Problem Set 4

1. A sequence of real numbers $\{x_k\}_{k=1}^{\infty} \to \infty$, if for every positive integer M, we can find a N such that for all $n \geq N$, $x_n > M$. Show that
(a) $\lim_{x\to 0} \frac{1}{x} = \infty$
(b) $\lim_{x\to \infty} \frac{1}{x} = 0$

2. f, g, and h are functions from $\mathcal{R}$ to $\mathcal{R}$. Suppose that $f(x) \leq g(x) \leq h(x)$ for all $x \neq p$ in some neighbourhood of p. Suppose also that $\lim_{x\to p} f(x) = \lim_{x\to p} h(x) = M$. Show that $\lim_{x\to p} g(x) = M$

3. If n is a positive integer, $a \in \mathcal{R}$ and $a > 0$ then show that there is exactly one positive b such that $b^n = a$.

4. Assume $f : \mathcal{R} \to \mathcal{R}$ is a continuous function. Show that $E^0 = \{x \in \mathcal{R} \mid f(x) = 0\}$ is a closed set. How about $E^+ = \{x \in \mathcal{R} \mid f(x) > 0\}$?

5. Answer the following questions for each function provided below:
(a) Is this function continuous? If not, identify all points where it is not continuous.
(b) Does this function have partial derivatives? If not, identify all points where it does not have partial derivative. Compute partial derivates where they exist.
(c) Is this function differentiable? If not, identify all points where it is not differentiable.
(i) $f(x_1, x_2) = \max\{x_1, x_2\}$
(ii) $f(x_1, x_2) = \frac{x_1, x_2}{(x_1^2 + x_2^2)}$ for all $(x_1, x_2) \neq (0, 0)$ and $f(0, 0) = 0$
(iii) $f(x_1, x_2, x_3) = x_1 x_2^2 x_3^3$

6. Sundaram:
Chapter 1: 46,47,57
Chapter 3: 10,13,15