Microeconomics: Barter Economy and its Outcomes

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Lecture 1

Introduction

This part of the course, we will

- study the nature interdependence in the decisions by economic agents
  - Consumers
  - Producers
- study market as medium for coordination of consumption across consumers
- study market as medium for coordination of production across producers
- study market as medium for coordination of consumption and production across consumers and producers
- examine the relationship between micro decisions and the collective (macro-economic) outcomes
- compare the market outcome with the other mechanisms for co-ordinating consumptions and production activities

Questions

Suppose, we have two goods - 10 units each. We want to divide them between two consumers.

- What is the best possible outcome under a command and control (i.e., socialist) economy?
- What is the best possible outcome under a decentralized economy with following features:
  - Complete freedom to trade
  - Individuals have complete information about each other
  - Trade/exchange of goods is costless and individuals are willing to cooperate with each other
- What is the best possible outcome under a decentralized economy with following features:
  - Complete freedom to trade
  - Individuals may not have full information about one another
  - However, they can buy and sell in a competitive market

Basics I

To start with, we will consider an exchange economy:

- There is no production - it has already taken place
- There are N individuals - an individual is indexed by i, $i = 1, 2, ..., N$
- There are M commodities - a commodity is indexed by j, $j = 1, 2, ..., M$
- Each individual is endowed with a bundle of commodities

For a 2×2 economy, let initial endowments be $(\mathbf{e}^1, \mathbf{e}^2)$, where

$$\begin{aligned} \mathbf{e}^1 &= (e_1^1, e_2^1) = (1, 9) \\ \mathbf{e}^2 &= (e_1^2, e_2^2) = (9, 1) \end{aligned}$$

Question

Suppose, we want to redistribute the available stock of the goods. How many possibilities are there?

Basics II

Definition

Allocation: An allocation is a distribution of available goods among the individuals.

E.g.,

- when $\mathbf{e}^1 = (1, 9)$ and $\mathbf{e}^2 = (9, 1)$,
- a possible allocation is: $\mathbf{x}^1 = (3, 6)$, and $\mathbf{x}^2 = (7, 3)$.
- another possible allocation is: $\mathbf{x}^1 = (3, 6)$, and $\mathbf{x}^2 = (7, 4)$.

In general, let

- $\mathbf{x}^1 = (x_1^1, x_2^1)$ denote the bundle allocated to person 1
- $\mathbf{x}^2 = (x_1^2, x_2^2)$ denote the bundle allocated to person 2

Basics III

Allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2)$ is feasible

$$\begin{array}{rcl} x_1^1 + x_1^2 & \leq & e_1^1 + e_1^2 \\ x_2^1 + x_2^2 & \leq & e_2^1 + e_2^2 \end{array}$$

Allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2)$ is non-wasteful if

$$\begin{array}{rcl} x_1^1 + x_1^2 & = & e_1^1 + e_1^2 \\ x_2^1 + x_2^2 & = & e_2^1 + e_2^2 \end{array}$$

Question

Under what assumption on individuals utility functions, we would want to consider only the non-wasteful allocations?

Basics IV

For a N-person and M-good economy, suppose endowments are: where $\mathbf{e}^1 = (e_1^1, e_2^1, ..., e_M^1)$,..., $\mathbf{e}^i = (e_1^i, e_2^i, ..., e_M^i)$,...,$\mathbf{e}^N = (e_1^N, e_2^N, ..., e_M^N)$.

So, the **total endowment** of goods can be written as:

$$\text{good 1}: \quad e_1^1 + e_1^2 + ... + e_1^N = \sum_{i=1}^{N} e_1^i$$

$$\text{good j}: \quad e_j^1 + e_j^2 + ... + e_j^N = \sum_{i=1}^{N} e_j^i$$

$$\text{good M}: \quad e_M^1 + e_M^2 + ... + e_M^N = \sum_{i=1}^{N} e_M^i$$

For a N-person and M-good economy, the macro-economic endowment is:

$$\mathbf{e} = (\mathbf{e}^1, \mathbf{e}^2, ..., \mathbf{e}^N),$$

Basics V

For the entire economy, a macro-economic allocation is

$$\mathbf{x} = (\mathbf{x}^1, ..., \mathbf{x}^N),$$

where

$$\begin{aligned}
\mathbf{x}^1 &= (x_1^1, x_2^1, ..., x_M^1) \\
\mathbf{x}^i &= (x_1^i, x_2^i, ..., x_M^i) \\
\mathbf{x}^N &= (x_1^N, x_2^N, ..., x_M^N)
\end{aligned}$$

Allocation $\mathbf{x} = (\mathbf{x}^1, ..., \mathbf{x}^N)$ is non-wasteful w.r.t. good 1 if

$$\sum_{i=1}^{N} x_1^i = \sum_{i=1}^{N} e_1^i$$

Non-wasteful Allocations

Definition

Allocation $\mathbf{x} = (\mathbf{x}^1, ..., \mathbf{x}^N)$ is non-wasteful if

$$\underbrace{\sum_{i=1}^{N} x_j^i}_{total\ consumption} = \underbrace{\sum_{i=1}^{N} e_j^i}_{total\ availablility}$$

for all $j = 1, ..., M$

Set of (non-wasteful) allocations is the set

$$\mathbf{X} = \{\mathbf{x} = (\mathbf{x}^1, ..., \mathbf{x}^N)\}$$

where $\mathbf{x}^i$ is such that

- $x_j^i \geq 0$ for all i and j, for all $i = 1, ..., N$, for all $j = 1, ..., M$; and
- $\sum_{i=1}^{N} x_j^i = \sum_{i=1}^{N} e_j^i$ for all $j = 1, ..., M$.

Assumptions

We will make the following assumption about the choice sets and the individual preferences:

- The set of alternatives is the set of (non-wasteful) allocations
- Individuals have (self-interested) preferences defined over the set of allocations
- Each preference is/can be represented by a (self-interested) utility function
    - Each preference is complete, transitive and ????
- Each utility function is monotone, and ???

Ref: Advanced Microeconomic Theory by Geoffrey Jehle and Philip Reny (2nd Ed.)

Pareto Optimal Allocations I

Consider a two-person, two-good economy.

Definition

Allocation $(\mathbf{x^1}, \mathbf{x^2})$ is Pareto superior to the endowment, $(\mathbf{e^1}, \mathbf{e^2})$, if

$$\begin{aligned} u^i(\mathbf{x}^i) &\geq u^i(\mathbf{e}^i) \text{ holds for } i = 1, 2.\text{And} \\ u^j(\mathbf{x}^j) &> u^j(\mathbf{e}^j) \text{ holds for at least one } j. \end{aligned}$$

Remark

If a feasible allocation $(\mathbf{x^1}, \mathbf{x^2})$ is Pareto superior to $(\mathbf{e^1}, \mathbf{e^2})$, then

- $(\mathbf{e}^1, \mathbf{e}^2)$ CANNOT be Pareto Optimum
- but is $(\mathbf{x}^1, \mathbf{x}^2)$ Pareto Optimum?

Pareto Optimal Allocations II

Definition

Allocation $(\mathbf{x}^1, \mathbf{x}^2)$ is Pareto Optimum, if there does not exist another (feasible) allocation $(\mathbf{y}^1, \mathbf{y}^2)$ such that:

$$(\mathbf{y}^1, \mathbf{y}^2) \text{ is Pareto superior to } (\mathbf{x}^1, \mathbf{x}^2).$$

Remark

In general, there can be several Pareto Optimum allocations, which can be derived from the initial endowments.

Outcome under Barter

A Thought Experiment: Barter: Suppose, there is no market (price mechanism). However, Individuals have

- complete freedom to trade
- people are free exchange bundles of goods with one another
- but individuals are free to not to trade also - trade only if they wish to do so
- individuals have all the information about each other - endowments, preferences, etc.

Question

What is the best achievable outcome under Barter?

An Example

Example

Consider the following two-person, two-good economy:

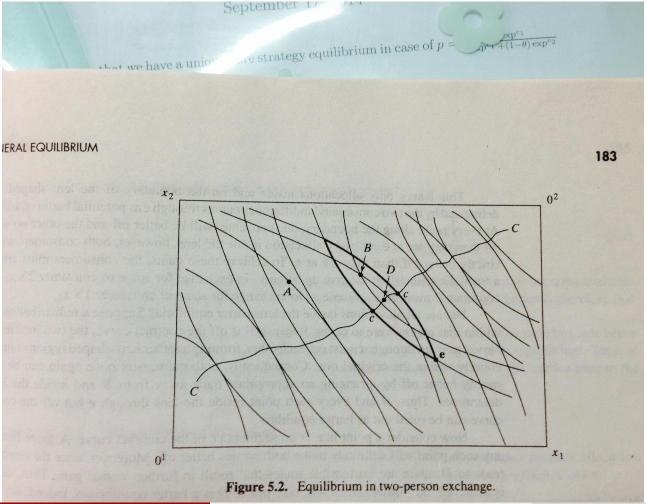
- Endowments: $\mathbf{e}^1 = (1, 9)$, and $\mathbf{e}^2 = (9, 1)$
- Preferences: $u^i(x, y) = x.y$. That is, $u^1(x_1^1.x_2^1) = x_1^1.x_2^1$ and $u^2(x_1^2.x_2^2) = x_1^2.x_2^2$
- Allocation: $\mathbf{x}^1 = (3, 3)$, and $\mathbf{x}^2 = (7, 7)$

Clearly, $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2)$ is Pareto superior to $\mathbf{e} = (\mathbf{e}^1, \mathbf{e}^2)$.

That is,

- $\mathbf{e} = (\mathbf{e}^1, \mathbf{e}^2)$ will be rejected in favour of $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2)$.
- Formally speaking, $\mathbf{e} = (\mathbf{e}^1, \mathbf{e}^2)$ will be blocked by allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2)$.

‘Rejectable’ Allocations



'Rejectable' Allocations

- Assume all exchanges are voluntary.

For a two-person two-good economy: Allocation $\mathbf{y} = (\mathbf{y^1}, \mathbf{y^2})$ will be blocked/rejected, if any of the following holds:

1. $u^1(\mathbf{e}^1) > u^1(\mathbf{y}^1)$; or
2. $u^2(\mathbf{e}^2) > u^2(\mathbf{y}^2)$; or
3. There exists a feasible allocation $(\mathbf{x^1}, \mathbf{x^2})$ that is Pareto superior to $(\mathbf{y^1}, \mathbf{y^2})$, i.e., for some $(\mathbf{x^1}, \mathbf{x^2})$

$$\begin{aligned} u^i(\mathbf{x}^i) &\geq u^i(\mathbf{y}^i). \text{ for } i = 1, 2.\text{And} \\ u^j(\mathbf{x}^j) &> u^j(\mathbf{y}^j) \end{aligned}$$

holds for at least one i.

‘Non-Rejectable’ Allocations

Remark

In the above example, recall $\mathbf{x}^1 = (3, 3)$ and $\mathbf{x}^2 = (7, 7)$. You can verify that:

- there is no other feasible allocation $\mathbf{y} = (\mathbf{y}^1, \mathbf{y}^2)$ for which the following hold (with at least one inequality)
  $u^1(\mathbf{y}^1) \geq u^1(\mathbf{x}^1)$, and $u^2(\mathbf{y}^2) \geq u^2(\mathbf{x}^2)$.
- so, allocation $(\mathbf{x}^1, \mathbf{x}^2)$ cannot be blocked by any individual or both of them together.

Question

For the above example,

- *How many unblocked allocations are there?*
- *What is the set of possible outcomes under Barter?*

Core Allocations: Properties I

For a two-person two-good economy, an allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2)$ belongs to the Core, only if

- Every i prefers $\mathbf{x}^i$ at least as much as $\mathbf{e}^i$, $i = 1, 2$
- Allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2)$ is Pareto Optimum

Question

For the above example, describe the set of Core allocations?