Market Equilibrium and the Core

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Lecture 4
Assume ‘well-behaved’ utilities. In that case,

- at the equilibrium allocation, \( \hat{x} = (\hat{x}^1, \hat{x}^2) \),
  the ICs are tangent to each other
- Therefore, the equilibrium allocation \( \hat{x} = (\hat{x}^1, \hat{x}^2) \) is Pareto Optimum.

**Question**

*Suppose, \( \hat{x} = (\hat{x}^1, \hat{x}^2) \) is a Competitive (market) equilibrium allocation*

- Are unilateral deviations from \( \hat{x} = (\hat{x}^1, \hat{x}^2) \) profitable?
- Can a subgroup profitably deviate from \( \hat{x} = (\hat{x}^1, \hat{x}^2) \)?
- Does the eq. allocation \( \hat{x} = (\hat{x}^1, \hat{x}^2) \) belong to the core?
For a $2 \times 2$ economy, suppose an allocation $\hat{x} = (\hat{x}^1, \hat{x}^2)$ along with a price vector $p = (p_1, p_2)$ is competitive equilibrium. Then,

- Individual $i$ prefers $x^i$ at least as much as $e^i$
- Indifference curves of the individuals are tangent to each other
- Allocation $\hat{x} = (\hat{x}^1, \hat{x}^2)$ is Pareto Optimum
- In view of the above, allocation $\hat{x} = (\hat{x}^1, \hat{x}^2)$ is in the Core.
Competitive Equilibrium and Core: $2 \times 2$ Economy
Competitive Equilibrium and Core I

Let

- \( W(u^i(.), e^i)_{N \times M} \) denote the set of Walrasian/competitive allocations.
- \( C(u^i(.), e^i)_{N \times M} \) denote the set of Core allocations.

We know that for a \( 2 \times 2 \) economy,

\[
  x \in W(u^i(.), e^i) \Rightarrow x \in C(u^i(.), e^i).
\]

Theorem

Consider an exchange economy \( (u^i(.), e^i)_{N \times M} \), where individual preferences are monotonic, i.e., \( u^i \) is increasing. If \( x \) is a WEA, then \( x \in C(u^i(.), e^i)_{N \times M} \).

Formally,

\[
  W(u^i(.), e^i)_{N \times M} \subseteq C(u^i(.), e^i)_{N \times M}.
\]
Proof: Take any \( x \) WEA. Let, \( x \) along with the price vector \( p \) be a WE. Suppose 
\[ x \notin C(e). \]

Therefore, there exists a ‘blocking coalition’ against \( x \). That is, there exists a set \( S \subseteq N \) and an ’allocation’ say \( y \), s.t. 
\[ \sum_{i \in S} y^i = \sum_{i \in S} e^i \] (1)

Moreover, 
\[ u^i(y^i) \geq u^i(x^i) \] for all \( i \in S \) (2)

and for some \( i' \in S \)
\[ u^i(y^{i''}) > u^i(x^{i'}) \] (3)

(1) implies 
\[ p \cdot \sum_{i \in S} y^i = p \cdot \sum_{i \in S} e^i \] (4)
(2) implies
\[ p.y^i \geq p.x^i = p.e^i, \quad \text{for all } i \in S \] (5)

(3) implies: for some \( i' \in S \)
\[ p.y^{i'} > p.x^{i'} = p.e^{i'}. \] (6)

(5) and (6) together give us:
\[ p. \sum_{i \in S} y^i > p. \sum_{i \in S} e^i \] (7)

But, (4) and (7) are mutually contradictory. Therefore,
\[ x \in C(e). \]
So, we have proved the First Fundamental Theorem of Welfare Economics:

Theorem

Consider an exchange economy \((u^i, e^i)_{i \in \{1, \ldots, N\}}\), where \(u^i\) is strictly increasing, for all \(i = 1, \ldots, N\).

Every WEA is Pareto optimum.
Competitive Equilibrium: Merits and Demerits

Question

- Is the price/market economy better than the barter economy, in terms of its functioning?
- Is the price/market economy better than the barter economy, in terms of the outcome achieved?

Question

- What are the limitations of a market economy?
- Can these limitations be overcome?