Market Equilibrium

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Lecture 3

Market Exchange: Basics

Let us introduce 'price' in our pure exchange economy. Let,

- There be N individuals and M goods
- $\mathbf{e}^i = (e^i_1, ..., e^i_M)$ denote endowment for individual i
- p_i denote the 'price' of ith good; $p_i > 0$ for all $i = 1, .., M$.
- So the price vector is

$$\mathbf{p} = (p_1, ..., p_M) >> \mathbf{0}.$$

Assume

- each good has a market and each individual is 'price-taker'.

For each individual,

- Total value of the initial endowment depends on the price vector
- An economic agent can buy any bundle of goods
- However, the total value of the bundle bought cannot exceed the total value of her endowment.

Market Exchange: 2×2 economy I

For person 1, the set of feasible allocations/consumptions is the set of $\mathbf{y}^1 = (y_1^1, y_2^1)$ such that:

$$p_1 y_1^1 + p_2 y_2^1 \leq p_1 e_1^1 + p_2 e_2^1.$$

Assuming monotonic preferences, Person 1 maximizes utility by choosing bundle $\mathbf{x}^1 = (x_1^1, x_2^1)$ s.t.

$$p_1 x_1^1 + p_2 x_2^1 = p_1 e_1^1 + p_2 e_2^1$$

Person 2 maximizes utility s.t.

$$p_1 x_1^2 + p_2 x_2^2 = p_1 e_1^2 + p_2 e_2^2.$$

Recall, within the Edgeworth box, for each allocation $(\mathbf{x}^1, \mathbf{x}^2)$, we have

$$x_1^1 + x_1^2 = e_1^1 + e_1^2, \text{ and } x_2^1 + x_2^2 = e_2^1 + e_2^2.$$

Market Exchange: 2 $\times$ 2 economy II

Note: The budget line for person 2 is: $p_1 x_1^2 + p_2 x_2^2 = p_1 e_1^2 + p_2 e_2^2$. However,

$$\begin{aligned}
p_1 e_1^2 + p_2 e_2^2 &= p_1 x_1^2 + p_2 x_2^2, \ i.e., \\
p_1 e_1^2 + p_2 e_2^2 &= p_1(e_1^1 + e_1^2 - x_1^1) + p_2(e_2^1 + e_2^2 - x_2^1), \ i.e., \\
0 &= p_1(e_1^1 - x_1^1) + p_2(e_2^1 - x_2^1), \ i.e., \\
p_1 x_1^1 + p_2 x_2^1 &= p_1 e_1^1 + p_2 e_2^1,
\end{aligned}$$

which is the budget line for the 1 person.

Preferences and Utilities: Assumptions

We assume:

- Preference relations to be continuous, strictly monotonic, and strictly convex
- The utility functions to be continuous, strictly monotonic and strictly quasi-concave

However, several of the results will hold under weaker conditions.

Question

What is the role of assumption that the utility functions are 'strictly quasi-concave'?

Let

- $u^1(.)$ denote the utility function for person 1
- $u^2(.)$ denote the utility function for person 2

Competitive Equilibrium: 2×2 economy I

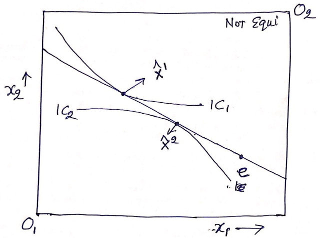
An allocation is $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ along with a price vector $\mathbf{p} = (p_1, p_2)$ is competitive equilibrium, if

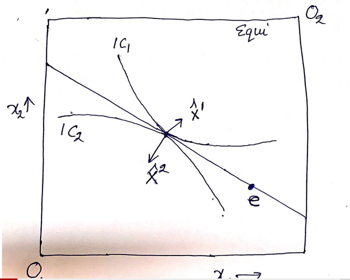
1. $\hat{\mathbf{x}}^1 = (\hat{x}_1^1, \hat{x}_2^1)$ maximizes $u^1(.)$ subject to $p_1 x_1^1 + p_2 x_2^1 = p_1 e_1^1 + p_2 e_2^1$
2. $\hat{\mathbf{x}}^2 = (\hat{x}_1^2, \hat{x}_2^2)$ maximizes $u^2(.)$ subject to $p_1 x_1^2 + p_2 x_2^2 = p_1 e_1^2 + p_2 e_2^2$
3. $\hat{x}_1^1 + \hat{x}_1^2 = e_1^1 + e_1^2$
4. $\hat{x}_2^1 + \hat{x}_2^2 = e_2^1 + e_2^2$

For 'well-behaved' utilities:

- 1. Implies : In equi. IC of person 1 will be tangent to her budget line.
- 2. Implies : In equi. IC of person 2 will be tangent to his budget line
- We know that: both of the demanded bundles, i.e., $\hat{\mathbf{x}}^1$ and $\hat{\mathbf{x}}^2$ lie on the same line. **Why?**
- 3 and 4 imply that the demanded bundles, i.e., $\hat{\mathbf{x}}^1$ and $\hat{\mathbf{x}}^2$ coincide. **Why?**

Competitive Equilibrium: 2 × 2 economy





2×2 Competitive Equilibrium: Properties I

- Note at the equilibrium allocation, $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$, the ICs are tangent to each other
- Therefore, the equilibrium allocation $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ is Pareto Optimum.

Question

Suppose, $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ is a Competitive (market) equilibrium allocation

- *Are unilateral deviations from $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ profitable?*
- *Does the eq. allocation $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ belong to the core?*

Competitive Equilibrium: $N \times M$ economy I

Consider a $N \times M$ economy denoted by $(u^i(.), \mathbf{e})$, where

$$\mathbf{e} = (\mathbf{e}^1, \mathbf{e}^2, ..., \mathbf{e}^N).$$

An allocation $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, ..., \hat{\mathbf{x}}^N)$ along with a price vector $\mathbf{p} = (p_1, ..., p_M)$ is a competitive equilibrium, if the following conditions are satisfied:

First: For each $i = 1, ..., N$, $\hat{\mathbf{x}}^i$ maximizes $u^i(.)$, subject to $\mathbf{p}.\mathbf{x}^i = \mathbf{p}.\mathbf{e}^i$. That is, $\hat{\mathbf{x}}^i$ solves

$$\max_{\mathbf{x}^i} \{u^i(\mathbf{x}^i)\} \tag{1}$$

subject to $p_1 x_1^i + ... + p_M x_M^i = p_1 e_1^i + ... + p_M e_M^i$.

Second: For all $j = 1, ..., M$

$$\sum_{i=1}^{N} \hat{x}_j^i = \sum_{i=1}^{N} e_j^i \tag{2}$$

Competitive Equilibrium: $N \times M$ economy II

Definition

$(\hat{\mathbf{x}}; \mathbf{p})$, i.e., $(\hat{\mathbf{x}}^1, ..., \hat{\mathbf{x}}^N; \mathbf{p})$ is called a Competitive or Walrasian equilibrium, if $(\hat{\mathbf{x}}^i, \mathbf{p})$ together satisfy (1) and (2) simultaneously, for all $i = 1, ..., N$.

Definition

The set of Walrasian/Competitive Equilibria, $W(u^i(.), \mathbf{e}^i)_{N\times M}$, is given by

$$W(u^i(.), \mathbf{e}^i)_{N\times M} = \{\mathbf{x} = (\mathbf{x}^1, ..., \mathbf{x}^N) \mid \exists \mathbf{p} \text{ such that } (\mathbf{x}^i, \mathbf{p}) \text{ satisfy (1) and (2), } \}$$

simultaneously, for all $i = 1, ..., N$.

Competitive Equilibrium: $N \times M$ economy III

Remark

We will show that:

- Walrasian/Competitive equilibrium may not exist. However,
- If utilities fns are continuous, strictly increasing and strictly quasi-concave, there does exist at least one equilibrium.
- In general there can be more than one Competitive equilibrium.
- Walrasian/Competitive equilibrium depends on the vector of initial endowments, i.e., $\mathbf{e}$.

Some Observations I

Let $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, ..., \hat{\mathbf{x}}^N)$ be a Competitive equilibrium allocation.

Proposition

Suppose, $(\hat{\mathbf{x}}, \mathbf{p})$ is a competitive equilibrium. Then, $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, ..., \hat{\mathbf{x}}^N)$ is a feasible allocation.

Proposition

Suppose, $(\hat{\mathbf{x}}, \mathbf{p})$ is a competitive equilibrium. Take a bundle $\mathbf{y}^i$. If $u^i(\mathbf{y}^i) > u^i(\hat{\mathbf{x}}^i)$, then $\mathbf{p}.\mathbf{y}^i > \mathbf{p}.\mathbf{e}^i$. Formally,

$$
\begin{aligned}
u^i(\mathbf{y}^i) > u^i(\hat{\mathbf{x}}^i) \quad &\Rightarrow \quad \mathbf{p}.\mathbf{y}^i > \mathbf{p}.\mathbf{e}^i \\
u^i(\mathbf{y}^i) > u^i(\hat{\mathbf{x}}^i) \quad &\Rightarrow \quad \left[\sum_{j=1}^{J} p_j y_j^i > \sum_{j=1}^{J} p_j e_j^i\right]
\end{aligned}
$$

Some Observations II

Proposition

Suppose, $(\hat{\mathbf{x}}, \mathbf{p})$ *is a competitive equilibrium, and the individual preferences are monotonic, i.e.,* u^i *is increasing. Take a bundle* $\mathbf{y}^i$*. If* $u^i(\mathbf{y}^i) \geq u^i(\hat{\mathbf{x}}^i)$*, then* $\mathbf{p}.\mathbf{y}^i \geq \mathbf{p}.\mathbf{e}^i$*. Formally,*

$$\begin{aligned} u^i(\mathbf{y}^i) \geq u^i(\hat{\mathbf{x}}^i) &\Rightarrow \mathbf{p}.\mathbf{y}^i \geq \mathbf{p}.\mathbf{e}^i \text{ i.e.,} \\ \mathbf{p}.\mathbf{y}^i < \mathbf{p}.\mathbf{e}^i &\Rightarrow u^i(\mathbf{y}^i) < u^i(\hat{\mathbf{x}}^i) \end{aligned}$$

Competitive Equilibrium and Core I

Let

- $W(u^i(.), \mathbf{e}^i)_{N\times M}$ denote the set of Walrasian/competitive allocations.
- $C(u^i(.), \mathbf{e}^i)_{N\times M}$ denote the set of Core allocations.

For a 2×2 economy, suppose an allocation $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ along with a price vector $\mathbf{p} = (p_1, p_2)$ is competitive equilibrium. Then,

- Individual i prefers ($\hat{\mathbf{x}}^i$ at least as much as $\mathbf{e}^i$
- Indifference curves of the individuals are tangent to each other
- Allocation $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ is Pareto Optimum
- In view of the above, allocation $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ is in the Core.

Competitive Equilibrium and Core II

So, for a 2×2 economy,

$$\mathbf{x} \in W(u^i(.), \mathbf{e}^i) \Rightarrow \mathbf{x} \in C(u^i(.), \mathbf{e}^i).$$

Theorem

Consider an exchange economy $(u^i(.), \mathbf{e}^i)_{N \times M}$, where individual preferences are monotonic, i.e., u^i is increasing. If $\mathbf{x}$ is a WEA, then $\mathbf{x} \in C(u^i(.), \mathbf{e}^i)_{N \times M}$. Formally,

$$W(u^i(.), \mathbf{e}^i)_{N \times M} \subseteq C(u^i(.), \mathbf{e}^i)_{N \times M}.$$

Proof: Take any WEA, say $\mathbf{x}$. Let, $\mathbf{x}$ along with the price vector $\mathbf{p}$ be a WE. Suppose

$$\mathbf{x} \notin C(\mathbf{e}).$$

Therefore, there exists a 'blocking coalition' against $\mathbf{x}$. That is,

Competitive Equilibrium and Core III

there exists a set $S \subseteq N$ and an 'allocation' say $\mathbf{y}$, s.t.

$$\sum_{i \in S} \mathbf{y}^i = \sum_{i \in S} \mathbf{e}^i \tag{3}$$

Moreover,

$$u^i(\mathbf{y}^i) \geq u^i(\mathbf{x}^i) \text{ for all } i \in S \tag{4}$$

and for some $i' \in S$

$$u^i(\mathbf{y}^{i'}) > u^i(\mathbf{x}^{i'}). \tag{5}$$

(3) implies

$$\mathbf{p}.\sum_{i \in S} \mathbf{y}^i = \mathbf{p}.\sum_{i \in S} \mathbf{e}^i \tag{6}$$

Competitive Equilibrium and Core IV

(4) implies

$$\mathbf{p}.\mathbf{y}^i \geq \mathbf{p}.\mathbf{x}^i = \mathbf{p}.\mathbf{e}^i, \text{ for all } i \in S \tag{7}$$

(5) implies: for some $i' \in S$

$$\mathbf{p}.\mathbf{y}^{i'} > \mathbf{p}.\mathbf{x}^{i'} = \mathbf{p}.\mathbf{e}^{i'}. \tag{8}$$

(7) and (8) together give us:

$$\mathbf{p}.\sum_{i \in S} \mathbf{y}^i > \mathbf{p}.\sum_{i \in S} \mathbf{e}^i \tag{9}$$

But, (6) and (9) are mutually contradictory. Therefore,

$$\mathbf{x} \in C(\mathbf{e}).$$

Competitive Equilibrium and Pareto Optimality

So, we have proved the following:

Theorem

Consider an exchange economy $(u^i, \mathbf{e}^i)_{i \in \{1,..,N\}}$, where u^i is strictly increasing, for all $i = 1, .., N$.

Every WEA is Pareto optimum.

Competitive Equilibrium: Merits and Demerits

Question

- *Is the price/market economy better than the barter economy, in terms of its functioning?*
- *Is the price/market economy better than the barter economy, in terms of the outcome achieved?*

Question

- *What are the limitations of a market economy?*
- *Can these limitations be overcome?*