902: Issues in Economic Systems and Institutions

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Solutions to final exam. Winter Semester, 2015-16.

PART B

1. (a) (i) Babbling.

   (ii) 2-interval partition: $[0, \frac{3}{8})$, $[\frac{3}{8}, 1]$.

   Actions: $a_1 = \frac{3}{16}$, $a_2 = \frac{11}{16}$

   (iii) 3-interval partition: $[0, \frac{1}{12})$, $[\frac{1}{12}, \frac{5}{12})$, $[\frac{5}{12}, 1]$.

   Actions: $a_1 = \frac{1}{24}$, $a_2 = \frac{6}{24}$, $a_3 = \frac{17}{24}$.

   (b) For any n-partition equilibrium to survive when the censor is present, a necessary and sufficient condition is that when the censor hears m_1 (corresponding to the left-most interval), he does not want to misrepresent it as m_2 (corresponding to the next interval to the right). If this IC is satisfied, all other ICs will be satisfied (see lecture slides). The condition for this IC to be violated is (after slight manipulation of the incentive constraint):

   $$a_1 + \widehat{b} > \frac{1}{2}(a_1 + a_2)$$

   The most informative (3-interval) equilibrium breaks down if (using the relevant actions derived above and the inequality) $\widehat{b} > \frac{5}{48}$.

   (c) Using the same inequality and borrowing the equilibrium actions for the 2-interval equilibrium, we get: $\widehat{b} > \frac{1}{4}$. When this is true, only the babbling equilibrium remains.

   (d) Define $\overline{\theta}_i$ $(i = R, S)$ to be the threshold value such that player i prefers action 1 if and only if the expected value of θ is greater than or equal to $\overline{\theta}_i$. Easy to check:

   $$\overline{\theta}_R = \frac{1}{2}; \quad \overline{\theta}_S = \frac{7}{16}$$

   The sender should commit to the threshold $\overline{\theta} = \overline{\theta}_S = \frac{7}{16}$. If the receiver is responsive to the message under this kind of commitment, it is clearly optimal because the sender gets the decision rule he would implement if he were the decision maker. We only need to check that when the receiver gets the higher message, she would want to choose $a = 1$. Note

$$\mathbf{E}\left(\theta|\theta \geq \frac{7}{16}\right) = \frac{23}{32} > \frac{1}{2} = \overline{\theta}_R$$

which lies above the receiver's threshold for choosing $a = 1$.

2. (a) The proposed equilibrium is that the bad advisor always reports $m = 0$, while the good advisor truthfully reports her signal $m = s$. Using Bayes' rule and these strategies, the reputations are:

$$\begin{aligned}
\widehat{\mu}(1,0) &= \widehat{\mu}(1,1) = 1 \\
\widehat{\mu}(0,0) &= \frac{\mu p}{\mu p + (1-\mu)} = \frac{1}{1 + \left(\frac{1-\mu}{\mu}\right)\left(\frac{1}{p}\right)} \\
\widehat{\mu}(0,1) &= \frac{\mu(1-p)}{\mu(1-p) + (1-\mu)} = \frac{1}{1 + \left(\frac{1-\mu}{\mu}\right)\left(\frac{1}{1-p}\right)}
\end{aligned}$$

Since $p > \frac{1}{2}$, $\frac{1}{p} < \frac{1}{1-p}$. Comparing the denominators of the expression, we establish that $\widehat{\mu}(0,1) < \widehat{\mu}(0,0)$. Also, since $\frac{p}{\mu p+(1-\mu)} < 1$, it follows that $\widehat{\mu}(0,0) < \mu$. The advisor's reputation worsens because even when the state is 0, the good advisor sends the politically incorrect message ($m = 0$) less often than the bad advisor (who sends it always), because sometimes the good advisor gets the wrong signal and sends the message $m = 1$ even under this state.

(b) We need to satisfy the IC of the good advisor, i.e., when she sees $s = 0$, she wants to send $m = 0$ rather than $m = 1$ (the other IC will always be satisfied because when $s = 1$, payoffs from the decision as well as the reputation are strictly higher from reporting it truthfully). First, the advisors own posterior beliefs after observing her signals is given by:

$$\Pr(\theta = 0|s = 0) = \Pr(\theta = 1|s = 1) = p$$

Using this, the good advisor's IC is:

$$\alpha. [p\widehat{\mu}(1,0) + (1-p)\widehat{\mu}(1,1)] + (1-p).1 \leq \alpha [p\widehat{\mu}(0,0) + (1-p)\widehat{\mu}(0,1)] + p.1$$

Use the expressions above to get the upper bound on α.

(c) The bad advisor's non-trivial IC is that when she sees $s = 1$, she wants to lie and say $m = 0$ rather than $m = 1$ (because lying is the postulated equilibrium behaviour). This implies

$$\alpha.\left[(1-p)\widehat{\mu}(1,0) + p\widehat{\mu}(1,1)\right] \leq \alpha\left[(1-p)\widehat{\mu}(0,0) + p\widehat{\mu}(0,1)\right] + 1$$

which gives us another bound on α. One implicit assumption here is that after the message $m = 0$, the decision maker indeed wants to choose $a = 0$. To check that this is true, observe that under this kind of equilibrium

$$\begin{aligned}\Pr(\theta = 0|m = 0) &= \frac{\frac{1}{2}\left[\mu p + (1-\mu).1\right]}{\frac{1}{2}\left[\mu p + (1-\mu).1\right] + \frac{1}{2}\left[\mu(1-p) + (1-\mu).1\right]} \\ &= \frac{1}{1 + \frac{\mu(1-p)+(1-\mu)}{\mu p + (1-\mu)}} > \frac{1}{2}\end{aligned}$$

Intuitively, since there is some probability the message $m = 0$ came from the good advisor, the posterior on $\theta = 0$ will be higher than the prior, so the receiver will find it optimal to choose $a = 1$.

(d) A political correctness (babbling) equilibrium always exists where everyone sends $m = 1$ regardless of the state-of-the-world, where the politically incorrect message ($m = 0$), which is off-the-equilibrium-path, causes severe reputation damage. For parameters identified above, there also exists an equilibrium where the good advisor speaks truthfully. Which equilibrium is selected will depend on culture.

(e) See slides.

3. (a) Standard.

   (b) Standard.

   (c) The posterior beliefs on the returns being +1 after 1, 2 and 3 net high signals are $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{8}{9}$ respectively. Under these beliefs, the expected payoff difference between investing and not investing is 0.33, 1.4, and 1.66 respectively. The reward for foregoing this by being a contrarian is 0.4. Therefore, a cascade will start once the net signals and consecutive identical choices reaches 3. After that, even a truly contrarian signal will not lead to a contrarian choice.

(d) Standard.

(e) Standard.