Microeconomic Theory: Lecture 2 (Addendum)
Time Inconsistent Preferences

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Time Inconsistent Preferences

- Odysseus and the sirens.
- The smoker's dilemma: wants to quit but cannot.
- Procrastination: more than just laziness.
- The agent seemingly has multiple selves with conflicting preferences.
- Prediction: how will such an agent behave?
- Ethics: which of several conflicting preferences should others respect?
- Welfare: how to evaluate such an agent's welfare?
- Paternalism and welfarism become less distinct concepts.

Two Choice Problems

- **Problem 1:** Which do you prefer?
  - (A) Rs 1 lakh now
  - (B) Rs 1 lakh + Rs 100 next week
- **Problem 2:** Which do you prefer?
  - (C) Rs 1 lakh one year from now
  - (D) Rs 1 lakh + Rs 100 a year and one week from now
- Most people answer: $A \succ B$ and $D \succ C$.
- Suppose you choose D over C. But a year later, you will want to reverse your choice!
- This pattern found in humans, rats and pigeons (Ainslie (1974)).

The Cake Eating Problem with Geometric Discounting

- A consumer has a cake of size 1 which can be consumed over dates $t = 1, 2, 3 \ldots$
- The cake neither grows nor shrinks over time (exhaustible resource like petroleum).
- The consumer's utility at date t is

$$U_t = u(c_t) + \delta u(c_{t+1}) + \delta^2 u(c_{t+2}) + \ldots$$

- $u(.)$ is instantaneous utility (strictly concave), $\delta \in (0, 1)$ is the discount factor.
- At date 0, the consumer's problem is to choose a sequence of consumptions $\{c_t\}_{t=0}^{\infty}$ to solve

$$\max_{\{c_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t u(c_t) \quad \text{subject to} \quad \sum_{t=0}^{\infty} c_t = 1$$

Time Consistency of the Optimal Path

- Let $\{c_t^*\}_{t=0}^{\infty}$ be the optimal consumption path at date 0.
- If the consumer gets the chance to revise her own plan at date t, will she do so (i.e. is the consumer dynamically consistent)?
- Suppose at some date t, the amount of cake left is c. At any $\widehat{t} < t$, the consumers' optimal plan for t onwards is:

$$\max_{\{c_\tau\}_{\tau=t}^{\infty}} \sum_{\tau=t}^{\infty} \delta^{\tau-\widehat{t}} u(c_\tau) \text{ subject to } \sum_{\tau=t}^{\infty} c_\tau = c$$

- The Lagrangian is

$$\mathcal{L}(\mathbf{c}, \lambda) = \sum_{\tau=t}^{\infty} \delta^{\tau-\widehat{t}} u(c_\tau) + \lambda \left[c - \sum_{\tau=t}^{\infty} c_\tau \right]$$

Time Consistency of the Optimal Path

- First-order condition:

$$\delta^{\tau-\widehat{t}} u'(c_\tau^*) = \lambda$$

- Eliminating λ:

$$\underbrace{\frac{u'(c_\tau^*)}{u'(c_{\tau+1})}}_{\text{intertemporal MRS}} = \underbrace{\delta}_{\text{discount factor}}$$

- Note that this is independent of $\widehat{t}$, the date at which the plan is being made.
- The consumer will not want to change her plans later.

Logarithmic Utility

- Suppose $u(c) = \log c$.
- From the first-order condition

$$c_{t+1}^* = \delta c_t^* \Rightarrow c_t^* = \delta^t c_0^*$$

- Using the budget constraint

$$c_0^* + \delta c_0^* + \delta^2 c_0^* + \ldots = 1 \Rightarrow c_0^* = 1 - \delta$$

$$c_t^* = \delta^t (1 - \delta)$$

- In every period, consume $1 - \delta$ fraction of the remaining cake, and save δ fraction.

Quasi-Hyperbolic Discounting and Cake Eating

- Suppose

$$U_t = u(c_t) + \beta \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} u(c_\tau)$$

- The Lagrangian for the date 0 problem is:

$$\mathcal{L}(\mathbf{c}, \lambda) = u(c_t) + \beta \sum_{\tau=t+1}^{\infty} \delta^{\tau-t} u(c_\tau) + \lambda \left[1 - \sum_{\tau=t}^{\infty} c_\tau\right]$$

- First-order conditions:

$$u'(c_0^*) = \lambda^*$$

$$\beta\delta^{\tau-t} u'(c_\tau^*) = \lambda^*$$

Time Inconsistency of the Optimal Path

- Eliminating λ^*:

$$\text{MRS}_{0,1} = \frac{u'\left(c_0^*\right)}{u'\left(c_1^*\right)} = \beta\delta$$

$$\text{MRS}_{t,t+1} = \frac{u'\left(c_t^*\right)}{u'\left(c_{t+1}^*\right)} = \delta \quad \text{for all } t > 0$$

- However, when date t arrives, the consumer will want to change the plan and reallocate consumption such that

$$\text{MRS}_{t,t+1} = \beta\delta$$

- Realizing that she may change her own optimal plan later, the self aware consumer will adjust her plan at date 0 itself.
- Alternatively, the consumer may try to *commit* and restrict her own future options (e.g. Christmas savings accounts).

Quasi-Hyperbolic Discounting: Time Consistent Path

- Find a subgame perfect equilibrium in stationary strategies: at any date t, if the amount of cake left is y, the t-th self saves $s(y)$ of the cake and eats the rest. $s(.)$ is the equilibrium savings function.
- The "value function" generated by this savings function is

$$v(y) = \sum_{t=0}^{\infty} \delta^t u\left(s^t(y) - s^{t+1}(y)\right) \quad \text{where } s^0(y) = y$$

- In equilibrium, the anticipated savings function must be self-fulfilling:

$$s(y) = y - \arg\max_c \left[u(c) + \beta\delta v(y-c)\right]$$

- The FOC:

$$u'\left(y - s(y)\right) = \beta\delta v'\left(s(y)\right) \tag{1}$$

Solution for Logarithmic Utility

- Postulate a constant savings rate: $s(y) = sy$. Then:

$$\begin{aligned} v(y) &= \sum_{t=0}^{\infty} \delta^t \ln\left[s^t(1-s)y\right] \\ &= \ln s \sum_{t=0}^{\infty} t\delta^t + [\ln(1-s) + \ln y] \sum_{t=0}^{\infty} \delta^t \\ &= \frac{\delta \ln s}{(1-\delta)^2} + \frac{\ln(1-s) + \ln y}{(1-\delta)} \end{aligned}$$

- Now using (4), the equilibrium savings rate $\widehat{s}$ can be solved:

$$\frac{1}{1-\widehat{s}} = \frac{\beta\delta}{(1-\delta)\widehat{s}} \Rightarrow \widehat{s} = \frac{\beta\delta}{1-\delta+\beta\delta}$$

Welfare

- Is there some savings rate $s \neq \widehat{s}$ which would have made all selves better off relative to equilibrium? Answer: yes!
- Express t-th self's utility as a function of s and y_t:

$$\begin{aligned} U_t(s; y_t) &= \ln\left[(1-s)y_t\right] + \beta \sum_{\tau=1}^{\infty} \delta^{\tau} \ln\left[s^{\tau}(1-s)y_t\right] \\ &= \frac{(1-\delta+\beta\delta)}{(1-\delta)}\left[\ln(1-s) + \ln y_t\right] + \frac{\beta\delta}{(1-\delta)^2}\ln s \qquad (2) \end{aligned}$$

- For fixed y_t, choose $s = s^*$ which maximizes this:

$$s^* = \frac{\beta\delta}{\beta\delta + (1-\delta)(1-\delta+\beta\delta)}$$

- Note that the solution is independent of y_t and $s^* > \widehat{s}$.

Pareto Inefficiency

- **Claim:** If the consumption path follows the savings rate s^* instead of $\widehat{s}$, then *all* selves are better off.
- **Proof:** Expression in (5) is maximized at $s = s^*$ for a given y_t, and is increasing in y_t. Since $s^* > \widehat{s}$, there is a bigger y_t left at every t if s^* is followed instead of $\widehat{s}$.
- Suppose each self is forced to save slightly more than the equilibrium rate $\widehat{s}$. Three effects on each self:
    - she receives more cake from previous selves (positive)
    - future selves' choices are better for her (positive)
    - her own savings suboptimal (negative, but 0 first-order effect)
- Since the first two effects dominate the last, she is better off.
- Forced saving coercively imposed by another agent (government, employer, friend, bank) is unambiguously good for this agent.

Compulsory Pension Schemes: Paternalism?

- Welfarist/libertarian: cash better, because the recipient is the best judge of what she needs.
- Paternalist answer: kind (buy a meal, fund a hospital). Recipients may waste the money on alcohol or weapons.
- Governments and corporate employers often have compulsory pension schemes (e.g., Social Security). Why not leave people to save for their future as they see fit?
- Seems like a clash between the two philosophies again: paternalism vs. welfarism. But is it?
- Forced saving could make the agent better off than complete freedom *by her own judgement*.
- Coercion (restriction on behaviour) and paternalism (imposition of preferences) are not synonymous.
- Is freedom an end in itself or a means to an end?