Barter Exchange and Core: Lecture 2

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Course 001

The Question

In the last lecture, we discussed the following questions:

Question

How can we redistribute the endowments such that:

- *Every individual prefers the reallocated bundle received over her initial endowment*
- *No subset of individuals can do better for themselves using their own endowments*
- *Every subset of individuals prefers the reallocated bundle to what they can manage to have on their own.*

Possible Outcomes under Barter

We

- Assume all exchanges are voluntary.

For a two-person two-goods economy, we saw:

Allocation $\mathbf{y} = (\mathbf{y^1}, \mathbf{y^2})$ will be blocked/rejected, if any of the following holds:

1. $u^1(\mathbf{e}^1) > u^1(\mathbf{y}^1)$; or
2. $u^2(\mathbf{e}^2) > u^2(\mathbf{y}^2)$; or
3. There exists a feasible allocation $(\mathbf{x^1}, \mathbf{x^2})$ that is Pareto superior to $(\mathbf{y^1}, \mathbf{y^2})$, i.e., for some $(\mathbf{x^1}, \mathbf{x^2})$

$$\begin{aligned} u^i(\mathbf{x}^i) &\geq u^i(\mathbf{y}^i). \text{ for } i = 1, 2. \text{And} \\ u^i(\mathbf{x}^i) &> u^i(\mathbf{y}^i) \end{aligned}$$

holds for at least one i.

‘Core’ Allocations

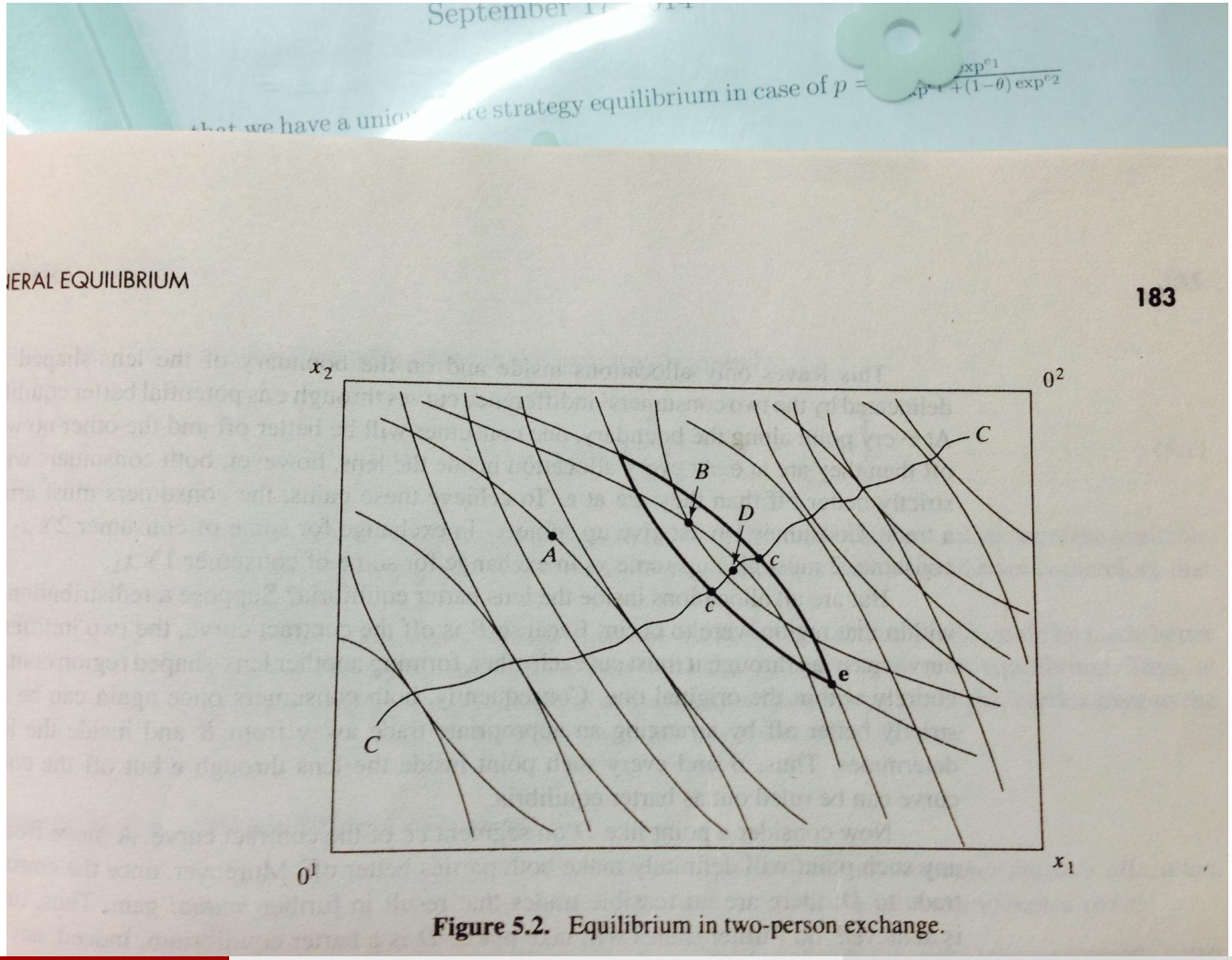

Figure 5.2. Equilibrium in two-person exchange.

Possible Outcomes under Barter

For the following example :

- Endowments: $\mathbf{e}^1 = (1, 9)$, and $\mathbf{e}^2 = (9, 1)$
- Preferences: $u^i(x, y) = x.y$. That is, $u^1(x_1^1.x_2^1) = x_1^1.x_2^1$ and $u^2(x_1^2.x_2^2) = x_1^2.x_2^2$
- Allocation: $\mathbf{x}^1 = (3, 3)$, and $\mathbf{x}^2 = (7, 7)$

We saw

- $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2)$ is Pareto superior to $\mathbf{e} = (\mathbf{e}^1, \mathbf{e}^2)$.
- $\mathbf{e} = (\mathbf{e}^1, \mathbf{e}^2)$ will be rejected in favour of $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2)$.
- Formally speaking, $\mathbf{e} = (\mathbf{e}^1, \mathbf{e}^2)$ will be blocked by allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2)$.
- Allocation $\mathbf{x}^1 = (3, 3)$, and $\mathbf{x}^2 = (7, 7)$ cannot be blocked
- Allocation $\mathbf{z}^1 = (7, 7)$, and $\mathbf{z}^2 = (3, 3)$ cannot be blocked
- Allocation $\mathbf{w}^1 = (5, 5)$, and $\mathbf{z}^2 = (5, 5)$ cannot be blocked

Core Allocations: Properties I

For a two-person two-good economy, an allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2)$ belongs to the Core, only if

- Every i prefers $\mathbf{x}^i$ at least as much as $\mathbf{e}^i$, $i = 1, 2$
- Allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2)$ is Pareto Optimum

Question

For the above example,

- *What is the size of the Core?*
- *Does Core denote the set of possible outcomes under Barter?*

Question

Does the set of Pareto optimum allocations depend on the initial endowments?

Core of a 3×2 economy I

Example

Consider the following three-person, two-good economy:

- Endowments: $\mathbf{e}^1 = (1, 9)$, $\mathbf{e}^2 = (9, 1)$, and $\mathbf{e}^3 = (5, 5)$
- Preferences: $u^1(x_1^1.x_2^1) = x_1^1.x_2^1$;
  $u^2(x_1^2.x_2^2) = x_1^2.x_2^2$;
  and $u^3(x_1^3.x_2^3) = x_1^3.x_2^3$

Now, consider the following allocation:

$$\mathbf{x}^1 = (3, 3), \mathbf{x}^2 = (7, 7), \text{ and } \mathbf{x}^3 = (5, 5).$$

Core of a 3×2 economy II

Question

- *Is the allocation* $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3)$ *feasible?*
- *Is allocation* $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3)$ *Pareto-superior to* $\mathbf{e} = (\mathbf{e}^1, \mathbf{e}^2, \mathbf{e}^3)$*?*
- *Is allocation* $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3)$ *Pareto Optimum?*
- *Does allocation* $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3)$ *belong to the Core?*

Consider a Coalition of 1 and 3, i.e., $S = \{1, 3\}$. Let $\mathbf{y}$ be such that

$$\mathbf{y}^1 = (2, 5) \text{ and } \mathbf{y}^3 = (4, 9).$$

Recall, $\mathbf{e}^1 = (1, 9)$ and $\mathbf{e}^3 = (5, 5)$.

- So the set $S = \{1, 3\}$ is better off rejecting the allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3)$, as defined above

Core of a 3×2 economy III

- We can say that $S = \{1, 3\}$ forms a 'blocking' coalition against the allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3)$,
- So, allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3)$ is Not an Unblocked allocation, and hence does not belong to the Core
- Hence $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3)$ does not belong to the

Remark

A Pareto Optimum allocation

- may not belong to the Core
- will not belong to the Core if there exists a blocking coalition

Example 4×2 economy I

Example

There are four individuals and two goods. Utility functions are: $u^i(x_1^i x_2^i) = x_1^i x_2^i$, *for* $i = 1, ..., 4$. *Endowments are:*

$\mathbf{e}^1 = (1, 9)$, $\mathbf{e}^2 = (9, 1)$, $\mathbf{e}^3 = (1, 9)$, *and* $\mathbf{e}^4 = (9, 1)$, *respectively.*

Now, consider the allocation:

$$\mathbf{x}^1 = (3, 3) = \mathbf{x}^3 \text{ and } \mathbf{x}^2 = (7, 7) = \mathbf{x}^4.$$

Again, the allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3, \mathbf{x}^4)$ is Pareto optimum but does not belongs to the Core of 4×2 economy of Example 4.

Let $S = \{1, 2, 3\}$, $\mathbf{y}^1 = (3, 4) = \mathbf{y}^3$ and $\mathbf{y}^2 = (5, 11)$.

You can verify that S forms a Blocking coalition against $(\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3, \mathbf{x}^4)$.

Blocking Coalition

Here is a general definition of Blocking Coalition for $N \times M$ economy.

Definition

Let $S \subseteq \{1, ..., N\}$. S is called a blocking coalitions for $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, ..., \mathbf{x}^N)$ if there is some vector $\mathbf{y}$ such that

$$\begin{aligned} \sum_{i \in S} y_j^i &= \sum_{i \in S} e_j^i \text{ for all } j = 1, ..., M \\ u^i(\mathbf{y}^i) = u^i(y_1^i, ..., y_M^i) &\geq u^i(x_1^i, ..., x_M^i) = u^i(\mathbf{x}^i) \text{ for all } i \in S \\ u^i(\mathbf{y}^i) = u^i(y_1^i, ..., y_M^i) &> u^i(x_1^i, ..., x_M^i) = u^i(\mathbf{x}^i) \text{ for some } i \in S \end{aligned}$$

Core of Barter Exchange

Consider a pure exchange economy $(u^i(.), \mathbf{e}^i)_{i \in N}$. For this economy,

Definition

Core is a set of allocations, $\mathbf{C}(u^i(.)_{i \in N}, \mathbf{e})$, such that if $\mathbf{x} \in \mathbf{C}(u^i(.)_{i \in N}, \mathbf{e})$, then $\mathbf{x}$ CANNOT be blocked by any coalition.

Remark

The size of the Core, i.e., outcome of barter depends on the 'nature' of the economy:

- the nature of individual preferences
- the initial endowment/wealth
- the number of individuals in the economy

Identifying the Core

Question

How to find the Core allocations?

If there are 25 individuals, you have to check $2^{25} - 1$ as potential coalitions that may block an allocation.

Question

Does the Core always exist?

- Scarf (1963) showed that when indifference curves are convex, the Core is non-empty
- Size of the Core shrinks with number of agents - Edgeworth (1881); Debreu and Scarf (1963); Aumann (1964), etc.

The Core in Real World

Question

In real world,

- *Will bargaining among individuals always lead to one of the allocations in the Core?*
- *Are there factors that can frustrate successful bargaining among individuals?*

Question

- *Can the market lead to the same set of outcomes as the Barter?*
- *Can outcome under market be better than under the Barter?*

Barter Vs Market I

1. Informational and logistical requirements
   - Barter requires
     - Search costs - to identify suitable trading partners
     - Successful negotiations
   - Market requires
     - No search costs
     - No cooperation - only decision making at individual level
2. Relative Efficiency
   - Barter
     - Pareto efficient outcome is unlikely, for large set of individuals
   - Competitive Market
     - Pareto efficient outcome more likely, especially for large set of individuals

   The claims are valid with or without production

Barter Vs Market II

3. Effect of Policy Interventions
    - Barter
        - Policy intervention only through reallocation of endowments
    - Market
        - Policy intervention through reallocation of endowments as well as direct transfers of 'purchasing power'

Remark

In real world,

- Neither Barter nor Market can guarantee the intended outcome
- Some endowments are not transferable - E.g. ????