Market Equilibrium and the Core

Ram Singh

Lecture 4

Competitive Equilibrium and Core: 2×2 Economy I

Assume 'well-behaved' utilities. In that case,

- at the equilibrium allocation, $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$, the ICs are tangent to each other
- Therefore, the equilibrium allocation $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ is Pareto Optimum.

Question

Suppose, $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ is a Competitive (market) equilibrium allocation

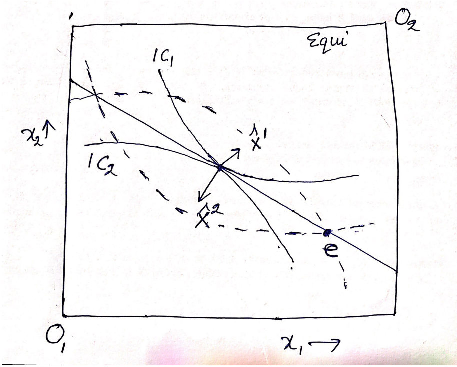
- *Are unilateral deviations from $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ profitable?*
- *Can a subgroup profitably deviate from $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$?*
- *Does the eq. allocation $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ belong to the core?*

Competitive Equilibrium and Core: 2×2 Economy II

For a 2×2 economy, suppose an allocation $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ along with a price vector $\mathbf{p} = (p_1, p_2)$ is competitive equilibrium. Then,

- Individual i prefers $\mathbf{x}^i$ at least as much as $\mathbf{e}^i$
- Indifference curves of the individuals are tangent to each other
- Allocation $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ is Pareto Optimum
- In view of the above, allocation $\hat{\mathbf{x}} = (\hat{\mathbf{x}}^1, \hat{\mathbf{x}}^2)$ is in the Core.

Competitive Equilibrium and Core: 2 $\times$ 2 Economy



Competitive Equilibrium and Core I

Let

- $W(u^i(.), \mathbf{e}^i)_{N\times M}$ denote the set of Walrasian/competitive allocations.
- $C(u^i(.), \mathbf{e}^i)_{N\times M}$ denote the set of Core allocations.

We know that for a 2×2 economy,

$$\mathbf{x} \in W(u^i(.), \mathbf{e}^i) \Rightarrow \mathbf{x} \in C(u^i(.), \mathbf{e}^i).$$

Theorem

Consider an exchange economy $(u^i(.), \mathbf{e}^i)_{N\times M}$, where individual preferences are monotonic, i.e., u^i is increasing. If $\mathbf{x}$ is a WEA, then $\mathbf{x} \in C(u^i(.), \mathbf{e}^i)_{N\times M}$. Formally,

$$W(u^i(.), \mathbf{e}^i)_{N\times M} \subseteq C(u^i(.), \mathbf{e}^i)_{N\times M}.$$

Competitive Equilibrium and Core II

Proof: Take any $\mathbf{x}$ WEA. Let, $\mathbf{x}$ along with the price vector $\mathbf{p}$ be a WE. Suppose

$$\mathbf{x} \notin C(\mathbf{e}).$$

Therefore, there exists a 'blocking coalition' against $\mathbf{x}$. That is, there exists a set $S \subseteq N$ and an 'allocation' say $\mathbf{y}$, s.t.

$$\sum_{i \in S} \mathbf{y}^i = \sum_{i \in S} \mathbf{e}^i \tag{1}$$

Moreover,

$$u^i(\mathbf{y}^i) \geq u^i(\mathbf{x}^i) \text{ for all } i \in S \tag{2}$$

and for some $i' \in S$

$$u^i(\mathbf{y}^{i'}) > u^i(\mathbf{x}^{i'}). \tag{3}$$

(1) implies

$$\mathbf{p}.\sum_{i \in S} \mathbf{y}^i = \mathbf{p}.\sum_{i \in S} \mathbf{e}^i \tag{4}$$

Competitive Equilibrium and Core III

(2) implies

$$\mathbf{p}.\mathbf{y}^i \geq \mathbf{p}.\mathbf{x}^i = \mathbf{p}.\mathbf{e}^i, \text{ for all } i \in S \tag{5}$$

(3) implies: for some $i' \in S$

$$\mathbf{p}.\mathbf{y}^{i'} > \mathbf{p}.\mathbf{x}^{i'} = \mathbf{p}.\mathbf{e}^{i'}. \tag{6}$$

(5) and (6) together give us:

$$\mathbf{p}.\sum_{i \in S} \mathbf{y}^i > \mathbf{p}.\sum_{i \in S} \mathbf{e}^i \tag{7}$$

But, (4) and (7) are mutually contradictory. Therefore,

$$\mathbf{x} \in C(\mathbf{e}).$$

Competitive Equilibrium and Pareto Optimality

So, we have proved the First Fundamental Theorem of Welfare Economics:

Theorem

Consider an exchange economy $(u^i, \mathbf{e}^i)_{i \in \{1,..,N\}}$*, where* u^i *is strictly increasing, for all* $i = 1, .., N$*.*

Every WEA is Pareto optimum.

Competitive Equilibrium: Merits and Demerits

Question

- *Is the price/market economy better than the barter economy, in terms of its functioning?*
- *Is the price/market economy better than the barter economy, in terms of the outcome achieved?*

Question

- *What are the limitations of a market economy?*
- *Can these limitations be overcome?*