Factor Price Equalization and Stolper-Samuelson Theorem

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Microeconomic Theory

Lecture 13
Consider a ‘small’ open economy

Two goods are produced; food and cloth/car, \( f \) and \( c \)

Two FOPs are used; labour and capital, \( l \) and \( t \)

Production technology; constant returns to scale (?)

Production functions; strictly quasi-concave (?)

\[
\begin{align*}
\text{Food} & : \quad y^f = y^f(z^f_l, z^f_t) \\
\text{Car/cloth} & : \quad y^c = y^c(z^c_l, z^c_t)
\end{align*}
\]

In view of constant returns assumption,
the optimum mix of inputs (cost minimizing combination of FOPs) does not depend on the level of operation.

Therefore, to search for optimum mix of FOPs, we can take output level to be one:

That is, we can focus on the following optimization:

Given vector of factor prices \((w, r)\),

\[
\min \{ w\hat{a}_{ij} + r\hat{a}_{jt} \}
\]

\[
y^j(\hat{a}_j, \hat{a}_t) = 1,
\]

where \(j = f, c\).

In view of the assumption on \(y^j\), (1) has a unique solution. Let

- \(a^f\), \(a^f_t\) solve (1) for food sector
- \(a^c\), \(a^c_t\) solve (1) for car/cloth sector
That is,

\[ 1 = y^f(a_f^i, a_t^f) \]
\[ 1 = y^c(a_c^i, a_t^c) \]

Clearly, for \( j = f, c \) we have

\[ a_f^i = z_f^i(w/r) \]
\[ a_f^t = z_f^t(w/r) \]
\[ a_c^i = z_c^i(w/r) \]
\[ a_c^t = z_c^t(w/r) \]
International Trade: Basic Set-up IV

Now, the production process is characterized by:

\[
a_i^f = z_i^f(w/r) \quad (2)
\]
\[
a_t^f = z_t^f(w/r) \quad (3)
\]
\[
a_i^c = z_i^c(w/r) \quad (4)
\]
\[
a_t^c = z_t^c(w/r) \quad (5)
\]
\[
\bar{l} = y^f a_i^f + y^c a_i^c = y^f z_i^f(w/r) + y^c z_i^c(w/r) \quad (6)
\]
\[
\bar{t} = y^f a_t^f + y^c a_t^c = y^f z_t^f(w/r) + y^c z_t^c(w/r) \quad (7)
\]
\[
p^f = w a_i^f + r a_t^f = w z_i^f(w/r) + r z_t^f(w/r) \quad (8)
\]
\[
p^c = w a_i^c + r a_t^c = w z_i^c(w/r) + r z_t^c(w/r) \quad (9)
\]
Factor Intensity

Definition

Food sector is labour intensive if,

\[(\forall (w, r) >> (0, 0)) \left[ \frac{a_l^f}{a_t^f} > \frac{a_l^c}{a_t^c} \right] \]

Assumption

- Suppose for all possible pair of input prices we have

\[\frac{a_l^f}{a_t^f} > \frac{a_l^c}{a_t^c} \text{ or } \frac{a_l^f}{a_t^f} < \frac{a_l^c}{a_t^c} \]

- WLOG, assume F sector is labour intensive (??): That is,

\[(\forall (w, r) >> (0, 0)) \left[ \frac{a_l^f}{a_t^f} > \frac{a_l^c}{a_t^c} \right], \text{i.e., } (a_t^c a_l^f - a_t^f a_l^c) > 0 \]
Assume

- There is flow of factors within an economy
- No trade in factors between economies
- Production technology is the same across countries
- There is free flow of goods across countries
- So, the output prices are the same across countries

then,

- Free trade is a substitute for free flow of FOP.
Theorem

Factor Price Equalization Theorem. Suppose, the factor intensity assumption holds. For any given output price vector and technology, the factor prices will be the same across countries.

Let, \( s = \frac{w}{r} \). So,

\[
p = \frac{p^f}{p^c} = \frac{wa_i^f + ra_i^f}{wa_i^c + ra_i^c}
\]

\[
= \frac{sa_i^f + a_i^f}{sa_i^c + a_i^c}
\]

\[
p = g(s)
\]

Sign of \( \frac{dp}{ds} = \text{sign of } (a_i^c a_i^f - a_i^f a_i^c) > 0. \)
Therefore $g'(s) > 0$. Let $s = h(p)$, where

$$h() = g^{-1}().$$

Clearly, $h'(s) > 0$

Question

- What is the ceteris paribus effect of change in factor endowments on output levels?
- What is the ceteris paribus effect of change in output prices on factor prices?
Stolper-Samuelson Theorem I

Theorem

Stolper-Samuelson Theorem. Given the factor intensity assumption, an increase in relative price of c leads to increase in relative price of t, and vice-versa.

Let

- $\theta_f = \frac{w_a f}{p_f}$, the share of labour in food sector
- $\theta_t = \frac{r_a f}{p_f}$, the share of capital in food sector
- $\theta_c = \frac{w_a c}{p_c}$, the share of labour in cloth sector
- $\theta_t = \frac{r_a c}{p_c}$, the share of capital in cloth sector
Stolper-Samuelson Theorem II

Clearly,

\[ p^f y^f(= 1) = wa^f_i + ra^f_t \]
\[ y^f(= 1) = \frac{wa^f_i}{p^f} + \frac{ra^f_t}{p^f} \]

\[ \theta^f_i + \theta^f_t = 1 \text{ and } \theta^c_i + \theta^c_t = 1. \]

Recall \( (a^c_i a^f_i - a^f_i a^c_i) > 0. \) So, we have

\[ \frac{w.r}{p^f.p^c} (a^c_i a^f_i - a^f_i a^c_i) = \frac{w.a^f_i.r.a^c_i}{p^f.p^c} - \frac{w.a^c_i.r.a^f_t}{p^c.p^f} \]
\[ = \theta^f_i \theta^c_i - \theta^f_t \theta^c_t \]
\[ = \theta^f_i (1 - \theta^c_i) - (1 - \theta^f_i) \theta^c_i \]
\[ = \theta^f_i - \theta^c_i > 0. \]  \hspace{1cm} (12)
Stolper-Samuelson Theorem III

That is, the share of \( l \) is higher in \( l \) intensive sector, and vice-versa. The least cost condition gives us

\[
wd\hat{a}_l^i + r\hat{a}_t^i = 0, \text{ i.e., (13)}
\]

\[
w\frac{da_l^i}{a_l^i} + r\frac{da_t^i}{a_t^i} = 0, \text{ i.e., (14)}
\]

\[
\theta_l^i\hat{\theta}_l^i + \theta_t^i\hat{\theta}_t^i = 0, \text{ (15)}
\]

where \( \hat{\theta}_l^i = \frac{da_l^i}{a_l^i} \), etc.

Differentiating (8) and (9) we get

\[
dp_f = a_f^l dw + a_t^f dr + wda_l^f + rda_t^f \quad (16)
\]

\[
dp_c = a_c^l dw + a_t^c dr + wda_l^c + rda_t^c \quad (17)
\]

In view of (13), we get
Stolper-Samuelson Theorem IV

\[ dp^f = a^f_i dw + a^f_t dr \]
\[ dp^c = a^c_i dw + a^c_t dr. \]
That is,

\[ \frac{dp^f}{p^f} = \frac{a^f_i dw}{p^f} + \frac{a^f_t dr}{p^f} = \frac{a^f_i w}{p^f} \frac{dw}{w} + \frac{a^f_t r}{p^f} \frac{dr}{r} \]
\[ \frac{dp^c}{p^c} = \frac{a^c_i dw}{p^c} + \frac{a^c_t dr}{p^c} = \frac{a^c_i w}{p^c} \frac{dw}{w} + \frac{a^c_t r}{p^c} \frac{dr}{r}. \]

That is,

\[ \hat{p}^f = \theta^f_i \hat{w} + \theta^f_t \hat{r} \]
\[ \hat{p}^c = \theta^c_i \hat{w} + \theta^c_t \hat{r}. \]
Stolper-Samuelson Theorem V

That is,

$$\hat{p}^c - \hat{p}^f = (\theta^f_i - \theta^c_i)(\hat{r} - \hat{w})$$  \hspace{1cm} (18)

That is,

$$\hat{r} - \hat{w} = \frac{1}{\theta}(\hat{p}^c - \hat{p}^f)$$  \hspace{1cm} (19)

where $\theta = \theta^f_i - \theta^c_i < 1$.

Theorem

*Given the factor intensity assumption, an increase in relative price of good c leads to more than proportionate increase in relative price of t, and vice-versa.*
Effect of Labour Supply I

Differentiating (6) and (7) we get

\[ d\bar{l} = a_l^f dy^f + a_l^c dy^c \quad (20) \]
\[ d\bar{t} = a_t^f dy^f + a_t^c dy^c \quad (21) \]

Note,

\[ \frac{a_l^f dy^f}{\bar{l}} = \frac{dy^f a_l^f y^f}{y^f} \frac{\bar{l}}{\bar{l}} = \hat{y}^f \lambda_l^f, \]

where \( \lambda_l^f = \frac{a_l^f y^f}{\bar{l}} \). So,

\[ \hat{l} = \frac{d\bar{l}}{\bar{l}} = \hat{y}^f \lambda_l^f + \hat{y}^c \lambda_l^c \quad (22) \]
\[ \hat{t} = \frac{d\bar{t}}{\bar{t}} = \hat{y}^f \lambda_t^f + \hat{y}^c \lambda_t^c \quad (23) \]
Effect of Labour Supply II

Clearly,

\[
\lambda_f^l + \lambda_c^l = 1 \\
\lambda_f^t + \lambda_c^t = 1
\]

Now, (22) and (23) will give us

\[
\hat{y}_f - \hat{y}_c = \frac{1}{\lambda} (\hat{l} - \hat{t})
\]

where \( \lambda = \lambda_f^l - \lambda_f^t > 0 \). So, an increase in labour force leads to

- relative increase in food production
- relative decrease in food price
- relative decrease in relative price of labour (wage)