004: Macroeconomic Theory

Introduction to Macroeconomics (Static Macro Models)

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Lecture Notes, DSE

Jan 2-12, 2018

What is Macroeconomics?

- Macroeconomics is one particular field of Economics that relates to the aggregate economy.
- Macroeconomics essentially deals with a **general equilibrium** set up where we simultaneously analyze the equilibrium scenarios in various markets, as determined by the behaviour of three sets of economic agents:
  - Households;
  - Firms;
  - Government.
- The functioning of these different set of agents are often interlinked and these interlinked actions generate certain outcomes for the aggregate economy.
- In Macroeconomics, we are concerned with these aggregative outcomes for the entire economy.

Modern Macroeconomics - The Central Issue:

- Macroeconomics is one field in Economics which carries utmost importance in the arena of policy making.
- Two central issues that motivate this field are as follows:
  - What causes aggregate output and employment levels in an economy to fluctuate/change over time?
  - How effective are various government policies in stabilizing the economy/generating steady growth?
- Despite its tremendous importance from the policy perspective, macroeconomics is one field which is fraught with controversies and debates.
- The focal point of all the major debates in macroeconomics over the years has been an ideological issue: should government intervene in the functioning of a private market economy or should it not?
- Or, to put it differently, does government intervention in macroeconomic matters improve welfare or does government intervention create more problems than it solves?

Macroeconomics: Various Schools of Thought

- There are various schools of thoughts - starting with Keynes and his predecessors (the Classics), and their modern reincarnations (Neo-classicals, New Keynesians, New Classicals) - each trying to make a case for its respective position using specific theoretical constructs (macro models).
- These models differ in terms of the underlying assumptions about the functioning of the aggregate economy and about individual behaviour.
- The primary objective of the course is to expose the students to these various schools of thoughts in terms of rigorous macro models and analyse the associated policy implications.

Modern Macroeconomics: Dynamic Analysis

- The secondary objective of the course is to familiarize the students with the major mathematical tools used in modern macro analyses across the world.
  - The core issues underlying these debates have remained unchanged, but the techniques used in expositing alternative schools of thoughts have undergone a dramatic change over the last two decades.
  - While the earlier theoretical expositions were based on 'static' models (involving a single time period), all the recent expositions use dynamic settings (involving tracing the economy/agents over a period of time) to analyse issues which also have dynamic implications (e.g. growth and inflation rather than commenting on just the current status of output, prices and employment).

Modern Macroeconomics: From Statics to Dynamics

- Analysing macro issues in a dynamic setting is a key element of Modern Macroeconomics.
- A part of this course would therefore entail some discussion of the basic dynamic tools (e.g, difference and differential equations) **and dynamic optimization techniques** (Dynamic Programming and/or Optimal Control)).
- These tools will then be applied to analyse the macro issues at hand.

Course Content:

- The course has two modules. The first module will be taught by Mausumi Das and the second module will be taught by Pami Dua.
- The first module begins with a brief discussion of the static macro models that are used to analyse short run issues.
- These static models, being static in nature, take various time dependent variables (e.g, capital stock, population) as exogenous and analyse the effectiveness of various policies under alternative assumptions but essentially in a static one-period framework. These static models are also aggregative in nature – often without any obvious micro-foundations.
- The first module then goes on to provide a micro-founded underpinning to these aggregative macro models and in the process also introduces a dynamic framework to analyse various medium and long run macroeconomic issues at hand.

Course Content: (Contd.)

- The first module focuses explicitly on output dynamics, i.e., issues related to economic growth (assuming prices to be constant). It uses dynamic techniques to analyse how the aggregate as well as per capita GDP growth rates respond to policy changes under alternative schools of thoughts.
- The second module also moves away from the static models but it explicitly looks at the price dynamics, i.e., issues related to inflation . This module uses dynamic techniques to analyse how the price dynamics (inflation) responds to policy changes under alternative schools of thoughts.

Preliminary Reading:

- Any student of macroeconomics **must** start by reading the following article by Mankiw to get a broad perspective about the field: **"The Macroeconomist as Scientist and Engineer": N. Gregory Mankiw, The Journal of Economic Perspectives, Vol. 20, No. 4 (Oct., 2006), pp. 29-46.**
- Other (more specific) readings will be provided as we move from topic to topic.
- Exact chapters of various books and other supplementary readings will be specified during the course.
- Lecture Notes on selected topics will be put up in the course folder at the department website and the department server.
- Problem sets will be circulated upon completion of various broad topics to help students apply the concepts taught in the class.

Mode of Evaluation:

- 2 midterms (of 15 marks each) be held during the semester.
- A final examination (of 70 marks) will be held at the end of the semester.

Help Outside the Class Room:

- Regular tutorials will be held by 2 tutors to facilitate understating of the concepts and techniques taught in the class and also to assist with the problem sets.
- I am available for clarification outside the class during the following contact hours:
  - Tuesdays: 3-4 pm
  - Fridays: 3-4 pm
- I can also be reached by e-mail at: mausumi@econdse.org

Static Macro Models: Short Run

- In the next few lectures, we shall briefly discuss the static, one-period macro models.
- These models were very much in vogue in the decade of 1950s and 60s and majorly contributed to the academic and policy debates until 1970s.
- Subsequently these theories were challenged on several grounds - both theoretical and empirical. We shall provide a critique of these static frameworks and then move on to more modern macro frameworks which are essentially dynamic in nature.

Mr. Keynes & the Classics Revisited:

- There are two main variants of the static macro models: (a) The Keynesian Framework; (b) The Classical/Neoclassical Framework.
- We shall start by re-visiting these two basic static macro frameworks.
- Both frameworks summarise the aggregate economy in terms of three markets:
  - The Goods Market
  - The Labour Market
  - The Money (or alternatively, the Bond) Market
- Each market is represented by a pair of equations that capture the demand and the supply side respectively. These equations represent aggregative behaviour that are **not** necessarily derived from any explicit micro-founded analysis.
- The **major** difference between the frameworks arises from the description of the labour market.

Role of various agents in the macroeconomy:

- Recall that there are three sets of agents in this economy: households; firms and the government. Their respective roles are as follows:
  - **Households:**
    - All factors of production (capital, labour) are owned and supplied by the households. They also are the share holders of the firms (implying if the firms earn any positive profit then that profit would be distributed back to the households in the form of dividends). Thus the entire flow of output (unless taxed) goes back to the households in the form of income.
    - The households also decide how much to consume and how much to save out of their total income.
  - **Firms**:
    - Firms are engaged in actual production. They employ the factors owned by the households to produce the final commodity and pay the households in the form of wages and rents. If the firms earn any positive profit, that also eventually goes back to the households as dividends. However, to begin with, we shall assume a perfectly competitive market structure and CRS technology (**which means firms earn zero profit**).

Role of various agents in the Macroeconomy: (Contd).

- **Government:**
  - We consider a decentralized market economy, not a socially planned (or command) economy. So the government is not actively engaged in the production process. But it can intervene into the goods market by imposing taxes and/or contributing to the demand (government consumption).
  - It also directly operates in the money market, either by controlling the money supply or by setting the interest rate. However, to begin with,, we shall assume that it controls the money supply and allows the interest rate to be determined by the market.

Demand & Supply in the three markets:

- In the **goods market**:
  - demand comes from the households (consumption demand), firms (investment demand) and the government (government consumption);
  - supply comes from the firms.
- In the **labour market**:
  - demand comes from the firms;
  - supply comes from the household.
- In the **money market**:
  - demand comes from the households;
  - supply comes from the government.

The Classical/Neoclassical System (in equations):

- **The Goods Market:**
- Supply Equation:

$$Y = F(N, \bar{K}); F_{N,} F_K > 0; F_{NN}, F_{KK} < 0 \tag{1}$$

- Demand Equation:

$$Y = C(Y) + I(r) + \bar{G};\ 0 < C'(Y) < 1;\ I'(r) < 0 \tag{2}$$

- **The Labour Market:**
- Supply Equation:

$$W = Pg(N);\ g'(N) > 0 \tag{3}$$

- Demand Equation:

$$W = Pf(N);\ f'(N) < 0 \tag{4}$$

- **The Money Market:**
- Supply Equation:

$$M = \bar{M} \tag{5}$$

- Demand Equation:

$$M = PL(Y, r); L_Y > 0; L_r < 0 \tag{6}$$

The Classical System: Solution

- We have 6 equations in 6 variables (P, Y, W, N, r, M) which define the classical macroeconomic system.
- Solution consists of
  - Equilibrium values of the Price & Quantity in the Goods Market: P^*, Y^*
  - Equilibrium values of the Price & Quantity in the Labour Market: W^*, N^*
  - Equilibrium values of the Price & Quantity in the Money Market: r^*, M^*
- The equations being **interdependent**, we cannot solve for the equilibrium values of quantities & prices in each market separately. So we follow a more roundabout method.

The Classical System: Solution (contd.)

- First we club the SS & DD equations in the Labour Market and the SS equation in the Goods Market together:

$$Y = F(N, \bar{K}); F_{N,} F_K > 0; F_{NN}, F_{KK} < 0 \tag{1}$$

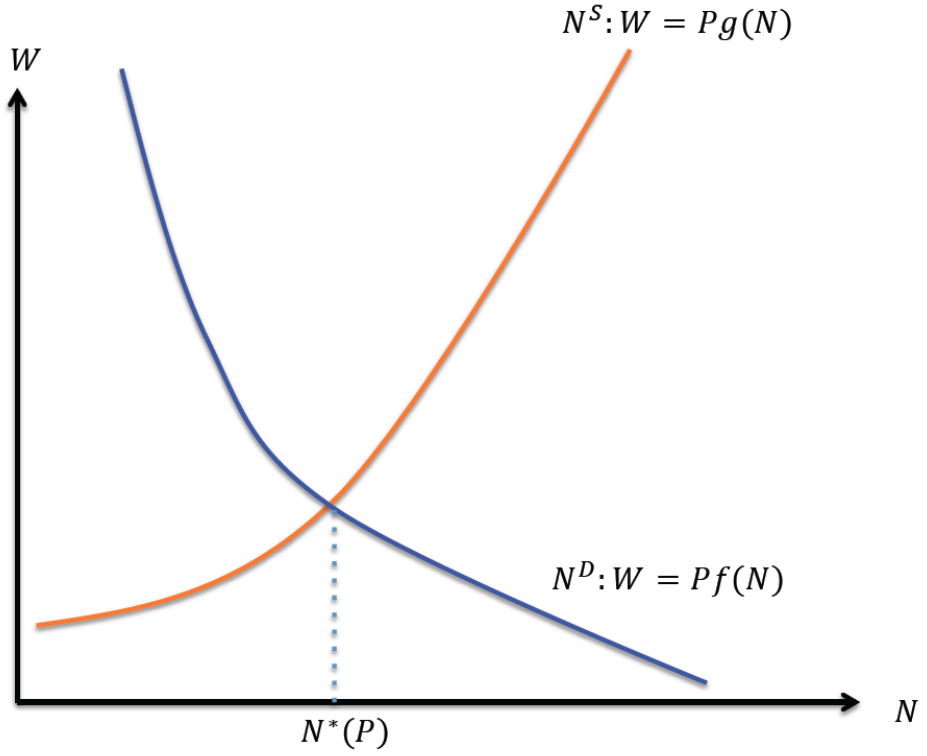
$$W = Pg(N); g'(N) > 0 \tag{3}$$

$$W = Pf(N);\ f'(N) < 0 \tag{4}$$

- This sub-system involves four endogenous variables: Y, N, W and P. We eliminate two of these variables to get a relationship between P and Y - which we call the aggregate supply curve (**AS**).
- Let us now see how this **AS** curve looks for the Classical system.

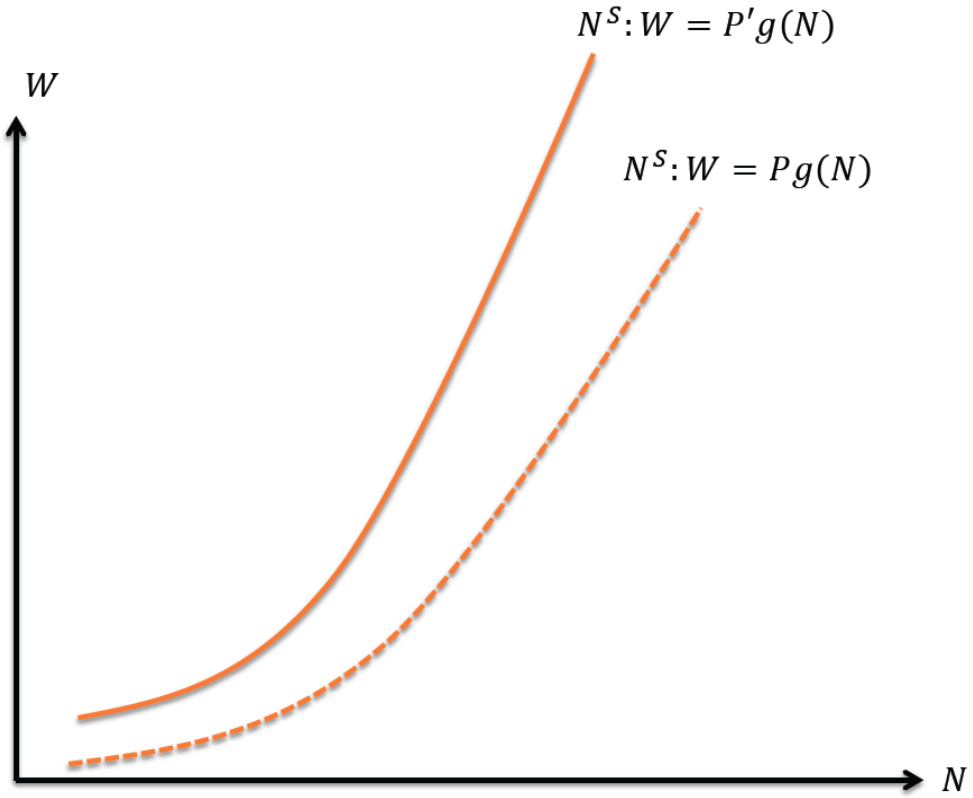
Derivation of the AS Schedule under the Classical System: Graphical Method

- In deriving the AS curve, we first focus on the two labour market equations.
- Plot (3) and (4) in the N-W plane (assuming some arbitrarily given value of P):



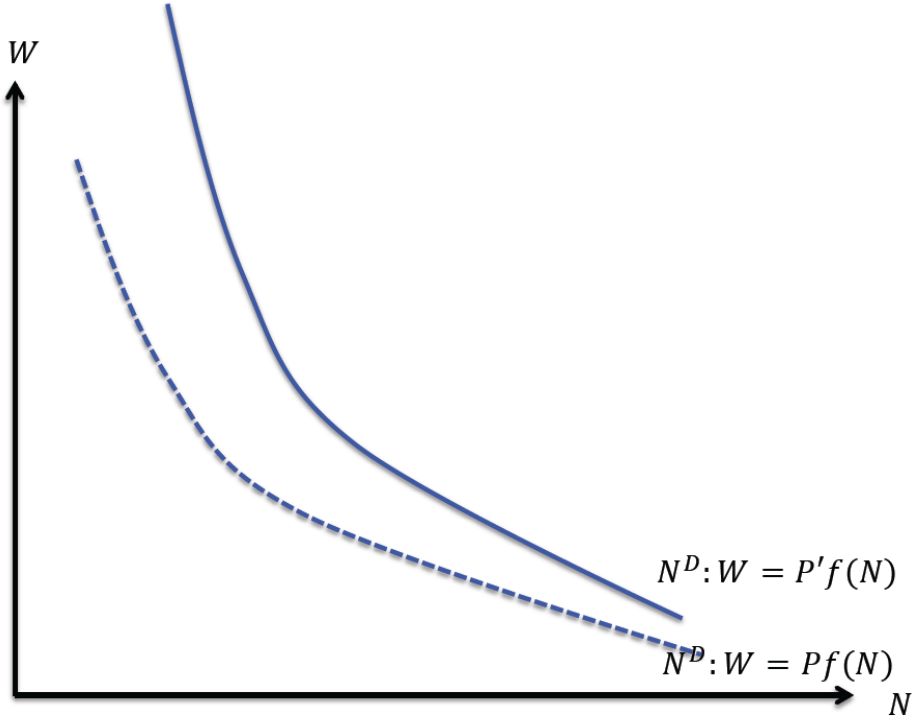
Derivation of the AS Schedule under the Classical System: Graphical Method (Contd.)

- Now increase P to a higher level, say P' :
  - The N^S curve shifts out proportionally - diverging away from the earlier curve for higher values of N (**Why?**)



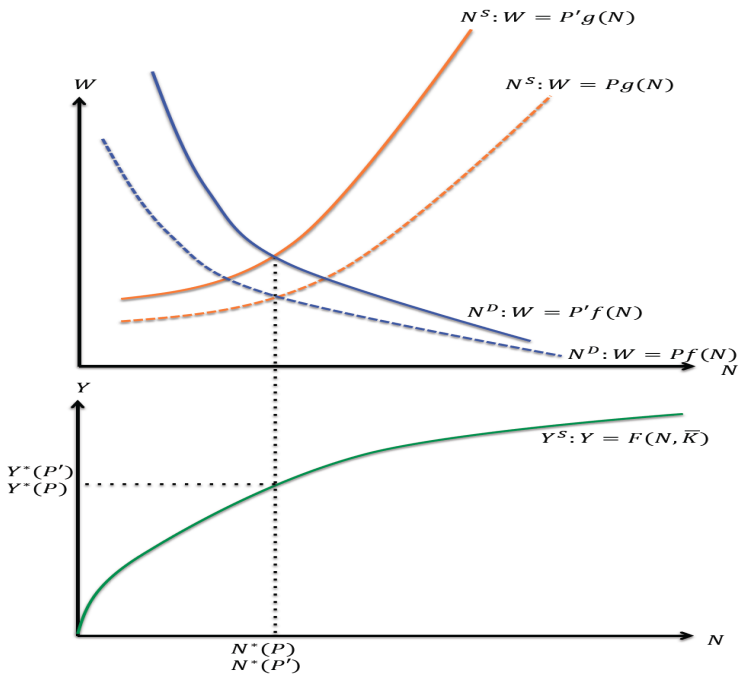
Derivation of the AS Schedule under the Classical System: Graphical Method (Contd.)

- Likewise as P increases to a higher level, say P' :
  - The N^D curve also shifts out proportionally - but it converges closer to the earlier curve for higher values of N (**Why?**)



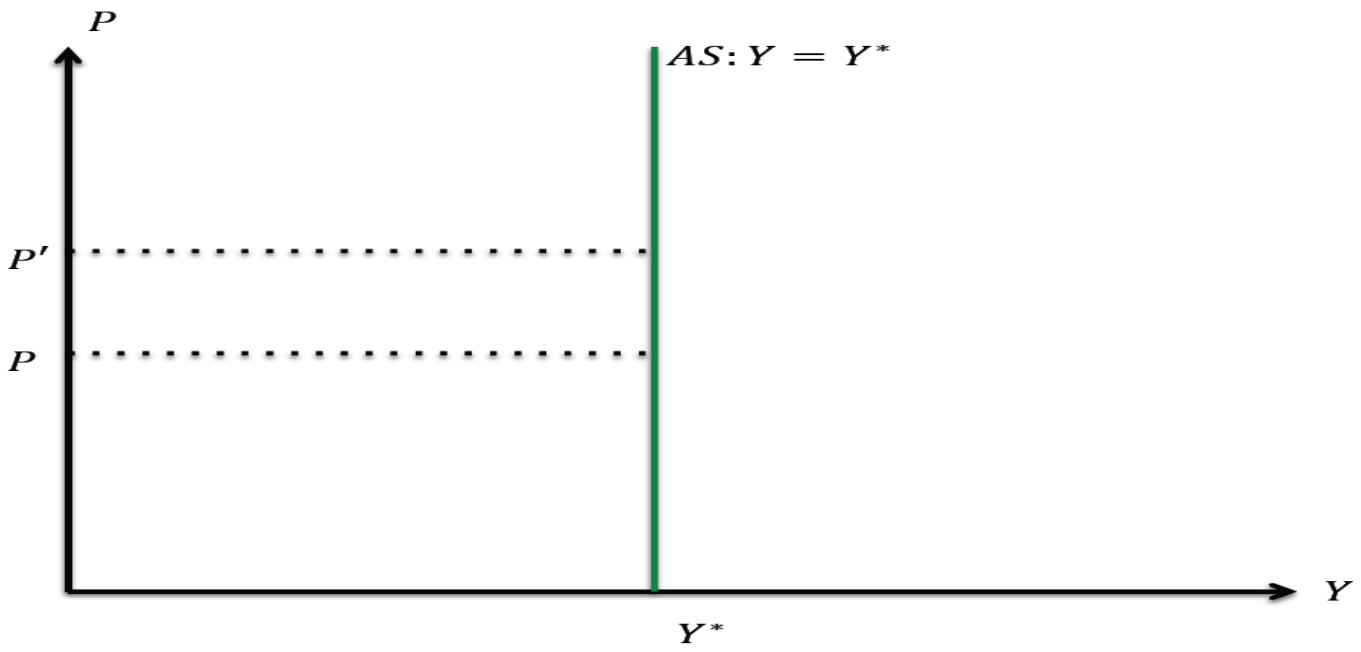
Derivation of the AS Schedule under the Classical System: Graphical Method (Contd.)

- However despite the fact that the shifts in N^S and N^D are not parallel, **the new point of intersection still remains the same** at N^* (**Why?**)
- Correspondingly, the output supplied remains fixed at Y^* :



Derivation of the AS Schedule under the Classical System: Graphical Method (Contd.)

- In other words, **the AS Schedule under the Classical System is Vertical**:



The Classical System: Solution (contd.)

- Let us now go back to the rest of the equations in the classical system.
- Let us club the SS & DD equations in the Money Market and the DD equation in the Goods Market together:

$$Y = C(Y) + I(r) + \bar{G}; 0 < C'(Y) < 1; I'(r) < 0 \tag{2}$$

$$M = \bar{M} \tag{5}$$

$$M = PL(Y, r); L_Y > 0; L_r < 0 \tag{6}$$

- This sub-system involves four endogenous variables: Y, r, M and P. We eliminate two of these variables to get another relationship between P and Y - which we call the aggregate demand curve (**AD**).
- How does the **AD** look under the Classical system? We discuss that below.

Derivation of the AD Schedule under the Classical System: Graphical Method

- In deriving the AD Schedule, first notice that the Money Supply Function is constant. This allows us to write the Money Market Equilibrium condition as:

$$\bar{M} = PL(Y, r); L_Y > 0; L_r < 0 \tag{7}$$

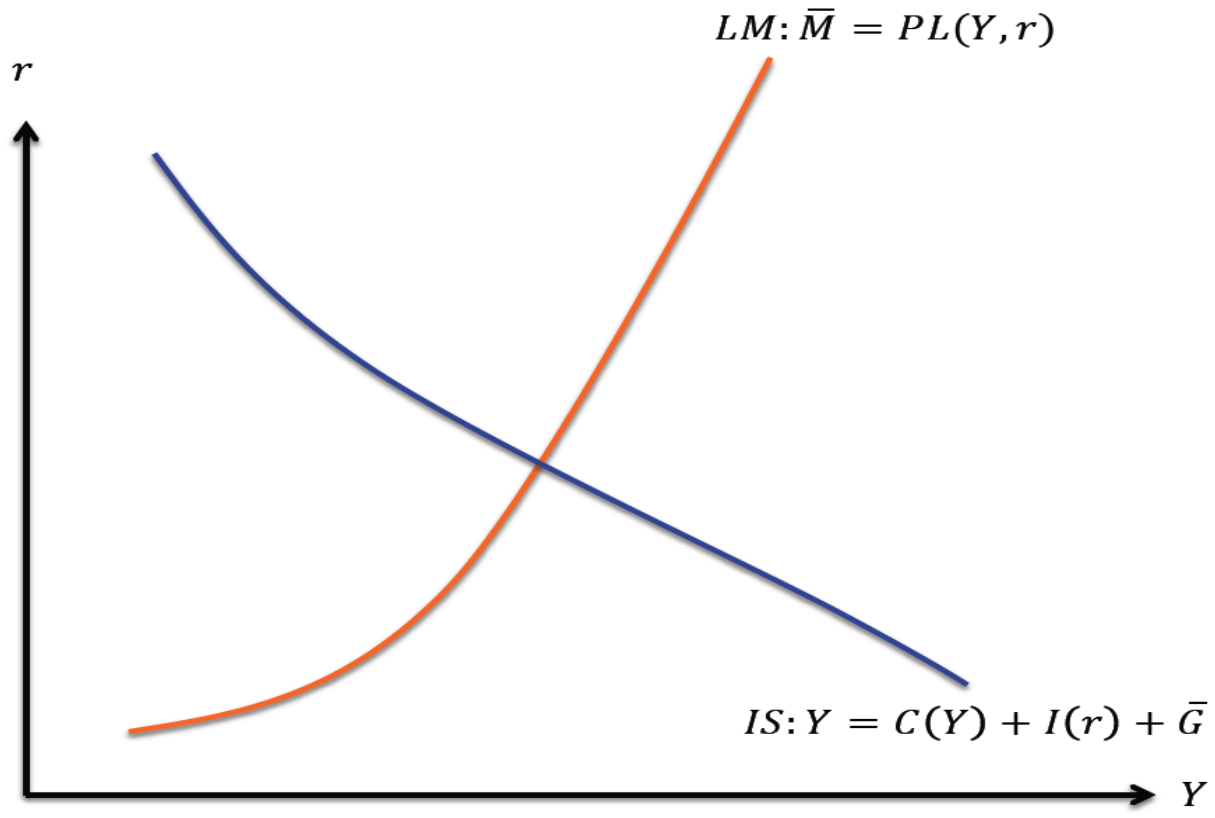
- This is the so-called **LM curve**, which represents a relationship between Y, r and P.
- On the other hand the Demand Equation for the Goods market represents another relationship between Y and r :

$$Y = C(Y) + I(r) + \bar{G}; 0 < C'(Y) < 1; I'(r) < 0 \tag{2}$$

- This is the so-called **IS curve.**

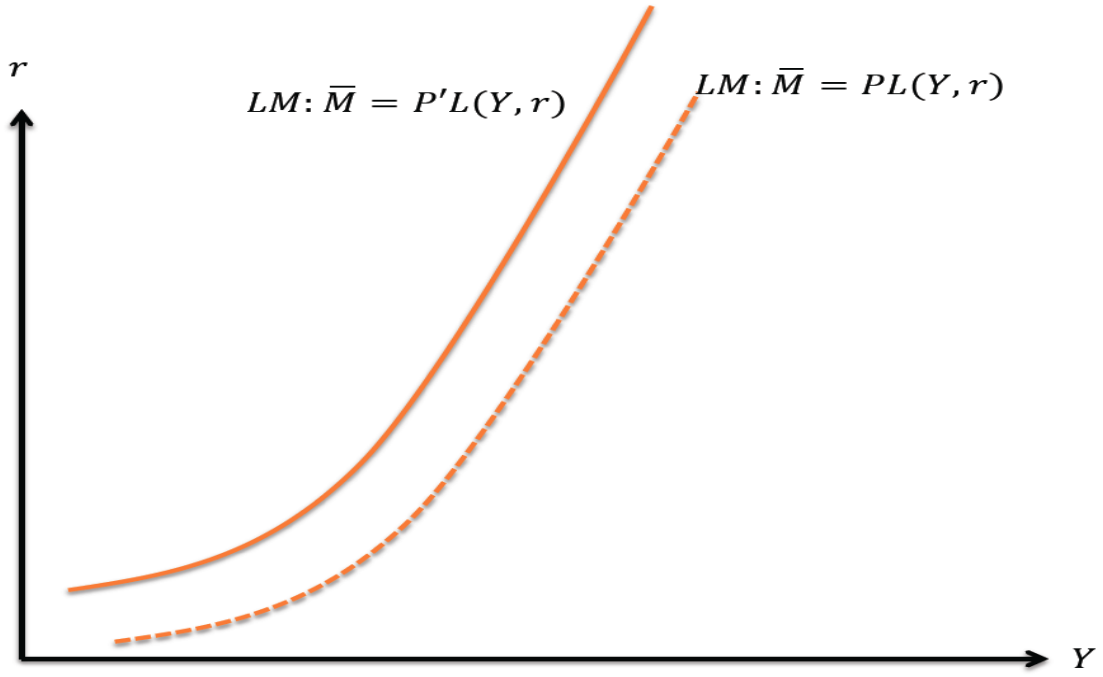
Derivation of the AD Schedule under the Classical System: Graphical Method (Contd.)

- Plot the IS and the LM curve in the Y-r plane (assuming some arbitrarily given value of P):



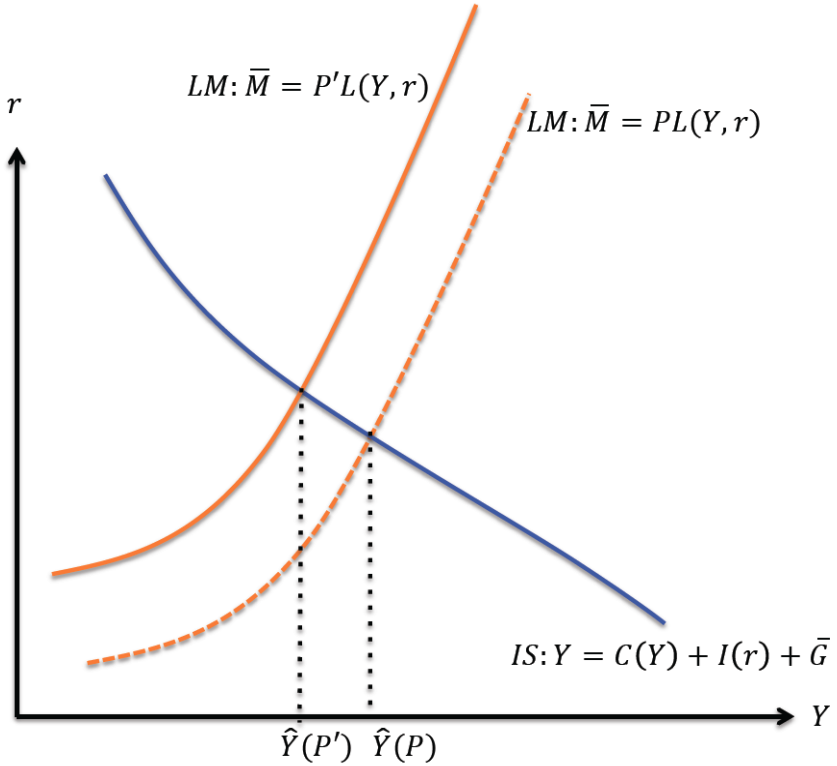
Derivation of the AD Schedule under the Classical System: Graphical Method (Contd.)

- Now increase P to a higher level, say P' :
- The LM curve shifts up.
- The IS Curve remains unchanged.



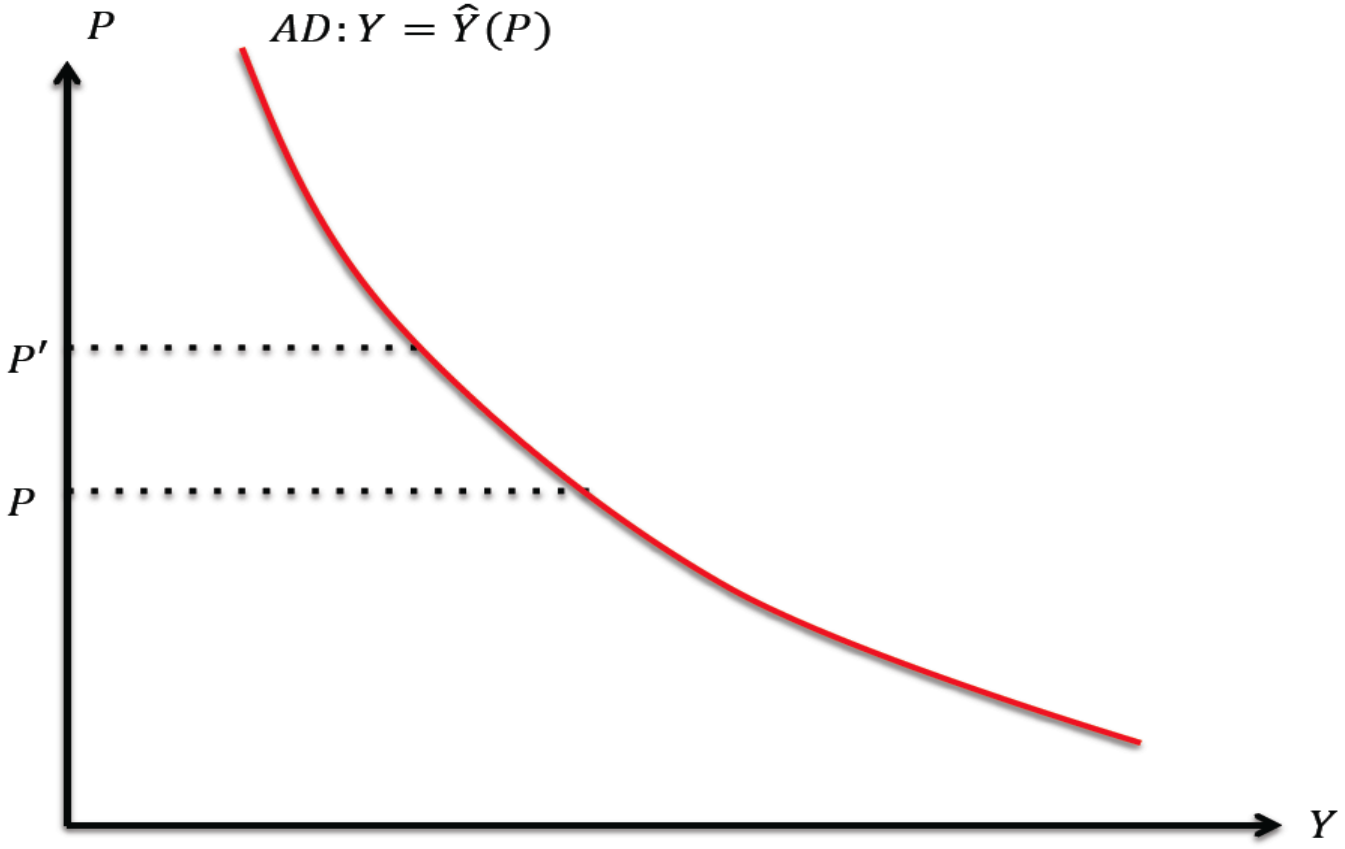
Derivation of the AD Schedule under the Classical System: Graphical Method (Contd.)

- Thus the new point of intersection shifts to the left:



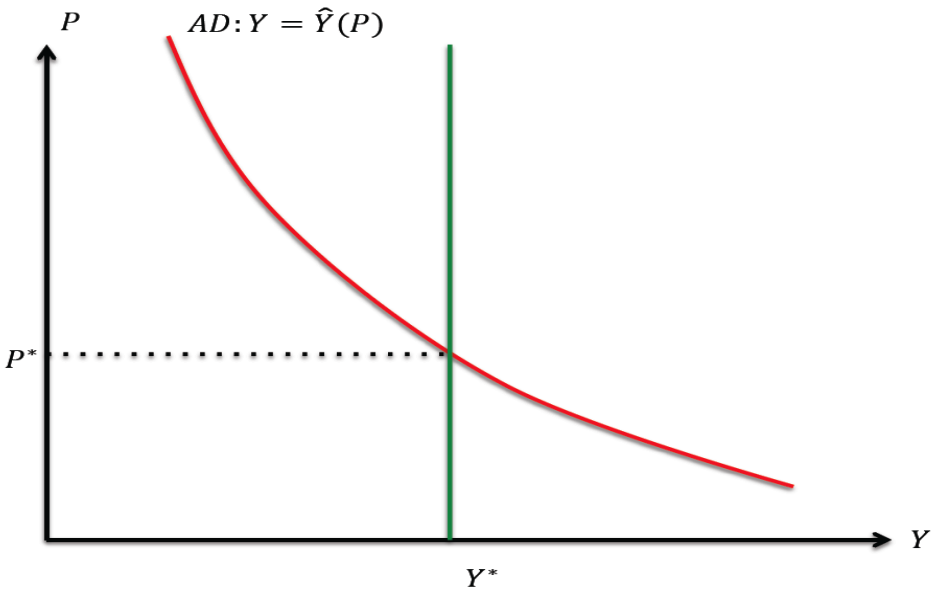
Derivation of the AD Schedule under the Classical System: Graphical Method (Contd.)

- In other words, **the AD Schedule under the Classical System is Downward Sloping**:



The Classical System: Solution (contd.)

- We then simultaneously plot the the **AS** and **AD** schedule in the Y-P plane to determine the equilibrium price level P^* and equilibrium output Y^* in the Goods Market.
- Once these two values are determined, other equilibrium values can be found by substituting these back in the other equations.
- Equilibrium price and quantity in the Goods Market - P^* and Y^* - as determined simultaneously by the intersection of the AS and the AD schedule - are shown below:



Digression: A Heuristic Justification of the Classical System

- Although we have specified all the equations here as ad hoc behavioural relationships without any explicit micro-foundations, they are not devoid of any economic logic. There are heuristic justifications for each of these equations - although these logics just appeal to common sense; they are not explicitly derived from agents' optimization exercise.
- We now examine these heuristic arguments one by one. In the process we also point out the limitations of these arguments.
- Let us first start with the labour market story underlying the classical system.

A Heuristic Justification of the Classical System (Contd.)

- In fact there is indeed a micro-founded labour market story implicit in the labour demand equation (4) of the classical system.
- Notice that labour is demanded by firms who are also engaged in the production and supply of the final commodity. Thus their labour demand decisions and production decisions are interrelated.
- The assumption implicit here is that all firms operate in a perfectly competitive market structure such that they take all prices as given.
- Profit maximization under perfect competition implies:

$$\underset{\{N\}}{\text{Max}}\ PF(N, \bar{K}) - WN$$

Heuristic Justification of the Classical System (Contd.)

- From the FONC, we get

$$F_N(N, \bar{K}) = \frac{W}{P}$$

which can be re-written as the labour demand function specified in (4):

$$W = Pf(N); \; f'(N) < 0 \; .$$

- Notice however that this profit-maximizing exercise is based on the *aggregate* production function. This begs the following question: who operates this aggregate production function?
- In a decentralized market economy obviously nobody actually operates with the aggregate production technology. So this outcome must come from aggregation of individual firms' optimization exercises.
- Is such aggregation always feasible? More importantly, will aggregation of firms' behaviour necessarily generate a labour demand function that looks as above? These are questions that we shall come back to when we discuss the micro-foundations explicitly.

Heuristic Justification of the Classical System (Contd.)

- Next, consider the labour supply equation in the Classical system (equation (3)):

$$W = Pg(N); g'(N) > 0 \tag{3}$$

- Inverting, we get:

$$N^S : N = \hat{g}\left(\frac{W}{P}\right); \hat{g} \equiv g^{-1}.$$

- The underlying logic here is that workers look at the real wage rate when they decide how much labour to supply. If the real wage rate goes up, then they are willing to supply more labour.
- Needless to say, this story presupposes an implicit labour-leisure choice of a household which positively responds to income. Does this necessarily hold? Again, without precise microfoundations we cannot say. (Remember the backward bending labour supply curve?)

Heuristic Justification of the Classical System (Contd.)

- The classical labour market story also presupposes that workers know the real wage rate when they decide about their labour supply, although prices are determined in the goods market.
- This implies some role of expectations (since goods prices would presumably not be known when wages were set in the labour market) which would influence the optimal labour supply decisions of the households. Once again without a precise theory of expectation formation, we do not know whether this assumption is justified or not.

Heuristic Justification of the Classical System (Contd.)

- Next, consider the goods market equations under the classical system.
- The goods market supply equation presupposes existence of an aggregate production function which is concave with respect to L, signifying that law of diminishing returns operates here just as it does for individual firms :

$$Y = F(N, \bar{K}); F_{N,} F_K > 0; F_{NN}, F_{KK} < 0 \tag{1}$$

- Does law of diminishing returns necessarily hold for the 'aggregate' production function? At this point we don't know. It needs a well-specified micro-founded story.

Heuristic Justification of the Classical System (Contd.)

- Next let us look at the goods market demand equation:

$$Y = C(Y) + I(r) + \bar{G}; 0 < C'(Y) < 1; I'(r) < 0 \qquad (2)$$

  - I should mention here that the goods demand condition that we have specified above does not repesent a truly the classical system; it is a product of the later Neoclassical synthesis which tried to combine the classical assumptions with some Keynesian insights. We shall discuss the truly classical system later as a special case.
- The consumption demand function merely states that as income goes up, households consume a part of their increased income.
  - Can one optimally derive a consumption/savings behaviour which depends only on income? Why does not savings depend on the rate of interest as well? The answers lie in the precise micro-foundations.
- Investment demand depends negatively on the interest rate. The heuristic argument here is that firms have to borrow to invest and a higher rate of interest means a higher cost of borrowing: hence investment demand falls.

Heuristic Justification of the Classical System (Contd.)

- Finally let us look at the money market in the classical system.
- The money demand is given by:

$$M = PL(Y, r); L_Y > 0; L_r < 0$$

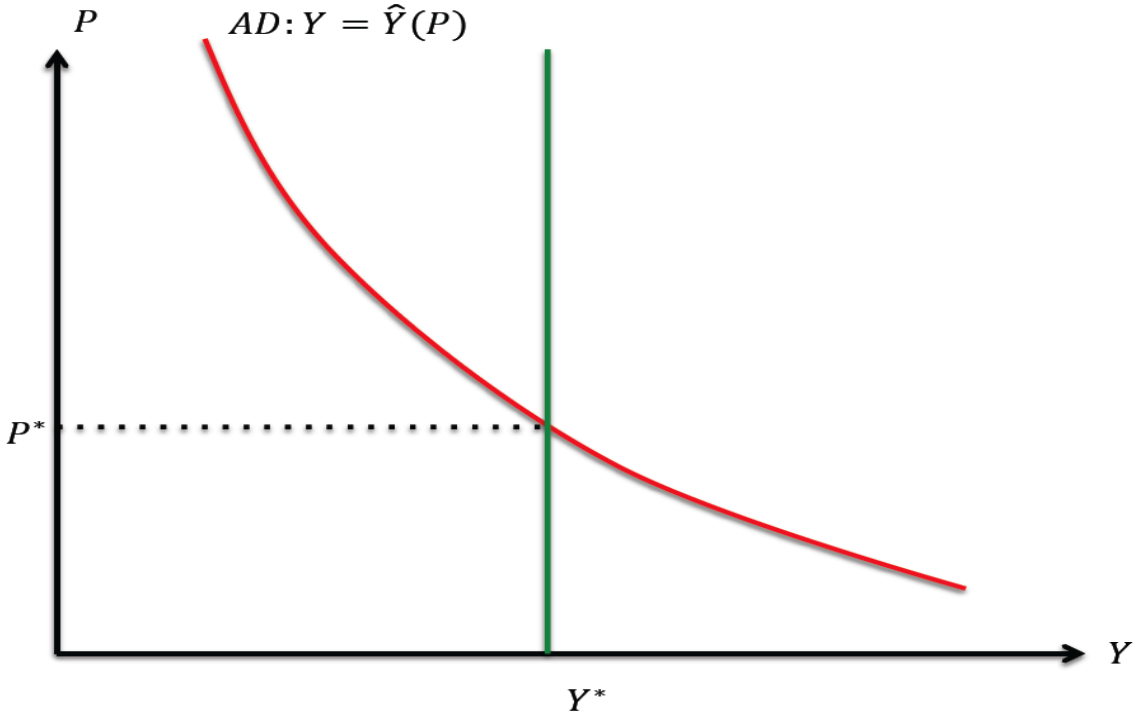
- This equation combines two motives for holding money:
  - Money that is held for transaction purposes: $M^d = KPY$; $K > 0$
  - Money that is held as an asset: $M^d = l(r)$; $l' < 0$
- Why should people hold money as an asset (and not bonds) when money yields zero real return?

Heuristic Justification of the Classical System (Contd.)

- The answer provided by Keynes was that people will hold money (vis-a-vis bond) if they expect bond prices to fall in future so that they can buy it cheap. Thus they can make some speculative gain out of it.
- But if bond prices are already very low (which means the interest rate is already very high) then people believe that it is unlikely to fall any further; hence they will be less willing to hold money.
- This would generate a negative relationship between demand for money (held as an asset) and the rate of interest. (**Question**: Which interest rate - **nominal or real**? **Why**?)
  - What kind of optimizing behaviour under risk/unceratinty and expectation would generate this outcome for the households? Only a micro-founded story can answer that question.
  - Also note once again that the money demand condition that we have specified above is not truly the classical money demand equation; it is a product of the latter Neoclassical synsthesis which tried to combine the classical assumptions with some Keynesian insights.

Equilibrium in the Classical System:

- As we have seen before, the equilibrium in the classical system is determined by the intersection of the AS and the AD schedule:



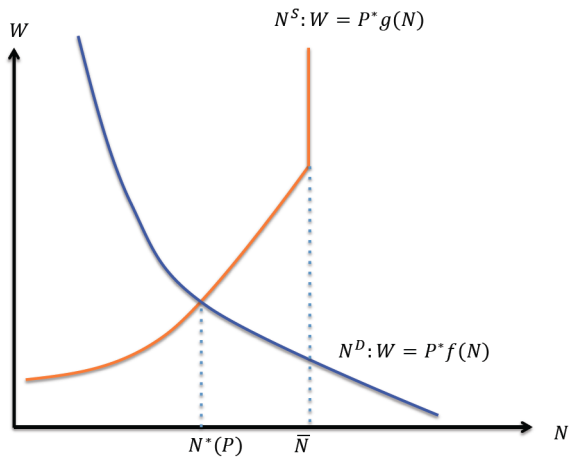
- Once we know Y^* and P^*, we can find out the corresponding equilibrium values of N^*, W^*, r^* and M^*.

Equilibrium vis-a-vis *Full* Employment

- Recall that the equilibrium level of employment in this model is defined by the point of intersection between the N^S and N^D curves drawn for the equilibrium price level P^*. The equilibrium nominal wage rate (W^*) is also simultaneously determined.
- This equilibrium level of employment represents the *market clearing* level of employment. It implies that given the equilibrium *real* wage rate $\left(\frac{W^*}{P^*}\right)$, everybody who is willing to supply labour indeed finds employment.
- In other words, there is *no 'involuntary' unemployment*.
- But to call it 'full employment' could be misleading!
- What is full employment?
  - Suppose there exists a maximum limit on the labour supply - say $\bar{N}$, such that it is not feasible to increase the labour supply beyond this level.
  - This maximum feasible level is some times referred to as the *full-employment* level of labour supply.

Equilibrium vis-a-vis *Full* Employment (Contd.)

- If there indeed exists such a maximum limit such that labour supply cannot be increased beyond this level - no matter how high the wage rate is, then the N^S curve becomes vertical at this point.
- But there is no *apriori* reason why the point of intersection between N^S and N^D will happen precisely at this vertical stretch.
- Thus the classical model is perfectly consistent with a scenario where there is no *full-employment* of the entire labour force. But any such unemployment must be *voluntary.*



Equilibrium vis-a-vis *Full* Employment (Contd.)

- **Question:** Let us begin with an AS-AD configuration such that at the corresponding equilibrium $N^* < \bar{N}$. In that case, will the equilibrium price level (P^*) re-adjust so that in equilibrium you always end up with $N^* = \bar{N}$?
- The answer is "no"! So the complete price flexibility in the Classical system does not necessarily lead to 'full employment'; it only eliminates 'involuntary unemployment'.

Effectiveness of Government Policies under the Classical System:

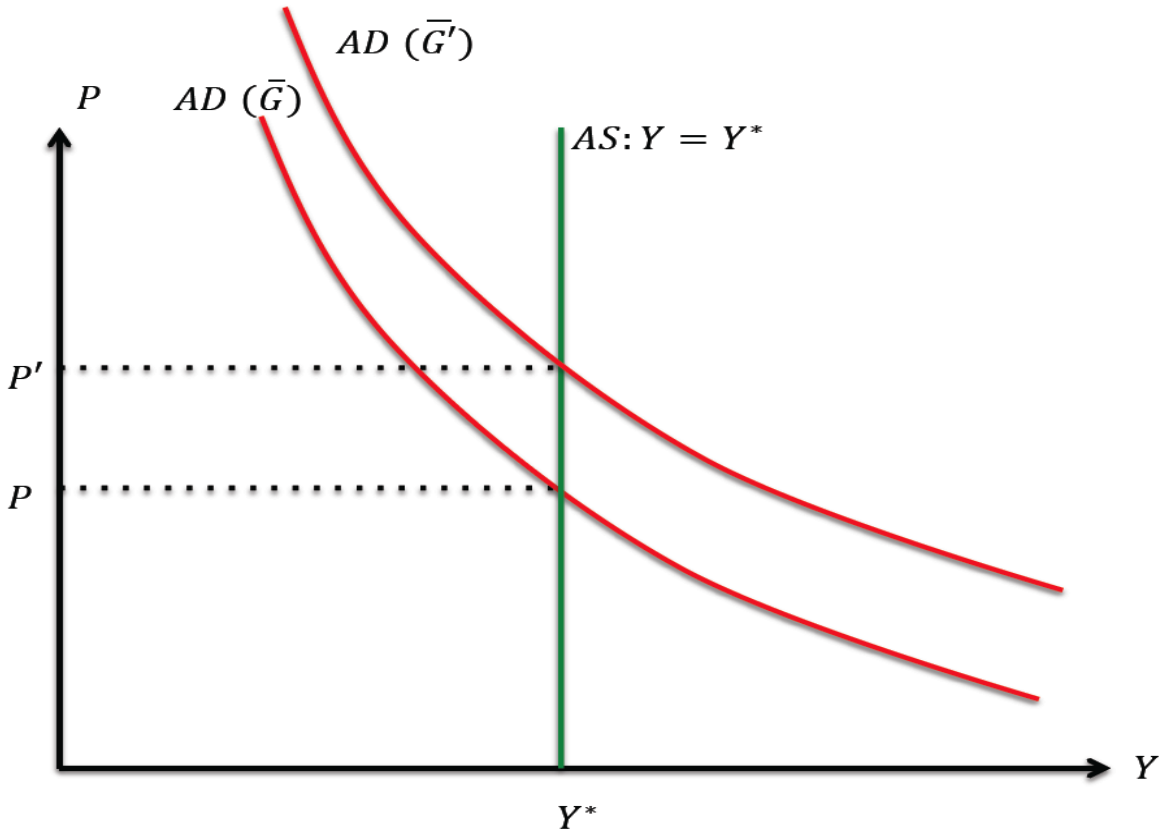
- We shall primarily focus on two kinds of government policies:
  - Fiscal Policy - which usually changes the amount of government expenditure ($\bar{G}$)
  - Monetary Policy - which usually changes the amount of money supply ($\bar{M}$)
- There could be other forms of government policies - e.g. taxes; government borrowing; government directly influencing the wage rate or price level in the goods/labour market or the interest rate in the money market. We shall talk about some of these latter policies as special cases.

Effectiveness of Government Policies under the Classical System (contd.):

- Notice that $\bar{G}$ enters only in the IS equation, while $\bar{M}$ enters only in the LM equation.
- In particular, an increase in $\bar{G}$ shifts the IS curve up while an increase in $\bar{M}$ shifts the LM curve down.
- Both policies lead to a rightward shift of the AD curve but leave the AS curve unchanged.
- Therefore the equilibrium output does not change.

Effectiveness of Government Policies under the Classical System (contd.):

- Thus the standard Fiscal and Monetary Policies (which affect only the demand side of the economy) are **completely ineffective** in raising the equilibrium output and employment under the classical system:



Proportional Income Tax - A Possible Exception?

- So far we had not introduced taxes in our model. Let us now introduce a **proportional income tax (t)** which is imposed **at the household level**.
- This changes the disposable income -available to the household for consumption:

$$C^d = C(Y - tY) = C(Y^d);\ 0 < C'(Y^d) < 1$$

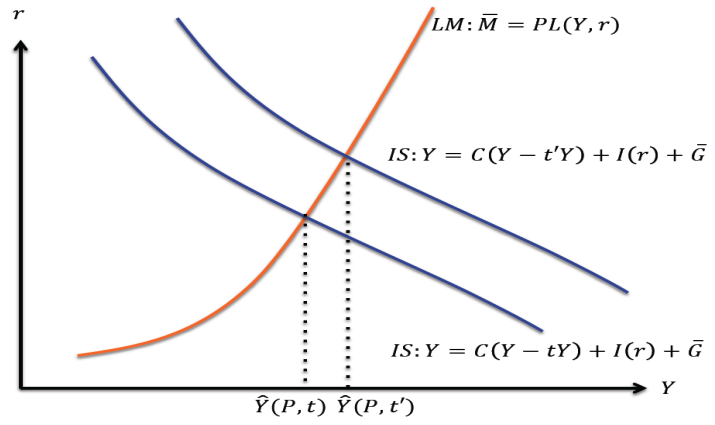
- Thus the demand equation in the Goods Market now becomes:

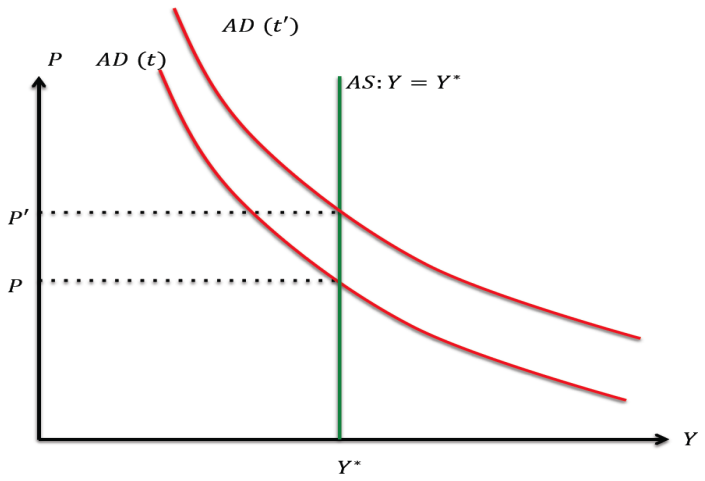
$$Y = C(Y^d) + I(r) + \bar{G}; 0 < C'(Y^d) < 1; I'(r) < 0$$

- The corresponding IS curve (representing the demand condition in the Goods Market in the Y-r plane) still looks the similar.
- But a change in a tax rate will now shift the IS curve, but not the LM curve. Thus the AD schedule gets affected.

Proportional Income Tax - A Possible Exception? (Contd.)

- Consider a tax cut such that the tax rate **decreases** from t to t':





Proportional Income Tax - A Possible Exception? (Contd.)

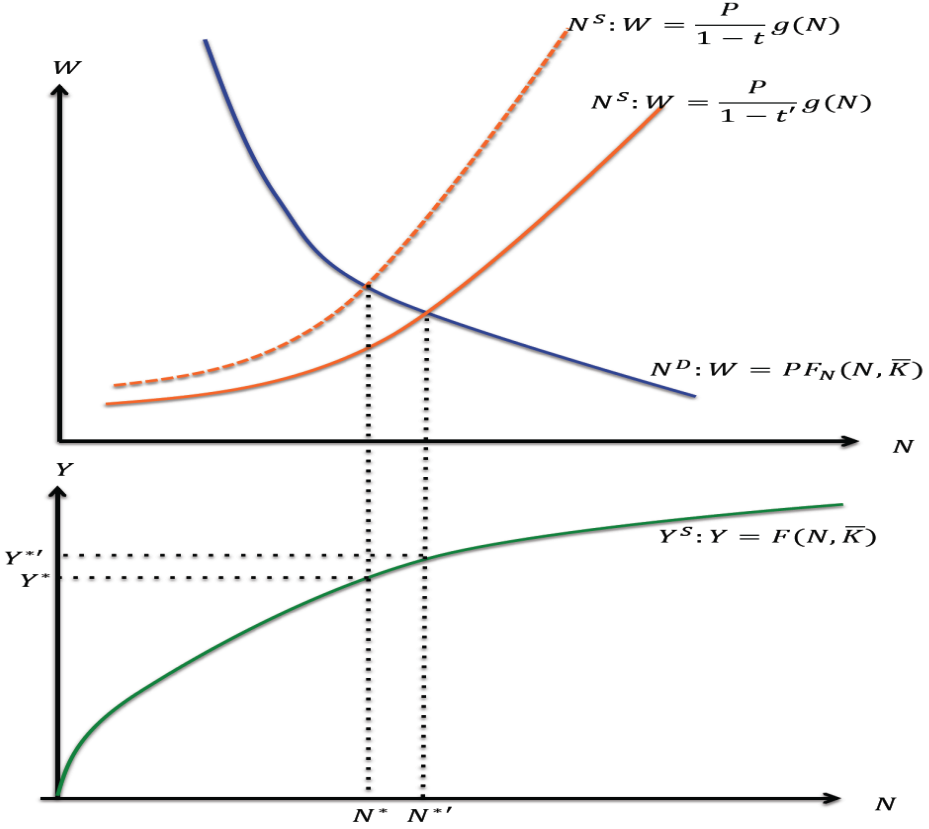
- As before the AD shifts to the right (because the IS curve has shifted up due to the tax cut).
- **But** this is not the end of the story!!
- Recall that the labour is supplied by the households.
- If household incomes are taxed then so would be wage income!So the **effective** wage rate - relevant for the households (and only for the households) is now $(1-t)W$ - **not** W!
- In other words, the supply equation in the labour market now becomes:

$$W = \frac{P}{(1-t)} g(N); \; g'(N) > 0$$

- The demand equation in the labour market however remains unchanged (**Why?**).

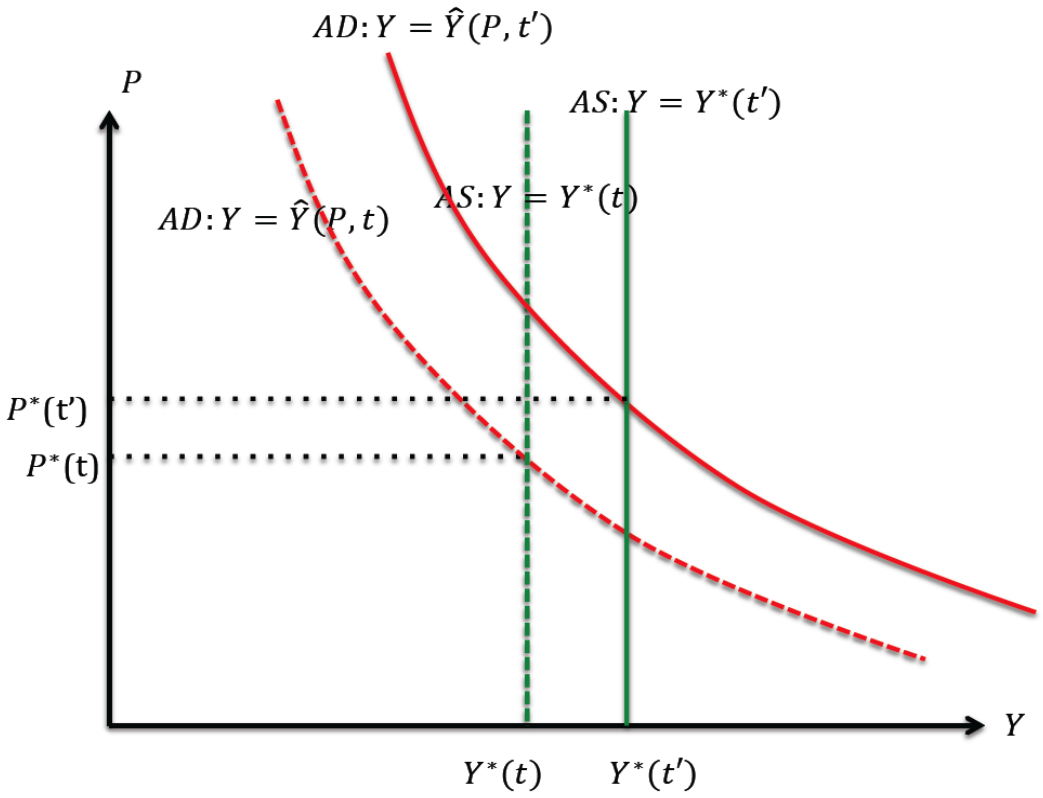
Proportional Income Tax - A Possible Exception? (Contd.)

- A tax cut shifts the labour supply schedule (in the W-N plane), but not the labour demand schedule. Hence the AS schedule gets affected too!



Proportional Income Tax - A Possible Exception? (Contd.)

- **Thus** a proportional income tax on household income - in particular **a tax cut** - could be effective in raising the equilibrium output and employment under the classical system:



Emergence of the Keynesian System: The Great Depression (1929-1941)

- The western capitalist economies (US,UK) were running more or less in line with the Classical/Neo-classical system till the 1920s - with a laissez faire government allowing private initiatives to dominate.
- Then came the Great Depression which created havoc - especially in the US economy.

The Great Depression (1929-1941): (Contd.)

- "The Great Depression of 1929 devastated the U.S. economy. Half of all banks failed. Unemployment rose to 25 percent and homelessness increased. Housing prices plummeted 30 percent, global trade collapsed by 60 percent and prices fell 10 percent. ....The economy shrank 50 percent in the first five years of the Depression. In 1929, economic output was $105 billion, as measured by gross domestic product. By 1933, the country had suffered five years of losses. It only produced $57 billion, half what it produced in 1929. New Deal spending boosted GDP growth 10.8 percent in 1934. It grew another 8.9 percent in 1935, a whopping 12.9 percent in 1936 and 5.1 percent in 1937. Unfortunately, the government cut back on New Deal spending in 1938, and the depression returned." (K. Amadeo (2017)).
- Shift in ideology: The dominant economic belief shifted from a pure free market economy to a mixed economy with much more emphasis on government spending for its success. This new school of thought owed its origin to Keynes' General Theory (1936).

The Keynesian System:

- The benchmark Keynesian system that we shall consider here will be almost analogous to the Classical system characterized above, except for the labour supply equation.
- The Keynesian model assumes that the labour market is unionized and the supply of labour is therefore determined by the rules/norms set by the labour union.
- The union sets the nominal wage rate at some level $\bar{W}$ by collective bargaining and once the wage is set, all workers supply their entire labour stock at this wage rate. (Why all workers would comply to such a rule is a different story and would require precise modelling of the union's and the workers' optimization problem(s). We shall come back to this point when we discuss the microfoundations of these assumptions).
- Thus the labour supply schedule now becomes perfectly elastic (a flat line) at the union-determined wage rate: $\bar{W}$.

The Keynesian System (in equations):

- **The Goods Market:**
- Supply Equation:

$$Y = F(N, \bar{K}); F_{N,} F_K > 0; F_{NN}, F_{KK} < 0 \tag{8}$$

- Demand Equation:

$$Y = C(Y) + I(r) + \bar{G}; 0 < C'(Y) < 1; I'(r) < 0 \tag{9}$$

- **The Labour Market:**
- Supply Equation:

$$W = \bar{W} \tag{10}$$

- Demand Equation:

$$W = Pf(N) \tag{11}$$

- **The Money Market:**
- Supply Equation:

$$M = \bar{M} \tag{12}$$

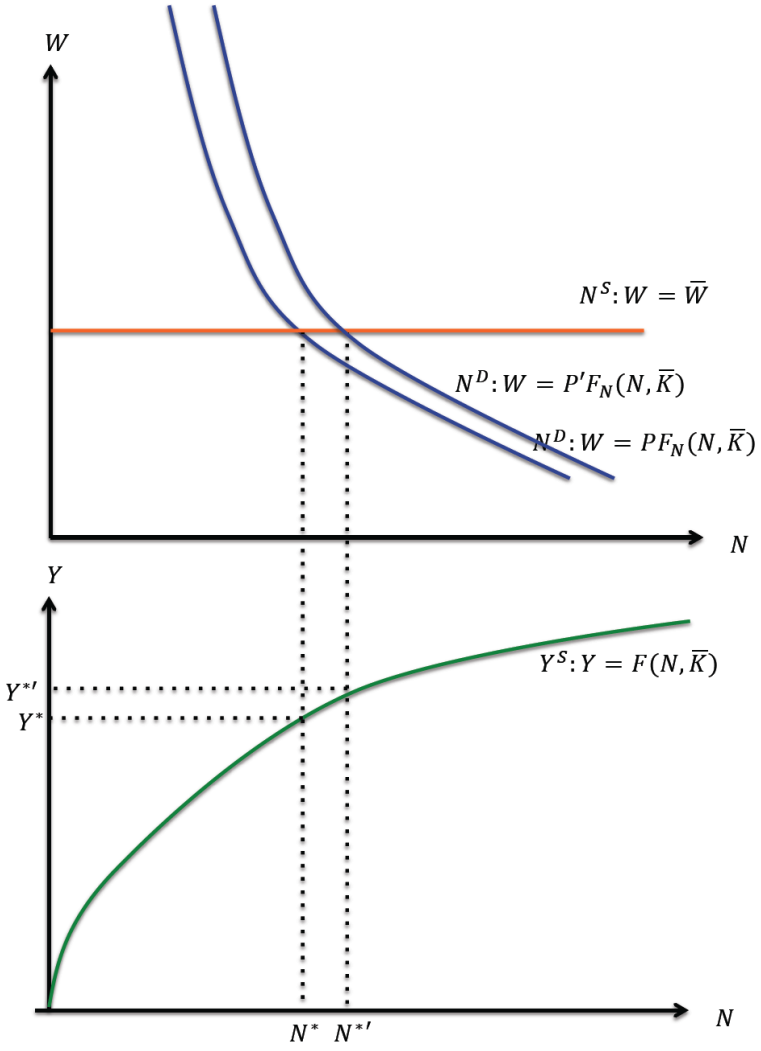
- Demand Equation:

$$M = PL(Y, r); L_Y > 0; L_r < 0 \tag{13}$$

The Keynesian Labour Market

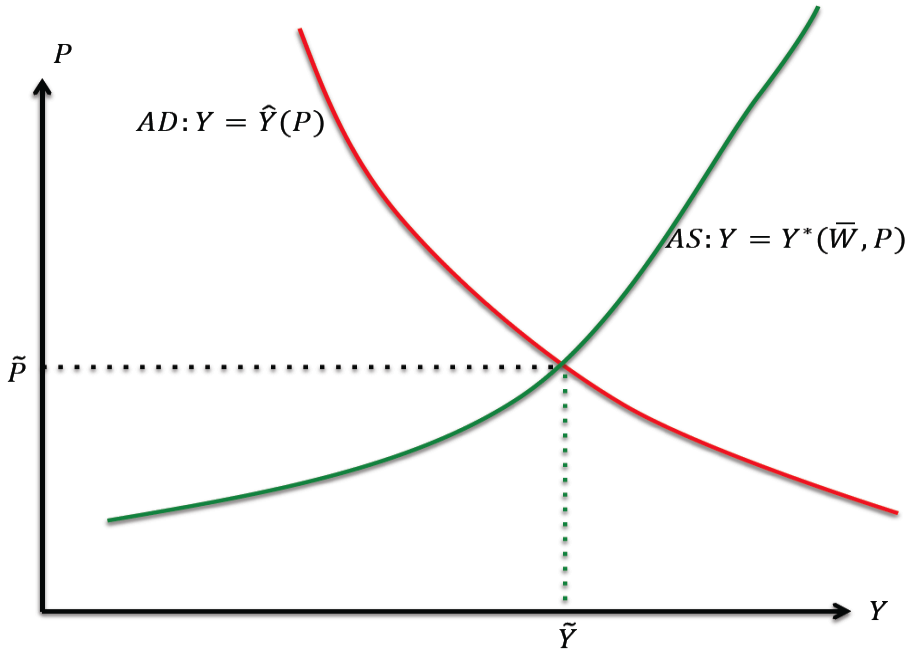
- As we have explained before, the only equation that differs between the two systems is the labour supply equation.
- The Keyenesian System assumes that labour supply is perfectly elastic at a given wage rate $\bar{W}$.
- The Labour Market:
  - Supply Equation:
$$W = \bar{W} \tag{14}$$
  - Demand Equation:
$$W = Pf(N) \tag{15}$$

Equilibrium in Keynesian Labour Market & the corresponding AS Schedule:



Equilibrium in Keynesian Labour Market & the corresponding AS Schedule:

- So the AS schedule is **upward sloping** under the Keynesian system.
- Notice however that as far as equations (2), (5) and (6) are concerned, nothing has changed, which means the AD curve in the Keynesian system is identical to the AD curve of the Classical System.
- The equilibrium in the Keynesian system is determined by the intersection of the (now upward sloping) AS and the AD schedule:



Effectiveness of Government Policies under the Keynesian System:

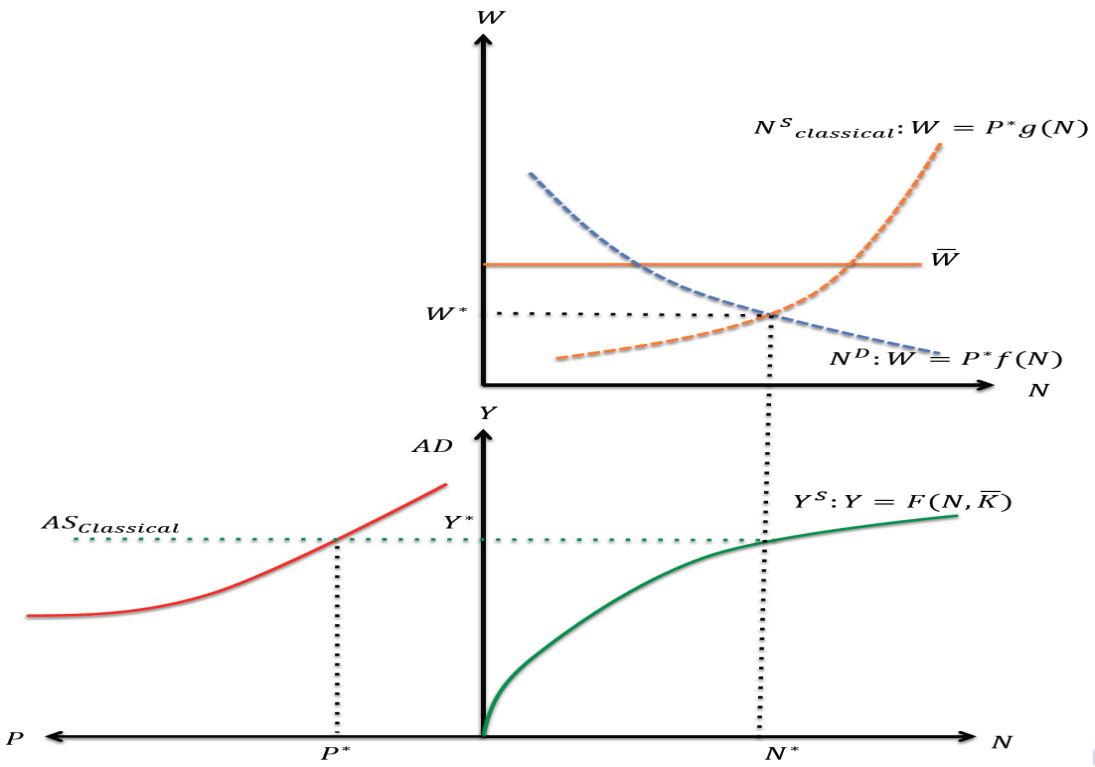
- **Question:** What does this tell you about the effectiveness of the standard monetary and fiscal policies ($\uparrow$ in $\bar{G}$ or $\bar{M}$)?

Equilibrium Employment: Comparing Keynes with Classics

- How does $\tilde{N}$ (equilibrium level of employment under the Keynesian System) compare with N^*(equilibrium level of employment under the Classical system)?
- Or equivalently: how does $\tilde{Y}$ (equilibrium output under the Keynesian System) compare with Y^*(equilibrium output under the Classical system)?
- To answer this question, we shall consider two cases:
  (a) $\bar{W} > W^*$
  (b) $\bar{W} < W^*$

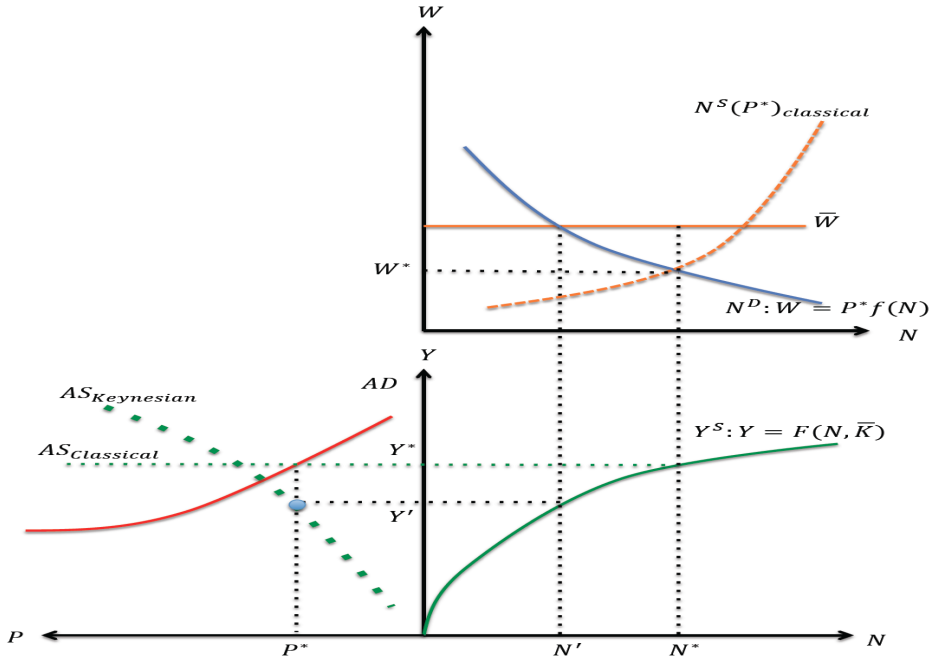
Equilibrium Employment: Keynes vis-a-vis Classics (Contd.)

- Let us start with case when $\bar{W} > W^*$.
- Let us first diagrammatically depict the Classical Equilibrium (P^*, Y^*, N^*, W^*) and then see whether this can still be an equilibrium when the nominal wage rate is arbitrarily fixed at some $\bar{W} > W^*$:



Equilibrium Employment: Keynes vis-a-vis Classics (Contd.)

- When labour supply is perfectly elastic at $W = \bar{W}$, the classical labour demand function (at P^*) intersects $\bar{W}$ at some N' - resulting in Y' amount of output being supplied. In other words, (P^*, Y') constitute a point on the Keynesian AS curve. But this cannot be an equilibrium point since at this point AD $>$ AS.

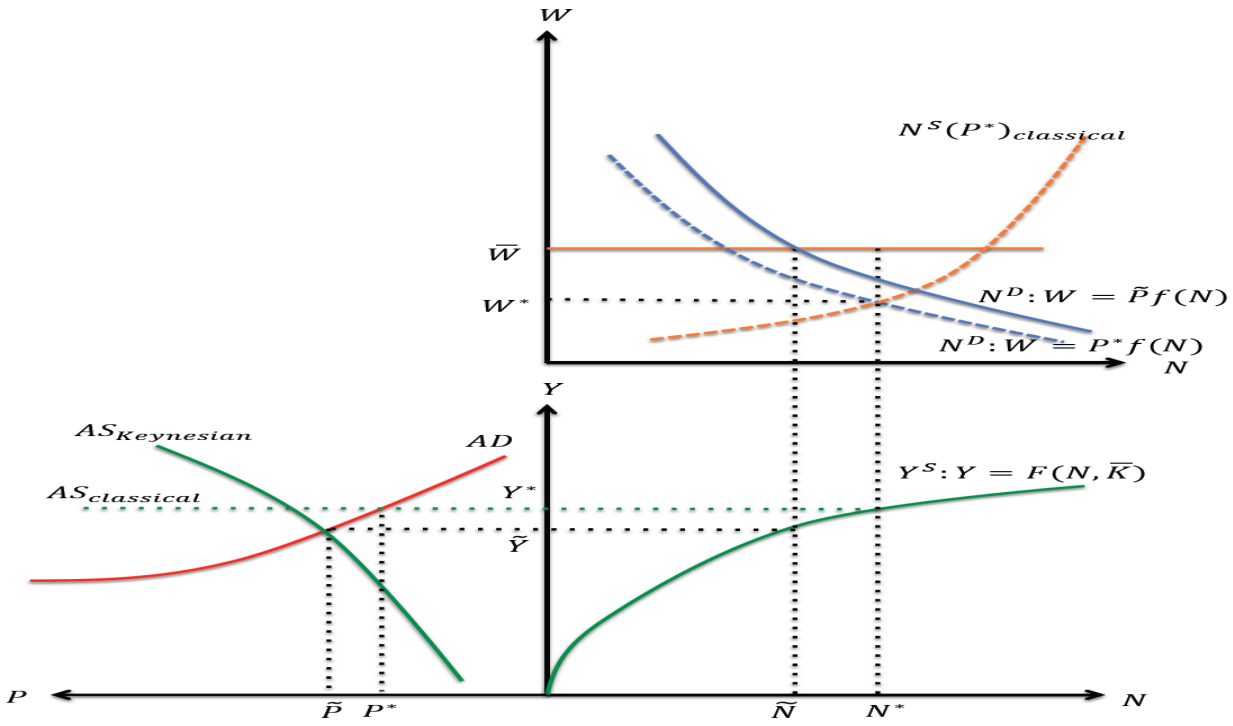


Equilibrium Employment: Keynes vis-a-vis Classics (Contd.)

- This tells us that when $\bar{W} > W^*$:
  - The Keynesian equilibrium price ($\tilde{P}$) must be *higher* than the Classical equilibrium price level (P^*).
  - But since higher P means lower demand (along a downward sloping AD curve), this implies that the Keynesian equilibrium output ($\tilde{Y}$) must be *less* than the Classical equilibrium output level (Y^*).
  - Consequently, equilibrium employment level under the Keynesian system ($\tilde{N}$) must be lower that the equilibrium employment level under the Classical system (N^*)

Equilibrium Employment: Keynes vis-a-vis Classics (Contd.)

- When $\bar{W} > W^*$, the relative position of $\tilde{N}$ vis-a-vis N^* is shown in the diagram below:

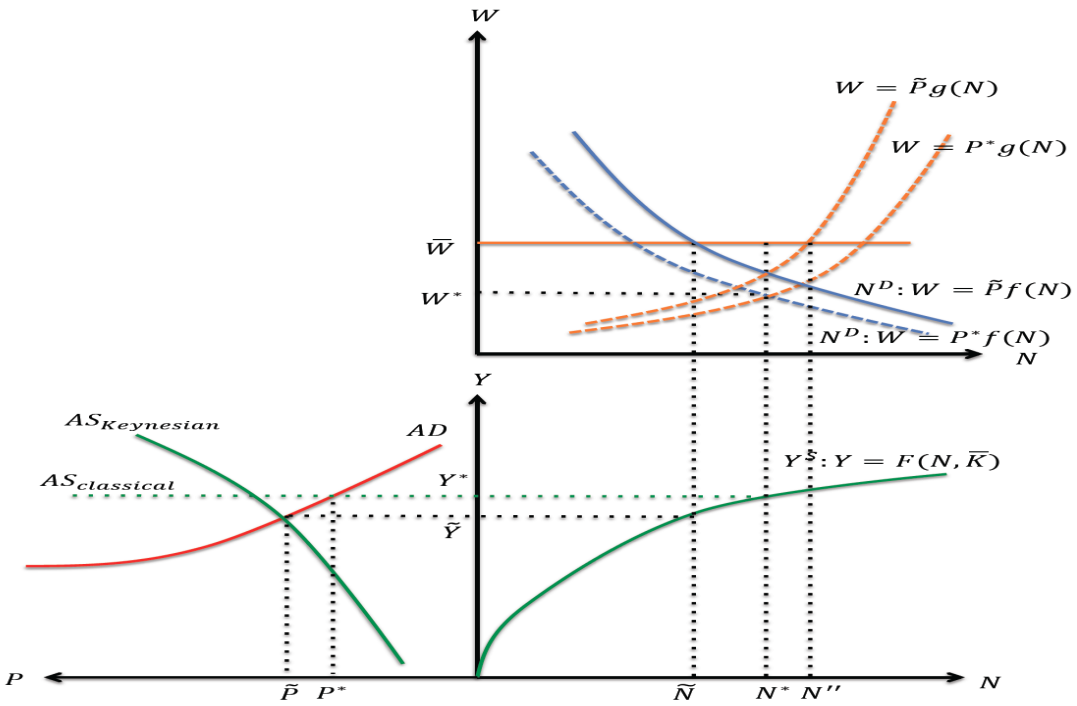


Keynesian System: Involuntary Unemployment?

- Is the reduction in employment from N^* to $\tilde{N}$ involuntary or involuntary?
- A flat Keynesian labour supply schedule at $W = \bar{W}$ does not allow us to properly identify the extent of voluntary vis-a-vis involuntary unemployement.
- To differentiate between voluntary and involuntary unemployment, we have to draw the classical labour supply schedule (that captures household' willingnes to work) for the given $\bar{W}$ and $\tilde{P}$.

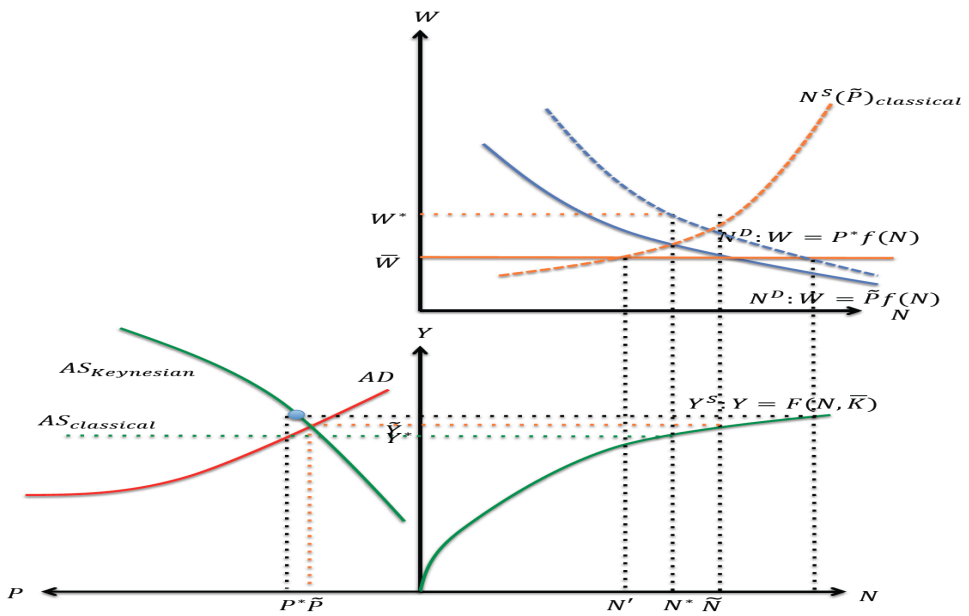
Keynesian System: Involuntary Unemployment? (Contd.)

- Once we draw this schedule, it is easy to see that indeed the entire gap of $(N^* - \tilde{N})$ represents involuntary unemployment.
- In fact, at $\bar{W}$ and $\tilde{P}$, the level of involuntary unemployment is even greater that $(N^* - \tilde{N})$, as shown by the difference $(N'' - \tilde{N})$ in the figure below. (Note however that N'' is **not** an equilibrium configuartion).



Keynesian System: Involuntary Unemployment? (Contd.)

- What happens is the Keynesian System if trade union fixes the nominal wage rate at a level such that $\bar{W} < W^*$?
- We can anlyse as before and show that now $\tilde{N} > N^*$. But notice that now part of this labour supply is 'forced labour'!!



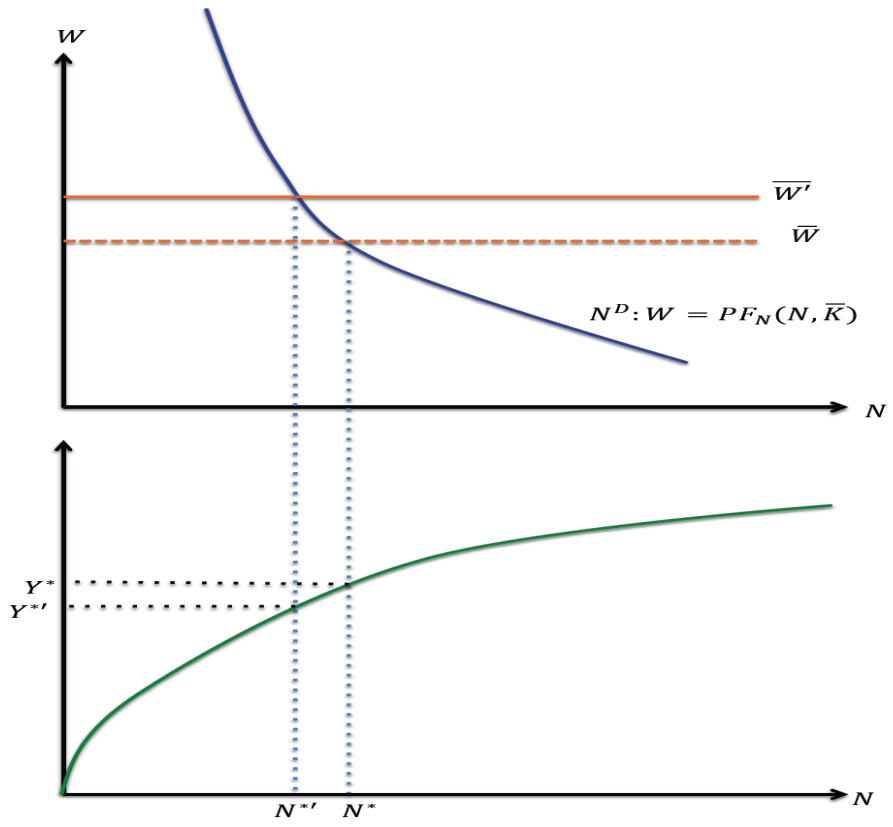
- If is not clear why a worker would be part of trade union if the union forces him to work beyond his optimal choice. So from now on we shall focus only on the case where $\bar{W} > W^*$.

Keynesian System: An Increase in Nominal Wage Rate

- Suppose now the labour union becomes stronger and is able to negotiate a higher nominal wage $\bar{W}'$.
- What happens to equilibrium output in the Keynesian System if the nominal wage rate changes (increases) from $\bar{W}$ to $\bar{W}'$?
- In particular, would the households be better off in terms of income?

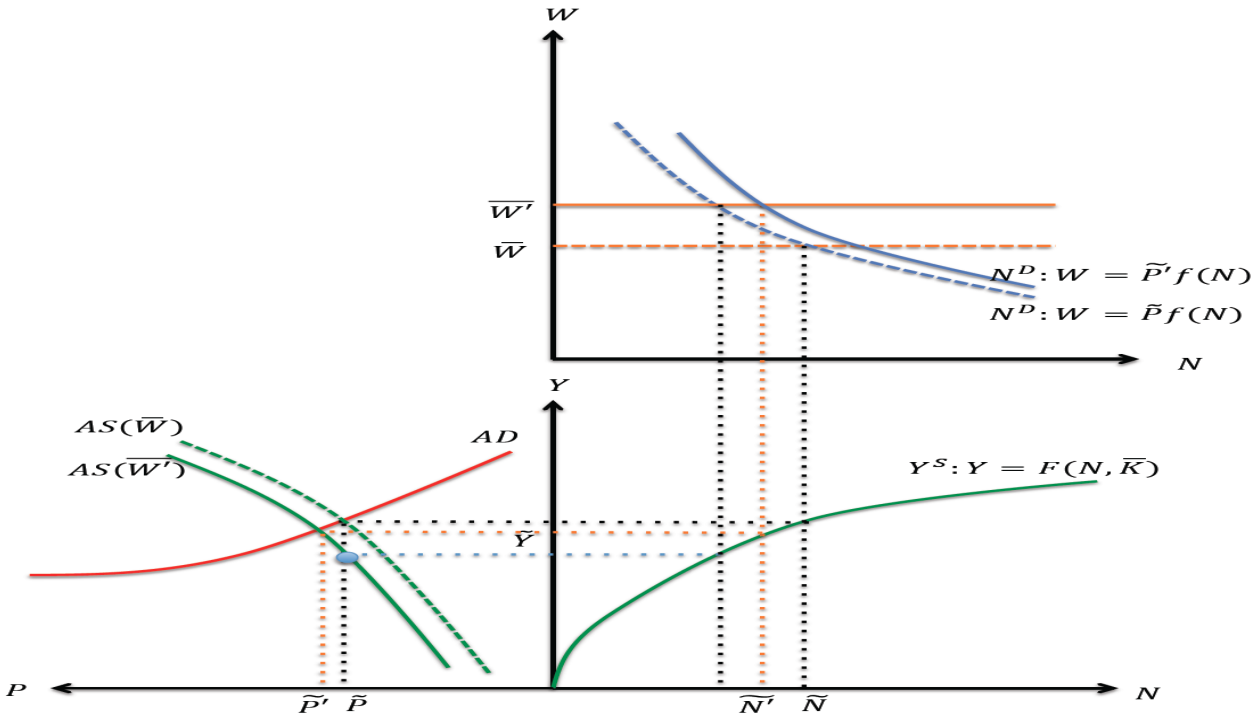
Keynesian System: An Increase in Nominal Wage Rate (Contd.)

- What happens when $\bar{W}$ goes up:



Keynesian System: Effect of a Rise in Nominal Wage Rate

- The AS schedule shifts to the left - reducing the equilibrium level of output and increasing the equilibrium price level.



- **Question:** What about the **real wage rate**? Would the real wage rate be higher/lower or remain the same in the new equilibrium?

Keynesian System: Effect of a Rise in Nominal Wage Rate (Contd.)

- As it turns out, in the new equilibrium the real wage rate actually increases. (In other words, although the price level in the new equilibrium will be higher, it will not be high enough to completely outweigh the increase in the nominal wage rate). (**Why?**)
- Are the households better off? The answer is not clear! While real wage rate is indeed higher than before, total income has actually gone down. So the households for whom wage component is high will be better off but at the expense of others.

Keynesian System: Effect of a Rise in Aggregate Demand

- What happens to the real wage rate when $\bar{G}$ or $\bar{M}$ increases?
- This leads to a shift in the AD curve (with unchanged AS curve).
- So the price level rises in the new equilibrium, and with a constant $\bar{W}$, the real wage rate surely falls.
- Notice that the real wage rate in this model behaves in a **counter-cyclical** fashion: In periods of boom (high demand) we have higher equilibrium output (and employment) but it is associated with lower real wages. And opposite happens in periods of slump (low demand).
- In other words, in this version of the Keynesian model real wage rate and aggregate output (and employment) are *negatively* correlated.
- This feature of the model is not supported by the empirical facts. It has been observed that real wage rate typically moves in **pro-cyclical** manner. In periods of boom, employment, output and real wage rate - all move in the upward direction; opposite happens during recessions.

Extension of the Keynesian System: The Neo-Keynesians & Sticky Prices

- An extension of the general Keynesian structure was later proposed, which was able to address this issue, while retaining the other basic Keynesian features. This is the Neo-Keynesian extension.
- This extension assumes that not only that nominal wage is rigid, but so is the nominal price level.
- Sticky prices mean that the aggregate supply curve is horizontal at some $P = \bar{P}$.
- Notice that a horizontal AS schedule means that this system is completely demand-determined. At $\bar{P}$ whatever output demanded is always supplied. (Thus this set up is diametrically opposite to the supply-determined Classical System discussed earlier).

The Neo-Keynesians & Sticky Prices: (Contd.)

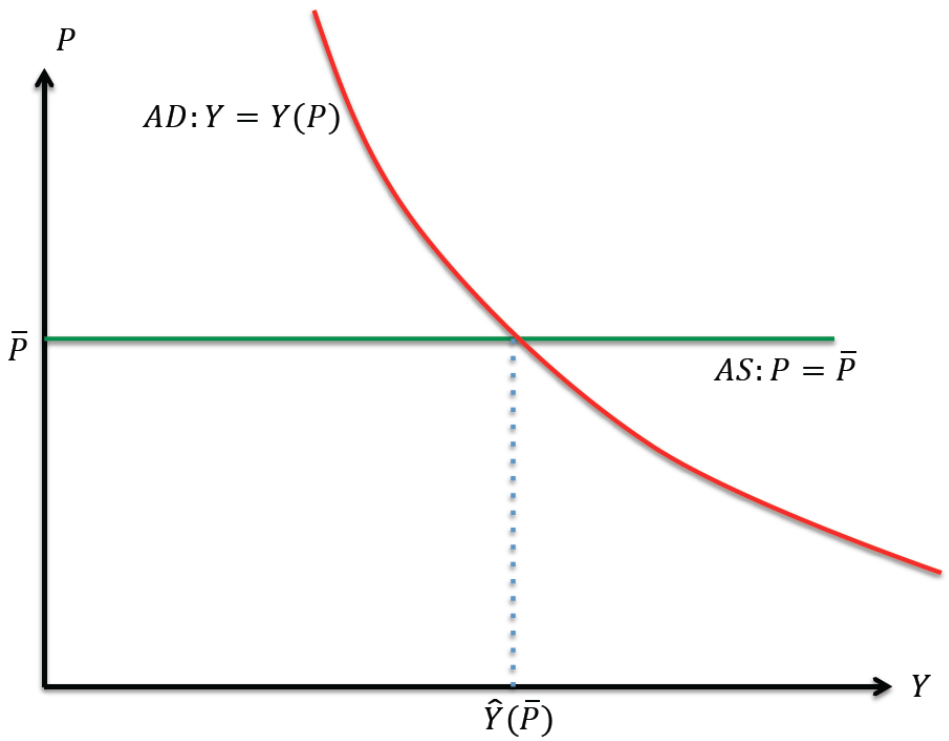
- The sticky price scenario is often justified by the assumption that there is **imperfect competition** in the final goods market; firms can set their own prices. Typically facing a constant nominal wage cost they set a price which is a mark up (λ) over the nominal cost such that $P = (1 + \lambda)W$.
- But often firms do not adjust the price level immediately in response to an increase in W.
- This could be because of a variety of reasons:
  - There could be adjustment costs associated with price change (menu cost) due to which it may not be optimal for the firms to change their prices immediately;
  - The firms might have limited bargaining power vis-a-vis the unions so that the union is able to extract a higher real wage (in which case λ adjusts keeping P unchanged).

The Neo-Keynesians & Sticky Prices: (Contd.)

- In what follows, we shall assume that $\bar{P}$ does not respond to a change in the wage rate for completely **exogenous reasons**, while keeping the earlier firm-side story in the background.
- It is as if the government (or some exogenous regulatory authority) has fixed the price level at some $\bar{P}$ and firms have no choice but to operate within that regulatory framework).

The Neo-Keynesians & Sticky Prices: (Contd.)

- In the sticky price scenario, the aggregate supply curve is horizontal.
- The equilibrium output is now completely determined by the position of the demand curve - in particular by the level of aggregate demand at the price level $P = \bar{P}$.

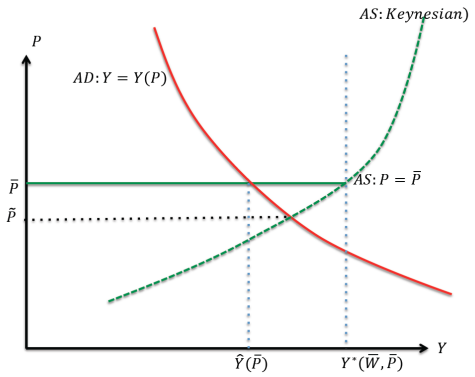


The Neo-Keynesians & Sticky Prices: (Contd.)

- The crucial question is: At what level would the price be set?
- Notice that we can plot the Keynesian (upward-sloping) AS schedule in the backdrop and identify two regions with reference to the Keynesian equilibrium price level ($\tilde{P}$):
  - The region above $\tilde{P}$ (above the intersection point of the Keynesian AS & AD schedules) which is **demand-constrained**;
  - The region below $\tilde{P}$ (below the intersection point of the Keynesian AS & AD schedules) which is **supply-constrained**.
- Can we ever have a Neo-Keynesian scenario where the $\bar{P}$ is set below $\tilde{P}$?

The Neo-Keynesians & Sticky Prices: (Contd.)

- Well, we can...but in that case the AS supply curve will **no longer be infinitely elastic**. In fact it will be vertical for any $\bar{P} < \tilde{P}$.
- Why? The reason is as follows:
  - Notice that for any arbitrary $(\bar{P}, \bar{W})$, the profit-maximizing level of output $Y^*(\bar{P}, \bar{W})$ actually coincides with a point on the AS curve under the Keynesian system:



  - This being the profit-maximization point (given $\bar{P}, \bar{W}$), firms would actually like to produce upto this level - *if they could*!
  - But they cannot do so **if the economy is demand constrained (since at this price level, the aggregate demand falls short of the profit maximising level of output).**

The Neo-Keynesians & Sticky Prices: (Contd.)

- But this argument does not hold if at this price level, the economy is actually supply constrained!
- When the economy is supply-constrained, the producers can choose output level that maximizes their profit (given $\bar{P}, \bar{W}$).
- That is, they can actually pick a point on the Keynesian AS curve (given $\bar{P}, \bar{W}$).
- Indeed facing a wage-price combination of $(\bar{P}, \bar{W})$, **a firm would never have any incentive to produce beyond** $Y^*(\bar{P}, \bar{W})$.
- This implies that we have to draw the Neo-Keynesian AS schedule (with sticky prices and sticky wages) a little differently that we did before:
    - At the given $\bar{P}$, It is no longer horizontal for all Y; it becomes vertical precisely at the point $Y^*(\bar{P}, \bar{W})$.

Labour Market under Sticky Prices:

- Let us now assume that $\bar{P}$ is such that the economy is indeed demand-constrained.
- We have seen that in this case, stickiness of the price level implies quantity adjustment by the firms: they produce exactly as much output as is demanded.
- This quantity adjustment will have implications for the labour demand function as well.
- When prices are sticky and the economy is demand-constrained, the labour demand function is given by:

$$N^D = \hat{N}(\bar{P}) : F(\hat{N}, \bar{K}) = \hat{Y}(\bar{P}).$$

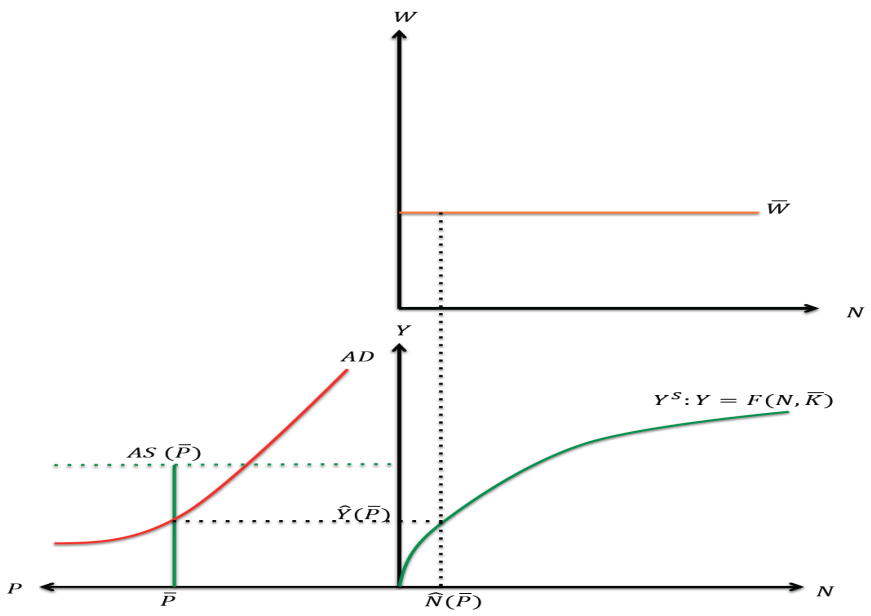
- Since the labour supply is horizontal at $\bar{W}$, $\hat{N}(\bar{P})$ also represents the equilibrium employment level under the Neo-Keynesian system.

The Neo-Keynesians & Sticky Prices: (Contd.)

- It is important to recognise here that due to the stickiness of the price that is set at a region where there is excess supply, the producers are **not** operating on their AS schedule.
- This implies that they are not supplying their profit-maximising level of output, which in turn means they are unable to operate on their optimal labour demand curve (which presupposes profit maximising behaviour). Thus the labour demand schedule now becomes irrelevant in determining equilibrium level of employment.

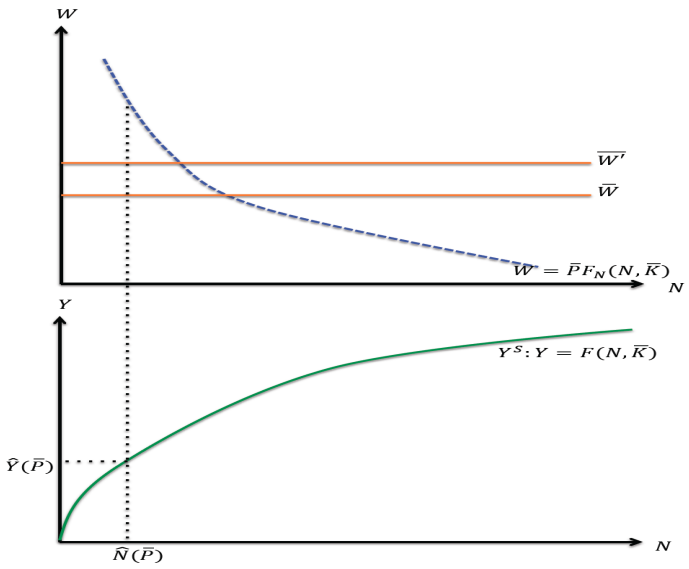
The Neo-Keynesians & Sticky Prices: (Contd.)

- The following diagram characterizes the equilibrium in the Neo-Keynesian system:



The Neo-Keynesians & Sticky Prices: (Contd.)

- Now let's see what happens when the nominal wage rate when $\bar{W}$ goes up:



- Since $\bar{P}$ does not respond to a change in $\bar{W}$ (by assumption), nothing changes in the Goods Market. The equilibrium output is still given by $\hat{Y}(\bar{P})$. Hence so is equilibrium employment: $\hat{N}(\bar{P})$.
- In fact nothing changes except that now the real wage is higher which implies that there is a re-distribution of income from profit-earners to wage-earners.

The Neo-Keynesians & Sticky Prices: (Contd.)

- Now consider a positive demand shock: suppose for some reason the aggregate demand schedule shifts to the right. This could policy-induced (e.g., a change in $\bar{G}$ or $\bar{M}$) or could be due to an independent shift in parameters. However let us assume that the economy is still demand-constrained.
- Notice that with sticky prices and rigid nominal wages, real wage rate remains that same, while output and employment goes up in the new equilibrium.
- So even though real wage now is not exactly pro-cyclical, at least it does not move in the opposite direction to output and employment! (In fact the total wage bill would actually go up.)

The Neo-Keynesians & Sticky Prices: (Contd.)

- There are other variants of the Neo-Keynesian framework that assume that union's bargaining power increases as level of employment goes up.
- In such a case, a positive aggregate demand shock (with sticky prices) would increase the real wage rate as well. (Aggreagte profit would also rise, although the mark up/profit margin would go down).
- Thus real wage and output (as well as employment) would now move in the same direction, making the real wage rate **pro-cyclical**- as is consistent with the empirical evidence.
- We shall come back to this case when we discuss the precise micro foundations of the Neo-Keynesian model.

Other Extensions of Keynes & the Classics:

- **Quantity Theory of Money** (A Special Case of the Classical System):
  - Money Demand Equation now becomes:

$$M = PkY;$$

    where k is a positive constant (related to the velocity of circulation of money)
  - Notice that this still generates a downward sloping AD schedule (**why**?), while the AS schedule remains vertical as before.
  - Nothing much changes in this special case of the Classical System in terms of equilibrium output/employment.
- **Question**: What is the role of the IS curve here?

Other Extensions of Keynes & the Classics: (Contd.)

- **Liquidity Trap/Interest Rate Targeting** (A Special Case of the Keynesian System)
  - Equation of the LM curve now becomes:

$$r = \bar{r}$$

    (At this interest rate money supply is perfectly elastic.)
  - The aggregate demand schedule in the Y-P plane is now vertical.
  - In this special Keynesian System, output is completely demand determined. Since the level of demand is now independent of the price level, there is only quantity adjustment - even though prices are fully flexible.
- **Question:** What happens if we import this assumption of liquidity trap/interest rate targeting to an otherwise Classical System?

Other Extensions of Keynes & the Classics: (Contd.)

- **Autonomous Investment** (Another Special Case of the Keynesian System):
  - Equation of the IS curve now becomes:

$$Y = C(Y) + \bar{I} + \bar{G}$$

  - The aggregate demand schedule in the Y-P plane is once again vertical.
  - In this special Keynesian System once again output is completely demand-determined. Again, there is only quantity adjustment - even though prices are fully flexible.
- **Question:** What happens if we import this assumption of autonomous investment to an otherwise Classical System?

Keynes & the Classics: What have we learnt so far?

- We have now seen different variants of the Keynesian and the Classical System.
- No matter which specific set of assumptions one takes, the starkest difference between these two categories of models (in terms of characterization of the equilibrium) is as follows:
  - In the Keynesian system, demand plays a crucial role in determining the equilibrium output.
  - In the Classical model, demand plays no role in determining the equilibrium output; it is completely supply driven.
- To the extent that government policies affect the demand side (and only the demand side) of the economy, such policies would work in the Keynesian system but would not work in the Classical System.
  - Notice however that any policy that affects the supply side of the economy will work in the Classical system as well as in the Keynesian system (but may not work in the two special cases of the Keynesian system).

Keynes & the Classics: Incomplete Information

- So far we have assumed that workers (households) as well as firms have complete information about the prices and wages that would prevail in the actual economy.
- In fact, the classical system that we have discussed represents the ideal scenario - there is no market imperfection or rigidity in any market, nor is there any incomplete information.
- We could treat this as our benchmark case - the best possible scenario (what would have happened is everything was perfect!)
- This ideal scenario - the benchmark case - is also somewhat unrealistic. The real world is characterized by various kinds of market imperfections or rigidities as well as incomplete information.
- We have already seen what happens if there are various kinds of rigidities.
- But even in the absence of market imperfections, things could be far from perfect simply because agents have incomplete information.
- We now turn to one such case.

Incomplete Information and Role of Expectations:

- Let us consider a modified version of the classical system, where the firms have full information about the wages and prices, but workers do not have complete information.
- In particular, let us assume that workers do not have complete information about the price level.(This is the Lucas model of incomplete information.)
  - The underlying logic is that since prices are determined in the goods market while nominal wages are set in the labour market, workers often do not have perfect knowledge about the price behaviour.
- If the workers do not have perfect knowledge about the price level that prevails, then they would make their calculations on the basis of their expectations about the price level.
- In other words, in this model with incomplete information, workers determine their labour supply on the basis of the **'expected' real wage**.

Incomplete Information and Role of Expectations: (Contd.)

- Notice that till now we have deliberately kept expectations out of the picture.
- But the moment we bring in incomplete information, expectations start playing a major role.
- In this modified Classical System, workers' labour supply schedule now depends on the expected real wage rate.
- If we incorporate this in our standard Classical system, then the AS schedule under the classical system may change its character - as we will see in a moment.

AS Schedule in the Classical System when Labour Supply depends on Expected Real Wage:

- When the workers determine their labour supply on the basis of the **'expected' real wage**, the labour supply equation is given by:

$$N^S : W = P^e g(N);\ g' > 0$$

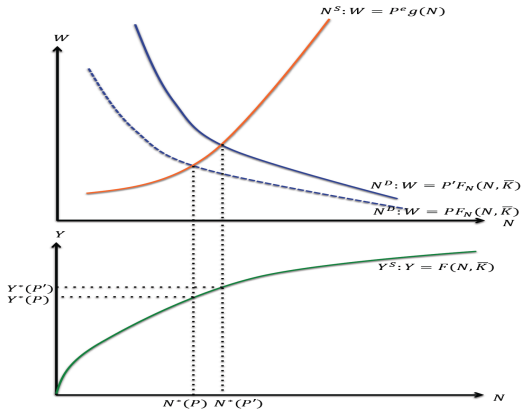
- The labour demand equation remains unchanged (because producers' are assumed to have complete information about the price of the product that they themselves would be selling):

$$W = PF_N(N, \bar{K})$$

- The labour market equilibrium now depends crucially on how price expectations are formed.
- If workers can perfectly anticipate the actual price level, then $P^e = P$ and we are back to the good old Classical world with a vertical AS curve.

AS Schedule in the Classical System when Labour Supply depends on Expected Real Wage (Contd.):

- If, on the other hand, P^e gets determined quite independent of the current price level (e.g., by past prices), then AS completely changes its character.



- The AS schedule is now upward sloping - just as it was in standard the Keynesian system!
- Thus standard fiscal and monetary policies would be effective in this 'modified' Classical system - although there is no wage or price rigidity!

How are Expected Prices Determined?

- It seems a little unrealistic to assume that the agent's expectations would be completely independent of the actual value of the variable.
- But to see exactly how they are related we have to look for some theories of expectation formation, which we shall discuss now.

Various Theories of Expectation Formation:

- **Static Expectations**:
  Today's expected value of the variable (x) depends on previous period's actual value. In particular:

$$x_t^e = x_{t-1}$$

- **Adaptive Expectations**:
  Today's expected value of the variable (x) depends on previous period's actual value and previous period's expected value. In particular:

$$x_t^e = x_{t-1}^e + \lambda \left[x_{t-1} - x_{t-1}^e \right]; \quad 0 < \lambda < 1$$

- Notice that Static Expectations is a special case of Adaptive Expectations (when $\lambda = 1$)

Various Theories of Expectation Formation (Contd.):

- **Perfect Foresight**:
  Agent's make a **guess** about the value of the variable and (by some devine power) the guess exactly matches its actual value. In particular:
  $$x_t^e = x_t$$
  Notice that guessing is **NOT** knowing!
- **Rational Expectations**:
  Agent applies mathematical tools of expectation formation, using the available information set, to come up with the expected value of the variable. In particular:
  $$x_t^e = E[x_t | I_{t-1}]$$
- Notice that under complete information and complete certainty, Perfect Foresight and Rational Expectations are equivalent.

Various Theories of Expectation Formation: (Contd.)

- We have now specified 4 different expectation formation rules.
- There are more sophisticated rules of expectation formation that explicitly incorporate a learning process (e.g, Bayesian Inference). However in this course we shall limit ourselves to the 4 simple rules specified above.
- We shall now apply these rule one by one to a particular macroeconomic system.
- A natural choice is Lucas' Incomplete Information model which brings in a role of expectations in the labour market.

Various Expectation Formation Rules: An Application (Classical Labour Market)

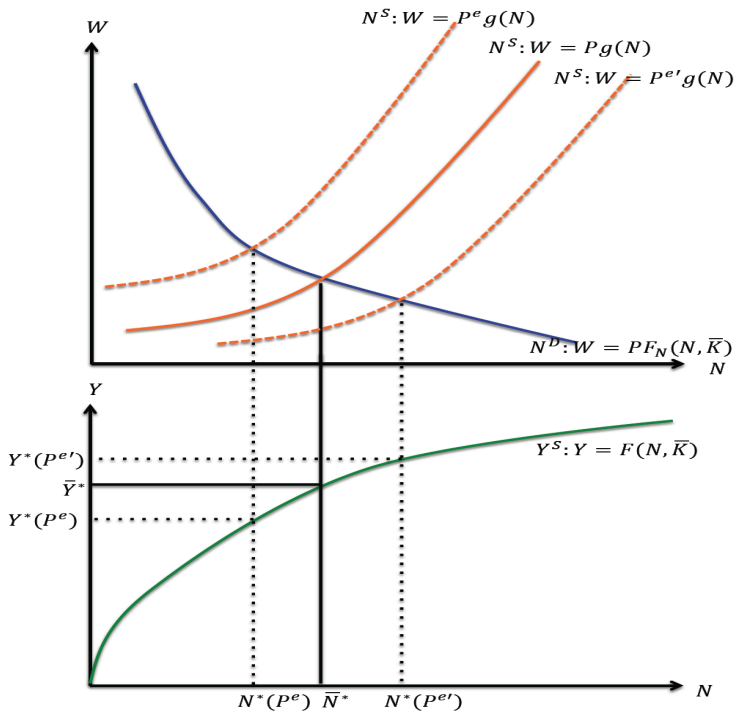
- Recall that in the Classical system, when both workers and producers base their supply/demand decisions on the actual real wage $\left(\frac{W_t}{P_t}\right)$, then the labour market equilibrium is given by:

$$\bar{N}^* : g(N) = F_N(N, \bar{K})$$

This $\bar{N}^*$ - which we shall call the **'Natural Level of Employment'** - is independent of the current price level (P_t).

An Application of Various Expectation Formation Rules (Contd.):

- On the other hand, when workers base their supply decisions on the expected real wage $\left(\frac{W_t}{P_t^e}\right)$, then the actual level of employment differs from the **natural rate** ($\bar{N}^*$) in the following way:



An Application of Various Expectation Formation Rules (Contd.):

- In other words,

$$N_t \gtreqless N^* \text{ according as } P_t^e \lesseqgtr P_t.$$

- Define $\bar{Y}^*$ as the **'natural level of output'** such that

$$\bar{Y}^* = F(\bar{N}^*, \bar{K}).$$

- When workers base their supply decisions on the expected real wage, then the actual output supplied differs from the **natural level** ($\bar{Y}^*$) in the following way:

$$Y_t^s \gtreqless \bar{Y}^* \text{ according as } P_t^e \lesseqgtr P_t.$$

- This allows us to write the Aggregate Supply schedule in the following way:

$$Y_t^s : Y_t = \bar{Y}^* + \hat{f}(P_t - P_t^e); \quad \hat{f}(0) = 0; \hat{f}' > 0.$$

An Application of Various Expectation Formation Rules (Contd.):

- This representation of the aggregate supply schedule is called the **Lucas Supply Function** (after Robert Lucas, who postulated that (in the short run) workers' may not have complete information about the price behaviour and hence their expectations may differ from the actual.)
- Without any loss of generality, let us assume that the Lucas Supply Function (i.e., the AS schedule under incomplete information) is linear:

$$Y_t^s : Y_t = \bar{Y}^* + \alpha \left[P_t - P_t^e \right]; \quad \alpha > 0. \tag{I}$$

- The equilibrium price level will of course depend on aggregate demand function, which we now turn to.

An Application of Various Expectation Formation Rules (Contd.):

- we know that the Aggregate Demand schedule is a decreasing function of the price level:

$$Y_t^d : Y_t = h(P_t);\ h' < 0$$

- We also know that aggregate demand increases corresponding to any increase in the the policy parameters $\bar{G}$, $\bar{M}$. Thus

$$Y_t^d : Y_t = h(P_t; \bar{G}; \bar{M});\ h' < 0;\ \frac{\partial Y}{\partial \bar{G}} > 0; \frac{\partial Y}{\partial \bar{M}} > 0$$

- Without any loss of generality, let us again assume that the AD schedule is linear:

$$Y_t^d : Y_t = -\mu P_t + \gamma \bar{G} + \mu \bar{M};\ \gamma, \mu > 0 \qquad \text{(II)}$$

- **Question:** In the AD schedule written above, why have we attributed same coefficient (μ) to both P_t and $\bar{M}$?

An Application of Various Expectation Formation Rules (Contd.):

- From the AS and the AD schedule (given by (I) and (II) respectively, we can solve for the equilibrium price level at time t as:

$$\begin{aligned} P_t \quad &: \quad \bar{Y}^* + \alpha\left[P_t - P_t^e\right] = -\mu P_t + \gamma\bar{G} + \mu\bar{M} \\ &\Rightarrow \quad P_t = \frac{1}{\alpha+\mu}\left[\gamma\bar{G} + \mu\bar{M} - \bar{Y}^* + \alpha P_t^e\right] \qquad \text{(III)} \end{aligned}$$

- Equation (III) gives us the precise relationship between actual price level and expected price level in this economy at every point of time.
- Let us now apply the different theories of expectation formation to equation (III) and see how the behaviour of the aggregate economy changes (if at all) over time.

An Application of Various Expectation Formation Rules (Contd.):

- We know that the equilibrium price level at time t is determined by the following equation:

$$P_t = \frac{1}{\alpha + \mu}\left[\gamma \bar{G} + \mu \bar{M} - \bar{Y}^* + \alpha P_t^e\right] \tag{III}$$

- **Under Static Expectations:**
  - $P_t^e = P_{t-1}$
  - Plugging this value of P_t^e in equation (III) we get a single difference equation in P_t, which will determine the movement of equilibrium price level over time:

$$P_t = \frac{\alpha}{\alpha + \mu} P_{t-1} + \frac{1}{\alpha + \mu}\left[\gamma \bar{G} + \mu \bar{M} - \bar{Y}^*\right]$$

An Application of Various Expectation Formation Rules (Contd.):

- **Under Adaptive Expectations:**
  - $P_t^e = P_{t-1}^e + \lambda \left[P_{t-1} - P_{t-1}^e \right]$
  - Plugging this value of P_t^e in equation (III) we get a system of two difference equations in two variables, P_t and P_t^e, which will simultaneously determine the movement of equilibrium price level as well as expected price level over time:

$$
\begin{aligned}
P_t &= \frac{\alpha\lambda}{\alpha+\mu} P_{t-1} + \frac{\alpha(1-\lambda)}{\alpha+\mu} P_{t-1}^e + \frac{1}{\alpha+\mu}\left[\gamma\bar{G} + \mu\bar{M} - \bar{Y}^*\right] && (1) \\
P_t^e &= \lambda P_{t-1} + (1-\lambda) P_{t-1}^e && (2)
\end{aligned}
$$

An Application of Various Expectation Formation Rules (Contd.):

- **Under Perfect Foresight:**
  - $P_t^e = P_t$
  - Plugging this value of P_t^e in equation (III) we get a unique solution for the equilibrium price (P_t), which must be the 'perfect foresight' solution to the system:

$$\left(1 - \frac{\alpha}{\alpha + \mu}\right) P_t = \frac{1}{\alpha + \mu}\left[\gamma \bar{G} + \mu \bar{M} - \bar{Y}^*\right]$$

$$\Rightarrow \quad P_t = \frac{1}{\mu}\left[\gamma \bar{G} + \mu \bar{M} - \bar{Y}^*\right] = P_t^e$$

An Application of Various Expectation Formation Rules (Contd.):

- **Under Rational Expectations:**
  - $P_t^e = E[\, P_t | \, I_{t-1}]$
  - Notice that under perfect certainty, the information set, I_{t-1}, would include the information that the equilibrium price level in every period is determined by:

$$P_t = \frac{1}{\alpha + \mu} \left[\gamma \bar{G} + \mu \bar{M} - \bar{Y}^* + \alpha P_t^e \right]$$

  - Hence when agents form their expectations, they will utilize this information. In other words:

$$\begin{aligned} E(P_t) &= E\left[\frac{1}{\alpha + \mu} \left[\gamma \bar{G} + \mu \bar{M} - \bar{Y}^* + \alpha E(P_t) \right] \right] \\ &= \frac{1}{\alpha + \mu} \left[\gamma \bar{G} + \mu \bar{M} - \bar{Y}^* \right] + \frac{\alpha}{\alpha + \mu} E(P_t) \\ \Rightarrow \quad & E(P_t) = \frac{1}{\mu} \left[\gamma \bar{G} + \mu \bar{M} - \bar{Y}^* \right] \end{aligned}$$

An Application of Various Expectation Formation Rules (Contd.):

- Notice that once we replace this value of $E(P_t)$ in the equilibrium price determination equation (III), we get

$$P_t = \frac{1}{\mu}\left[\gamma\bar{G} + \mu\bar{M} - \bar{Y}^*\right] = E(P_t)$$

- **Point to note**: The rational expectation solution and the perfect foresight solution are identical, although the underlying mechanisms of arriving at the two solutions are different!

Difference between Perfect Foresight and Rational Expectation Solutions Under Uncertainty:

- Let us now introduce some uncertainty in the system such that the price determination equation is given by:

$$P_t = \frac{1}{\alpha + \mu}\left[\gamma \bar{G} + \mu \bar{M} - \bar{Y}^* + \alpha P_t^e\right] + \epsilon_t \qquad \text{(IIIa)}$$

  where ϵ_t is a random variable with an expected value of $\bar{\epsilon}$.
- In this case the perfect foresight solution is given by:

$$P_t = \frac{1}{\mu}\left[\gamma \bar{G} + \mu \bar{M} - \bar{Y}^*\right] + \frac{\alpha + \mu}{\mu}\epsilon_t = P_t^e.$$

- On the other hand the Rational Expectation solutions for expected price and actual price are given by:

$$\begin{aligned} E(P_t) &= \frac{1}{\mu}\left[\gamma \bar{G} + \mu \bar{M} - \bar{Y}^*\right] + \frac{\alpha + \mu}{\mu}\bar{\epsilon} \\ P_t &= \frac{1}{\mu}\left[\gamma \bar{G} + \mu \bar{M} - \bar{Y}^*\right] + \frac{\alpha + \mu}{\mu}\epsilon_t \end{aligned}$$

Perfect Foresight and Rational Expectation Solutions Under Uncertainty (Contd.):

- Notice that since the realized value of the random term ϵ_t may not be equal to its expected value $\bar{\epsilon}$,
  - the solution for expected price level under perfect foresight and under rational expectation now differ.
  - In fact, under perfect foresight, the expected price level still coincides with its actual value.
  - However, under rational expectations, the expected price level now differs from the actual price level due to the existence of a random surprise term $(\epsilon_t - \bar{\epsilon})$.

Existence of Multiple Perfect Foresight/Rational Expectation Solutions:

- **Food for Thought:**
  - In this model (because the functional forms are assumed to be linear) we end up with 'unique' perfect foresight/rational expectation solutions.
  - It is conceivable that these equations are not linear. Then there may exist multiple solutions to the same equation.
  - In such a scenario, would agents' expectations be necessarily met even under the assumption of perfect foresight/rational expectation (with complete certainty and perfect information)?
  - The answer is "no" - **unless all agents coordinate** and everybody picks the same value among these multiple solutions! This in fact is one of the problematic areas of the perfect foresight/rational expectation hypothesis, which we shall come back to later in the course (in module 3).

Keynes & the Classics: A Critique

- As we have mentioned at the beginning of the class, these static (one period) macro models have two major short-comings:
  - They are all based on 'ad-hoc' assumptions. Prima facie it is not obvious that all these assumptions can be substantiated by explicit optimizing behaviour of agents;
  - They ignore all intertemporal (dynamic) issues, even when we know that savings, investment etc. will necessarily have implications for future and therefore completely ignoring the future in current decision making process does not seem right.
- In the next few sections, we shall try to address each of these criticisms one by one.

References

- **A word of caution:** I do not follow any particular textbook *ad verbatim*. Thus the refereneces are only suggestive; they are meant to be read as a supplementary reading and not as a substitute for the lecture notes.
- The book that I have followed more closely than any other:
  - **William Scarth: "Macroeconomics: The Development of Modern Methods for Policy Analysis", Edward Elgar, 2014, Chapter 1.**
    (Scanned copy of the chapter is available in the course web page)
- For a broad overview of the current topic, you may also consult the following book:
  - **Wendy Carlin & David Soskice: "Macroeconomics: Imperfections, Institutions & Policies", Oxford University Press, 2006, Chapter 2.**
    (Scanned copy of the chapter is available in the course web page)