Unilateral Externality: Two Dimensional Choice

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Lecture 1

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Externality: Leading Types

Technical/Technological Externality

- interdependence of payoff functions
- interdependence is technical - choice of one agent affects the payoff function(s) of other(s)
- interdependence is direct - choice of one agent directly affects the payoff function(s) of other(s)
- interdependence generally affects ‘productivity’

Pecuniary Externality

- interdependence of payoff functions
- interdependence is generally indirect
- is/can be channelized through a market relation/transaction

Two dimensional choice: A simple illustration I

Assume

- There are two economic agents - say, Agent/Firm 1 and Agent/Firm 2
- Production by Agent/Firm 1 'causes' externality for Agent/Firm 2
- Agent/Firm 1 makes two choices. E.g., a polluting plant chooses
  - the 'scale' of operation/activity, s
  - the care level x exercised during production

Let

- $\phi^1(s, x) = \phi^1(s) - sx$ be the payoff/profit function for Agent/Firm 1.
- $\phi^2(.)$ be the payoff/profit function for Agent/Firm 2.

$$\phi^2(.) = \bar{\phi}^2 - l(s, x),$$

$\bar{\phi}^2$ denote the maximum payoff (utility/profit) Agent 2 can achieve in absence of externality caused by Firm 1. $\bar{\phi}^2 > 0$.

Two dimensional choice: A simple illustration II

Assume

- Activity s is good for firm 1 but bad for firm 2.
- Care x is good for firm 2 but bad for firm 1.

$$\phi^1_s(s,x) > 0, \text{ for } s < \bar{s}(x), \text{ and } \phi^1_s(s,x) < 0, \text{ for} s > \bar{s}(x),$$

$$\phi^1_{ss}(s,x) < 0$$

$$l_s(s,x) > 0, \; l_{ss}(s,x) \geq 0,$$

i.e., $\phi^2_s(s,x) < 0, \; \phi^2_{ss}(s,x) < 0$. Further,

$$\phi^1_x(s,x) < 0, \; \phi^1_{xx}(s,x) \leq 0, \text{ and}$$

$$l_x(s,x) < 0, \; l_{xx}(s,x) > 0,$$

i.e., $\phi^2_x(s,x) > 0, \; \phi^2_{xx}(s,x) < 0$.

Laissez Faire

Firm/Agent 1 will solve

$$\max_{s,x}\{\phi^1(s,x)\}.$$

Since $\phi^1_x(s,x) < 0$, it will choose

$$x^*_p = 0$$

will opt s that solves the following FOCs:

$$\phi^1_s(s,0) = 0 \quad (1)$$

Let $s^*_p(0) > 0$ be the solution. That is, $\phi^1_s(s^*_p(0)) = 0$.

First-Best I

The First-Best (total profit maximization) problem is

$$\max_{s,x}\{\phi^1(s,x)+\phi^2(s,x)=\phi^1(s,x)+\bar{\phi}^2-I(s,x)\} \quad (2)$$

For this OP, the FOCs is:

$$\begin{aligned}\phi^1_s(s,x)+\phi^2_s(s,x) &= 0\\ \phi^1_x(s,x)+\phi^2_x(s,x) &= 0, i.e.,\end{aligned}$$

$$\begin{aligned}\phi^1_s(s,x)-I_s(s,x) &= 0 \quad (3)\\ \phi^1_x(s,x)-I_x(s,x) &= 0, i.e., \quad (4)\end{aligned}$$

Let s^* and x^* solve () and (), simultaneously Assume $(s^*,x^*)>(0,0)$. Therefore, we get

$$\begin{aligned}\phi^1_s(s^*,x^*)-I_s(s^*,x^*) &= 0\\ \phi^1_x(s^*,x^*)-I_x(s^*,x^*) &= 0, i.e.,\end{aligned}$$

First-Best II

Clearly, $x_p^* < x^*$.

Question

How does $s_p^*(0)$ compare with s^*? Under what conditions, we will get $s_p^*(0) > s^*$?

Simple Functional Form: Suppose,

$$\phi^1(s,x) = \phi^1(s) - sx$$

where $\phi^1_s(s) > 0$ and $\phi^1_{ss}(s) < 0$. Note: for this form,

$$\phi^1_x(s,x) < 0 \text{ and } \phi^1_{xx}(s,x) \leq 0.$$

Now, the First Best, (s^*, x^*) will solve the following FOCs

$$\begin{aligned} \phi^1_s(s) - x - l_s(s,x) &= 0 \\ -s - l_x(s,x) &= 0, \textit{i.e.}, \end{aligned}$$

First-Best III

That is,

$$\phi_s^1(s^*) = x^* + l_s(s^*, x^*) \tag{5}$$

$$-l_x(s^*, x^*) = s^*. \tag{6}$$

Also, for this functional form (1) reduces to:

$$\phi_s^1(s) = 0 \tag{7}$$

Since $\phi^1(s)$ is concave, from a comparing (5) with (7), you can see that $s_p^*(0) > s^*$ holds.

Corrective Measure: Quantity Regulation I

Suppose,

- The regulator sets standards for the externality generators
- The regulatory standard $(s^R, x^R) = (s^*, x^*)$. That is,
- Firm is allowed to produce up to s^* and required to choose care x^*
- Sever penalty for production beyond s^* or care below x^*

- In equi, Firm 1 will choose (s^*, x^*)
- The outcome is Kaldor-Hicks efficient.

Ex-post Liability

Recall

- $\bar{\phi}_2$ is the profit in the absence of externality, i.e., $\bar{\phi}_2 = \phi^2(s = 0)$

Suppose,

- The externality creator is required to compensate the 'victim' of externality
- Firm 1 pays a compensation equal to loss; i.e., equal $l(s, x) = \bar{\phi}_2 - \phi^2(s, x)$.

Now, in equilibrium, 1 will choose s and x that solves:

$$\max_{s,x}\{\phi^1(s, x) - l(s, x)\} \tag{8}$$

$$\begin{aligned} \phi^1_s(s, x) - l_s(s, x) &= 0 \quad (9) \\ \phi^1_x(s, x) - l_x(s, x) &= 0, i.e., \quad (10) \end{aligned}$$

equilibrium choice is $s = s^*$ and $x = x^*$.

Per unit Tax

Suppose, there is no care standard fixed for Firm 1. Further,

- activity level s is observable to the govt
- the per-unit (of activity) tax rate is $\bar{t}$

Now, Firm 1 will choose x^T and s^T that solve

$$\max_{s,x}\{\phi^1(s,x) - sx - s\bar{t}\}, i.e., \tag{11}$$

Clearly $x^T = 0$. However, s^T will solve

$$\begin{aligned} \phi^1_s(s) - \bar{t} &= 0 \\ \phi^1_s(s^T) &= \bar{t}. \end{aligned} \tag{12}$$

A comparison of (5) with (12) shows that depending on the level $\bar{t}$ we can have: Clearly

$$s^T >< s^*$$

Un-observable Activity Level: Regulation

Suppose,

- care level x is observable to the regulator
- activity level s is NOT observable to the regulator
- the regulator requires care level x^* - if Firm 1 invests x^*, it has no further obligations

Suppose,

- Firm 1 will adopt care standard x^*

Now, Firm will choose s^R to solve

$$\max_s \{\phi^1(s) - sx^*\}, i.e., \tag{13}$$

will chose s^R that solves

$$\phi^1_s(s) - x^* = 0, i.e., \phi^1_s(s^R) = x^* \tag{14}$$

From (5) and (14) we see that

$$s^R > s^*$$

Un-observable Activity: Liability

Suppose,

- care level x is observable to the court
- activity level s is NOT observable to the court
- the court requires care level x^* - at this care level, Firm 1 has no liability

Suppose,

- Firm 1 will adopt care level x^*

Now, Firm will choose s^L to solve

$$\max_s \{\phi^1(s) - sx^*\}, i.e., \tag{15}$$

will chose s^L that solves

$$\phi^1_s(s) - x^* = 0, i.e., \phi^1_s(s^L) = x^* \tag{16}$$

From (5) and (16) we see that

$$s^L > s^*$$