UNIVERSITY OF DELHI
M.A. Economics: Winter Semester 2018

Course 004: Macroeconomic Theory

Problem Set 2:
From Ad Hoc Micro Foundations to Dynamic General Equilibrium

1. Consider a representative agent who lives exactly for two periods. He works in the first period of his life and is retired in the second period of his life. The agent has one unit of time endowment in the first period - which he distibutes between work and leisure. Thus he earns a wage income on the time spent working at current market wage rate w_t (per unit of time). He has no other source of income in the first period of his life.

At the end of the first period, he consumes a part of his wage income and saves the rest. In the second period of his life, he earns an interest income on the savings made in the previous period (at the prevalent market interest rate r_{t+1}) . This constitute his sole second period earnings, which he consumes entirely at the end of the second period.

The utility function of the agent is given by:

$$\underset{\{C_{1h},C_{2h},\hat{L}_h\}}{Max.}(C_{1h})^{\alpha}(C_{2h})^{\beta}(\hat{L}_h)^{\gamma};\ \alpha+\beta+\gamma=1.$$

(i) Write down the first and second period budget constraint of the agent in nominal terms.

(ii) Solve the optimization problem of the agent to derive his optimal consumption level in both the periods as well as his optimal labour supply.

(iii) Specify the precise assumption(s) that will generate

(a) a Keynesian consumption function (where current consumption is only a function of current income and parameters);

(b) a labour supply function which is upward sloping in the real wage rate.

2. Consider an economy populated by S identical firms and H identical households.

The firm side story is as follows. Each firm (indexed by i) has access to a production function technology, given by:

$$Y_i = AN_i + B\bar{K}_i;$$

where A, B are positive constants, and $\bar{K}_i$ is the short run supply of capital which is given. The firm takes the market price (P) and market wage rate (W) as given and employes labour so as to maximise its profit.

(i) Let the nominal wage rate in this economy be constant at $\bar{W}$. Let us assume that each firm also has a capacity constraint such that it can employ maximum $\bar{N}$ amount of labour. Derive the labour demand schedule for the $i-$th firm as a function of the real wage rate $\left(\frac{\bar{W}}{P}\right)$. Diagrammatically depict the corresponding aggregate labour demand schedule for the entire economy with real wage rate $\left(\frac{\bar{W}}{P}\right)$ on the vertical axis.

Let us now turn to the household side story. Each household (indexed by h) consists of a single member. The representative agent belonging to household h lives exactly for two periods. He has a total endowment of 1 unit of time in the first period of his life, of which he works for $(1-\hat{L}_h)$ units of time and enjoys the rest as leisure. Working generates some wage income in the current period. Apart from the wage income the agent also has some non-wage income in the current period (defined in nominal terms), denoted by R. Out of his **total first period income**, he consumes some (C_{1h}) in the first period, and saves (S_h) the rest. Savings generates an (expected) interest income $((1+\tilde{r})S_h)$ in the second period - which he consumes entirely in that period. Apart from this interest income, the agent has no other sources of income in the second period of his life. The agent cares for his current consumption (C_{1h}), his future consumption (C_{2h}) and the amount of leisure enjoyed in the first period $(\hat{L}_h)$. Let the utility function of household h be:

$$\underset{\{C_{1h},C_{1h},\hat{L}_h\}}{Max.}(C_{1h})^{1/2}+(C_{2h})^{1/2}+(\hat{L}_h)^{1/2}.$$

(ii) Write down the life-time budget constraint of the agent in terms of current nominal wage rate $(\bar{W})$, nominal non-wage income in the curent period (R), current price level (P), expected future price level $(\tilde{P})$ and expected future interest rate $(\tilde{r})$.

(iii) Suppose households have **static expectations** such that $\tilde{P}=P$ and $\tilde{r}=r$. Now derive the optimal consumption and leisure choices of the household in terms of the current real wage rate $\left(\frac{\bar{W}}{P}\right)$, current non-wage income in real terms $\left(\frac{R}{P}\right)$ and current interest rate (r).

(iv) Assuming that $(2+r)\frac{\bar{W}}{P}>1$, derive the corresponding aggregate labour supply function for the entire economy. For any given value of r (and given $\frac{R}{P}$), diagrammatically depict the labour supply schedule with real wage rate $\left(\frac{\bar{W}}{P}\right)$ on the vertical axis. Suppose $\bar{N}>H$. From the aggregate labour demand and labour supply schedule, charaterise the labour market equilibrium. How does this equilibrium change when there is a change in r?

(v) Combine the labour market equilibrium with the aggregate production function to draw the AS schedule (total output supplied) as a function of r (i.e., in the (Y,r) plane).

(vi) Let us now look at the optimal solution for current consumption by household h. Derive the corresponding aggregate consumption function as a function of aggregate current income $Y\equiv\left[\left(\frac{\bar{W}}{P}\right)(1-\hat{L}_h)H+\frac{HR}{P}\right]$. Show that unlike the Keysenian consumption function, aggregate consumption now depends not only on current income (Y) by also on the current interest rate (r).

(vii) Let the investment demand be autonomous: $I=\bar{I}$. Also let the governement expenditure be fixed at $\bar{G}$ and there are no taxes. Derive the corresponding equation of the IS curve, depicting a relationship between Y and r. Notice that this modified IS equation now depicts the aggregate demand in the economy. Draw the corresponding AD schedule (total output demanded) as a function of r (i.e., in the (Y,r) plane).

(viii) From the modifed AS-AD schedules in the (Y,r) space, identify the equlibrium output and equilibrium interest rate in this economy. How does the equilibrium output of this modified AS-AD system respond to fiscal policy changes?

(ix) Show that money is neutral in the economy such that an increase in money supply leads only to a increase in price level with no impact on the equilibrium output.

3. Consider an economy inhabited by H identical households. Each household has a single inifnitely-lived member who decides on his optimal consumption stream so as to maximize his lifetime utility given by the following utility function:

$$\underset{\{c_t^h\}_{t=0}^{\infty}}{Max.}\sum_{t=0}^{\infty}\beta^t\frac{\left(c_t^h\right)^{1-\sigma}}{1-\sigma};\sigma<1.$$

The agent is not allowed to borrow from others; hence the only asset that he can invest in is physical capital stock (k_t^h). Existing capital stock depreciates fully in one period such that $\delta=1$.The agent has an endowment of one unit of labour and some capital stock (from previous period's investment) in every period which he supplies inelastically to the market to earn some wage and interest income. He consume a part of this income and invests the rest which forms his capital stock in the next period. Thus the period by period budget constraint of the agent is given by:

$$k_{t+1}^h = w_t + r_t k_t^h - c_t^h; \; k_t^h \geqq 0 \text{ for all } t \geqq 0; \; k_0^h \text{ given.}$$

(i) Write down the Bellman equation for the above dynamic optimization problem.

(ii) Assuming that a unique value function exists which is also differentiable, write down the corresponding first order condition (Euler equation).

(iii) Apply Envelope theorem and simplify to generate a system of dynamic equations in c_t^h and k_t^h.

(iv) Assume that

$$w_t = \bar{w} \text{ and } r_t = \bar{r} \text{ for all } t.$$

Derive the corresponding dynamic equations for the aggregate economy depicting the dynamics of average consumption and average capital stock in the economy.

(v) Show that along the optimal path, consumption of all households grow at the same rate even though their initial capital holding is different.

4. Consider an economy inhabited by H households, which are identical in every respect so that we can talk in terms of a representative household. The household has a single inifnitely-lived member. He has an initial capital endowment of k_0^h and an endowment of one unit of time which he optimally decides to allocate between labour (l_t^h) and leisure ($1-l_t^h$) in every period. The part of the time endowment which is allocated to labour generates some labour income for him at the prevailing wage rate of w_t per unit of labour time. He also earns an interest income on the capital stock owned at the prevailing interest rate of r_t.The agent is not allowed to borrow from others. Also, existing capital stock depreciates fully in one period.

The instantaneous pay-off function of the agent now depends both on consumption (c_t^h) as well as the amount of leisure enjoyed $(1 - l_t^h)$. Accordingly, his lifetime utility is represented by the following utility function:

$$\underset{\{c_t^h\}_{t=0}^{\infty}, \{l_t^h\}_{t=0}^{\infty}}{Max.} \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t^h\right)^{1-\sigma}}{1-\sigma} + \log(1 - l_t^h) \right].$$

(i) Write down the period by period budget constraint of the representative agent.

(ii) Write down the Bellman equation for the above dynamic optimization problem.

(iii) Assuming that a unique value function exists which is also differentiable, write down the corresponding first order conditions. (Notice that now there are two control variables: c_t^h and l_t^h).

(iv) Apply Envelope theorem and simplify to derive the equations charaterising the optimal dynamic paths of c_t^h, l_t^h and k_t^h.

(v) Show that along the optimal path, as current wage rate (w_t) rises, the effect on optimal labour supply could be ambiguous; it does not *necessarily* rise.