902: Issues in Economic Systems and Institutions

Department of Economics, Delhi School of Economics

Solutions to final exam. Winter Semester, 2016-17.

PART B

1. (a) Informed leader's optimal effort:

$$e_1^*(\theta) \arg\max_{e_1} \alpha_1\theta\,(e_1+e_2) - \frac{1}{2}e_1^2 = \alpha_1\theta$$

Uninformed follower's optimal effort:

$$e_2^* = \arg\max_{e_2} \alpha_2\frac{1}{2}(4+6)\,(e_1+e_2) - \frac{1}{2}e_1^2 = 5\alpha_1$$

(b) The answer is the solution to

$$\max_{\alpha_1,\alpha_2} \frac{1}{2}.4(4\alpha_1+5\alpha_2)+\frac{1}{2}.6(6\alpha_1+5\alpha_2)-\frac{1}{2}\left(\frac{1}{2}4\alpha_1^2+\frac{1}{2}6\alpha_1^2\right)-\frac{1}{2}5\alpha_2^2 \text{ subject to } \alpha_1+\alpha_2=1$$

The solution is

$$\alpha_1^* = \frac{5}{9};\ \ \alpha_2^* = \frac{4}{9}$$

(c) In a separating equilibrium, let e_h and e_l be the leader's effort choice when productivity is high and low respectively (i.e., $\theta = 6$ and 4). Since follower learns the value of θ, his effort choice is given by

$$\begin{aligned} e(e_h) &= 6\alpha_2 = 3 \\ e(e_l) &= 4\alpha_2 = 2 \end{aligned}$$

since $\alpha_2 = \frac{1}{2}$. Also, $e^l = \frac{1}{2}.4 = 2$, since the leader has no reason to signal extra hard when signaling low productivity. e^h must satisfy two incentive constraints that make sure the leader does not want to switch to the high (low) productivity effort level when actual productivity is low (high):

$$\text{IC } (\theta = 6)\text{:}\ \ \frac{1}{2}.6(e_h+3) - \frac{1}{2}e_h^2 \geq \frac{1}{2}.6(2+2) - \frac{1}{2}.2^2 \Rightarrow e_h \leq 3+\sqrt{7}$$

(we choose the higher root).

$$\text{IC } (\theta = 4): \ \frac{1}{2}.4(e_h + 3) - \frac{1}{2}e_h^2 \geq \frac{1}{2}.4(2+2) - \frac{1}{2}.2^2 \Rightarrow e_h \geq 4$$

Combining:

$$4 \leq e_h \leq 3 + \sqrt{7}$$

So the lowest effort level $\widehat{e}_h$ which will "separate" the high productivity type from the low productivity type is 4. In the separating equilibrium with lowest effort, the value of e_h is

$$\max\{\widehat{e}_h, e_1^*(6)\} = \min\{4, 3\}$$

[Note that if lowest effort level needed to separate from the low productivity type is lower than the high productivity type's optimal effort under full information, she will choose the latter].

(d) In this example, due to signaling motives, the leader supplies higher effort under incomplete information than under full information (but still less than first best). Hence, team surplus is higher.

2. (a) (i) Babbling. (ii) 2-interval partition: $[0, \frac{7}{18})$, $[\frac{7}{18}, 1]$.(iii) 3-interval partition: $[0, \frac{1}{9})$, $[\frac{1}{9}, \frac{4}{9})$, $[\frac{4}{9}, 1]$.

(b) Optimal ceiling

$$\arg\min_{\overline{a}} \int_0^{\overline{a}} b^2 d\theta + \int_{\overline{a}}^{1} (\theta - \overline{a})^2 d\theta = 1 - b = \frac{17}{18}$$

Calculate welfare under the optimal ceiling, using the formula above, and under cheap talk using the derivation in the slides.

(c) The equilibrium strategies are as follows: (i) the unbiased type ($b = 0$) sends the message $m = \theta$ for $\theta < \overline{\theta}$ and $m = \overline{m}$ for $\theta \geq \overline{\theta}$ (ii) the biased type ($b = 1$), who always prefers a higher action in the $[0, 1]$ interval, sends the message $m = \overline{m}$ for all θ. The receiver's response is

$$\begin{aligned} a(m) &= m \ \text{ if } m < \overline{\theta} \\ &= \overline{\theta} \ \text{ if } m = \overline{m} \end{aligned}$$

By Bayes' rule

$$\Pr(b = 1|m = \overline{m}) = \pi = \frac{\alpha.1}{\alpha.1 + (1-\alpha)(1-\overline{\theta})}$$

Note that the biased type sends the message $\overline{m}$ with higher probability than the unbiased type, hence beliefs about types have to be updated after $\overline{m}$. If the biased type has sent $\overline{m}$, expected value of θ is $\frac{1}{2}$. If the unbiased type has sent it, that expectation is $\frac{1+\overline{\theta}}{2}$. The message $\overline{m}$ must lead to an expected value of θ equal to $\overline{\theta}$.

$$\mathbf{E}(\theta|m = \overline{m}) = \pi.\frac{1}{2} + (1-\pi)\frac{(1+\overline{\theta})}{2} = \overline{\theta}$$

Use the last two equations to solve for $\overline{\theta}$. It is the solution to the quadratic

$$(1-\alpha)\overline{\theta}^2 + 2(2\alpha - 1)\overline{\theta} + 1 - 2\alpha = 0$$

3. (a) (i) Majority rule: if the voting threshold is $\overline{x}$, a yes vote implies the conditional expectation of the corresponding random variable is $\frac{1+\overline{x}}{2}$, while a no vote implies the conditional expectation is $\frac{\overline{x}-1}{2}$. Being pivotal means there is 1 yes vote and 1 no vote. If the voter's own random variable is $\overline{x}$, she should be indifferent between a yes and a no vote, i.e.,

$$x_0 + \overline{x} + \frac{1+\overline{x}}{2} + \frac{\overline{x}-1}{2} = 0 \Rightarrow \overline{x} = -\frac{x_0}{2}$$

(ii) Unanimity rule: being pivotal means other 2 votes are yes. Using the indifference condition:

$$x_0 + \overline{x} + 2\left(\frac{1+\overline{x}}{2}\right) = 0 \Rightarrow -\frac{1+x_0}{2}$$

(b) (i) Majority rule: investment happens if there are 2 or 3 yes votes, i.e., with probability

$$\begin{aligned} & 3\left(\frac{1-\overline{x}}{2}\right)^2\left(\frac{1+\overline{x}}{2}\right) + \left(\frac{1-\overline{x}}{2}\right)^3 \\ = & \frac{1}{4}\left(1+\frac{x_0}{2}\right)^2\left(2+\frac{x_0}{2}\right) \end{aligned}$$

(ii) Unanimity rule: investment happens if all 3 votes are yes, i.e., with probability

$$\left(\frac{1-\overline{x}}{2}\right)^3 = \left(\frac{3+x_0}{4}\right)^3$$

(c) (i) Under majority rule, probability of a yes vote (using $x_0 = 0$) is $\frac{1}{2}$. Expected profit:

$$3.\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)\left[2.\frac{1}{2}-\frac{1}{2}\right]+\left(\frac{1}{2}\right)^3 3.\frac{1}{2}=\frac{3}{8}$$

(ii) Under unanimity rule, probability of a yes vote (using $x_0 = 0$) is $\frac{1}{2}$. Expected profit:

$$\left(\frac{3}{4}\right)^3 \frac{1}{2}\left(1-\frac{1}{2}\right)=\frac{81}{256}$$

Higher expected profit under majority rule.