Problem Set 1

1. Prove that $(B - A) \cup (C - A) = (B \cup C) - A$.

2. A power set $P(S)$ of S is the set of all subsets of S, that is $P(S) = \{T \mid T \subseteq S\}$. Show that $A \subseteq B \Leftrightarrow P(A) \subseteq P(B)$.

3. Negate the following propositions:
(i) (Notations: F is the set of commodity bundles. $\prec$ is a preference relation) For all $x, y, z \in F$, $x \prec y$ and $y \prec z$ implies $x \prec z$.
(ii) (Notations: Set of agents in an economy is denoted by N. u_k is the utility function of agent k. Set of feasible outcomes is denoted by F.).
An outcome x is such that for all $y \in F$, if $u_i(y) > u_i(x)$ for some $i \in N$ then there exists $j \in N$ such that $u_j(y) < u_j(x)$.

4. Are the following propositions True?
(i) ($f : \mathcal{R} \to \mathcal{R}$) $f(x)$ is an increasing function $\Leftrightarrow$ $|f(x)|$ is an increasing function.
(ii) Two real numbers a and b are equal $\Leftrightarrow$ For every $\epsilon > 0$, $|a - b| < \epsilon$.

5. Let $f : A \to B$ and $g : B \to C$ be functions.
(i) Show that if $g \circ f$ is one-to-one and onto then f is one-to-one and g is onto.
(ii) Give an example where g is not one-to-one and f is not onto but $g \circ f$ is one-to-one and onto.

6. Consider $f : \mathcal{R}^2 \to \mathcal{R}^2$ defined by $f(x_1, x_2) = (x_1 x_2, x_1 + x_2)$. Find the range of f and draw it as a subset of $\mathcal{R}^2$.