|}\dfrac{dq}{dt}}{\dfrac{\partial s}{\partial q}\dfrac{dq}{dt}+\dfrac{\partial s}{\partial w}\dfrac{dw}{dt}}+\frac{dE(s)}{ds}$$

Assuming that $dq/dt<0$ and $dw/dt>0$,[6] and

[4]See R. Carbone and J. R. Sweigart (1976) and Carbone et al. (1978).

[5]The relevant measure of elasticity is that associated with the post-tax levels of the terms in equation (12).

[6]While these assumptions are common in the literature, they may not be universally applicable. To illustrate, consider the following expressions for dq/dt and dw/dt derived from the standard second-order conditions for profit maximization:

$$\text{(a)}\quad \frac{dq}{dt}=\left\{\frac{\partial s}{\partial q}\left[-\frac{\partial^2 c(q,w)}{\partial w^2}-\frac{\partial^2 s}{\partial w^2}t\right]-\frac{\partial s}{\partial w}\left[-\frac{\partial^2 c(q,w)}{\partial q\partial w}-\frac{\partial s}{\partial q\partial w}t\right]\right\}\div|D|$$

$$\text{(b)}\quad \frac{dw}{dt}=\left\{-\frac{\partial s}{\partial q}\left[-\frac{\partial^2 c(q,w)}{\partial q\partial w}-\frac{\partial^2 s}{\partial q\partial w}t\right]+\frac{\partial s}{\partial w}\left[\frac{d^2R}{dq^2}-\frac{\partial^2 c(q,w)}{\partial q^2}-\frac{\partial^2 s}{\partial q^2}t\right]\right\}\div|D|$$

where R is total revenue and D is the conventional matrix of partial derivatives formed from second-order conditions for profit maximization. Satisfying second-order conditions requires $|D|>0$ and

$$\frac{d^2R}{dq^2}-\frac{\partial^2 c(q,w)}{\partial q^2}-\frac{\partial^2 s}{\partial q^2}t<0$$

Further, $\partial s/\partial q>0$ and, if we assume diminishing marginal productivity for resources devoted to waste treatment, $\partial s/\partial w$, $\partial^2 c(q,w)/\partial w^2$, and $\partial^2 s/\partial w^2$ are all negative. However, we have no a priori information concerning the signs of $\partial^2 c(q,w)/\partial q\partial w$ and $\partial^2 s/\partial q\partial w$. If the sum of these two terms is negative, or if positive but small relative to the other terms in (a) and (b), then $dq/dt<0$ and $dw/dt>0$, as assumed. But, if the sum of $\partial^2 s/\partial q\partial w$ and $\partial^2 c(q,w)/\partial q\partial w$ is positive and large, it is possible that $dq/dt>0$ or $dw/dt<0$. Hence, it is possible, though the chance seems remote, that the optimal tax rate would be higher for monopolistic than for competitive firms.

observing that dq/dt approaches some finite number as $|\eta|$ approaches infinity, it is easily seen that as demand approaches the perfectly elastic state the value of the optimal tax rate approaches marginal external damages, $dE(s)/ds$. But when η is finite, a tax rate less than marginal external damages is required to achieve an optimal tradeoff between the external diseconomy and the welfare loss associated with monopoly restricted output. Thus, the optimal second best tax rate can be higher as demand is more price elastic.[7] Moreover, a positive tax will be appropriate only when marginal external damage, $dE(s)/ds$, exceeds

$$\frac{\dfrac{f(q)}{|\eta|}\dfrac{dq}{dt}}{\dfrac{\partial s}{\partial q}\dfrac{dq}{dt}+\dfrac{\partial s}{\partial w}\dfrac{dw}{dt}}$$

The conclusion that optimal second best tax rates can be higher for firms with more price elastic demand functions differs from the conclusion arrived at by Baumol and Oates. In expanding on Buchanan's initial discussion of monopoly and pollution taxes, they state:

> The more inelastic demand, the smaller, apparently, is the change in output resulting from a given change in the [emission] tax. This seems to suggest that other things being equal, we should adopt a higher emission tax where the product demand is *more price inelastic*. As is widely recognized, this role of elasticity is typical of many second-best problems of pricing and resource utilization.
> [p. 77, emphasis added]

Baumol and Oates fail to note here a difference between most second best problems and the special problem of taxing for pollution control. In the usual case, social losses from taxation are minimized by minimizing distortions in market-dictated resource utilization. However, the purpose of a tax on pollutants is to reduce the social cost of market imperfections where the starting point is not ideal. Corrective taxes reduce external costs but they add to the deadweight loss attributable to the monopolist's restricted output.

IV. Conclusions

In the search for efficient controls for externalities, economists have, in general, supported taxes over administrative regulations. The reasons for this choice are well known and grounded solidly in economic theory. However, in our enthusiasm for market-like solutions, we have been remiss in failing to sort out the complexities of optimal taxation. This paper has dealt with one factor which complicates the derivation of optimal taxes for the control of externalities. I have derived two main conclusions: (i) where polluters are imperfectly competitive, second best optimal tax rates may be less than marginal effluent harm; and (ii) the amount by which optimal tax rates fall short of marginal damages may increase as price elasticity of demand for the polluter's produce decreases.

[7]It should be noted, however, that the optimal tax rate need not increase monotonically with price elasticity of demand.

REFERENCES

William J. Baumol, "On Taxation and the Control of Externalities," *Amer. Econ. Rev.*, June 1972, *62*, 307–22.

______ **and Wallace E. Oates**, *The Theory of Environmental Policy*, Englewood Cliffs 1975.

J. M. Buchanan, "External Diseconomies, Corrective Taxes, and Market Structures," *Amer. Econ. Rev.*, Mar. 1969, *59*, 174–77.

______ **and W. C. Stubblebine**, "Externality," *Economica*, Nov. 1962, *29*, 371–84.

R. Carbone et al., "A Bargaining Resolution of the Efficiency versus Equity Conflict in Energy and Air Pollution Regulation," in J. S. Arnofsky and A. G. Rao, eds., *Energy Policy*, Amsterdam 1978.

______ **and J. R. Sweigart**, "Equity and Selective Pollution Abatement Procedures," *Manage. Sci.*, Dec. 1976, *23*, 361–80.

R. Coase, "The Problem of Social Cost," *J. Law Econ.*, Oct. 1960, *3*, 1–44.

C. J. Goetz and J. M. Buchanan, "External Diseconomies in Competitive Supply," *Amer. Econ. Rev.*, Dec. 1971, *61*, 883–90.

J. R. Marchand and K. P. Russell, "Externalities, Liability, Separability, and Resource Allocation," *Amer. Econ. Rev.*, Sept. 1973, *63*, 611–20.

E. A. Thompson and R. Batchelder, "On Taxation and the Control of Externalities: Comment," *Amer. Econ. Rev.*, June 1974, *64*, 467–71.