Market Outcomes: Efficient or Fair?

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Microeconomic Theory

Lecture 14

Fair Versus Efficient

Question

1. *What is a fair allocation?*
2. *Is a ‘fair’ allocation also an efficient allocation?*
3. *Is a ‘fair’ allocation Pareto efficient?*
4. *Can an allocation be fair as well as efficient?*
5. *Can competitive market lead to fair outcome?*
6. *How to choose from the set of efficient alternatives?*

Fairness: Two Definitions

Consider a $N \times M$ pure exchange economy. Let,

- $\mathbb{N}$ be the set of individuals.
- $(\mathbf{e}^1, ..., \mathbf{e}^N)$ be the vector of initial endowments.

Definition

Allocation $\mathbf{x} = (\mathbf{x}^1, ..., \mathbf{x}^N)$ is 'fair' if it is equal division of endowments, i.e., if

$$(\forall i, j \in \mathbb{N}) \left[\mathbf{x}^i = \mathbf{x}^j = \frac{\sum_{i=1}^{N} \mathbf{e}^i}{N} \right].$$

Definition

Allocation $\mathbf{x} = (\mathbf{x}^1, ..., \mathbf{x}^N)$ is 'fair' if it is Non-envious/envy-free. That is, if

$$\begin{aligned} (\forall i, j \in \mathbb{N})[\mathbf{x}^i \quad & R_i \quad \mathbf{x}^j], \ i.e., \\ (\forall i, j \in \mathbb{N})[u^i(\mathbf{x}^i) \quad & \geq \quad u^i(\mathbf{x}^j)]. \end{aligned}$$

Fair Vs Pareto Efficient

Question

1. *Are the above definitions equivalent to each other?*
2. *Is an 'Equal division' allocation 'Non-envious' ?*
3. *Is a Non-envious allocation also an Equal division allocation?*
4. *Is an 'Equal division' allocation Pareto efficient?*
5. *Is a Non-envious allocation Pareto efficient?*
6. *Between a Non-envious allocation and a Pareto efficient allocation, which one is socially desirable?*

Fair and Efficient I

Question

Is fair (equal) division of endowments a P.O allocation?

Consider a 2×2 pure exchange economy:

- The goods are; x and y.
- $u^1(.) = x^{\alpha}.y^{1-\alpha}$ and $u^2(.) = x^{\beta}.y^{1-\beta}$, and $\alpha = \beta = \frac{1}{2}$
- the total initial endowment vector is $(\bar{x}, \bar{y}) >> (0, 0)$.

Let

- x_1 and x_2 denote the amounts of good x allocated to individuals 1 and 2, resp. Let $x_1 = x_2 = \frac{\bar{x}}{2}$
- y_1 and y_2 denote the amounts of good y allocated to individuals 1 and 2, resp. Let $y_1 = y_2 = \frac{\bar{y}}{2}$

Fair and Efficient II

Clearly

$$\begin{aligned} \text{MRS}^1 &= \frac{y_1}{x_1} \\ \text{MRS}^2 &= \frac{y_2}{x_2} = \frac{\bar{y} - y_1}{\bar{x} - x_1} \end{aligned}$$

So the set of PO allocations is solution to

$$\frac{y_1}{x_1} = \frac{y_2}{x_2} = \frac{\bar{y} - y_1}{\bar{x} - x_1}$$

But,

$$\left(\frac{y_1}{x_1} = \frac{\bar{y} - y_1}{\bar{x} - x_1}\right) \Rightarrow \left(\frac{y_1}{x_1} = \frac{\bar{y}}{\bar{x}}\right)$$

Fair But Not Efficient

Consider a 3×3 exchange economy. Let

- $u^1(x, y, z) = 3x_1 + 2y_1 + z_1$
- $u^2(x, y, z) = 2x_2 + y_2 + 3z_2$
- $u^3(x, y, z) = x_1 + 3y_1 + 2z_1$
- $\mathbf{e}^1 = \mathbf{e}^2 = \mathbf{e}^3 = (1, 1, 1)$. So, $u^1(\mathbf{e}^1) = u^2(\mathbf{e}^2) = u^3(\mathbf{e}^3) = 6$.

Consider an allocation $\mathbf{y} = (\mathbf{y}^1, \mathbf{y}^2, \mathbf{y}^3)$, where

$$\mathbf{y}^1 = (3, \frac{2}{3}, 0), /\mathbf{y}^2 = (0, 0, 2), \ \ \mathbf{y}^3 = (0, \frac{7}{3}, 1).$$

- $\mathbf{y} = (\mathbf{y}^1, \mathbf{y}^2, \mathbf{y}^3)$ is Pareto efficient.
- However, $u^1(\mathbf{y}^1) = 31/3$, $u^2(\mathbf{y}^2) = 6$ and $u^3(\mathbf{y}^3) = 9$,
- So, $\mathbf{y} = (\mathbf{y}^1, \mathbf{y}^2, \mathbf{y}^3)$ is efficient but not fair on this criterion.

Fairness under Markets I

Proposition

When preferences are strongly monotonic and initial allocation is 'Equal', the competitive equilibrium is fair (Envy-free)

Let

- $\mathbf{e}^1 = \mathbf{e}^2 = ... = \mathbf{e}^N$ be the initial endowment vectors
- $(\mathbf{x}^{*1}, ..., \mathbf{x}^{*N})$ is a Walrasian equilibrium allocation.
- $\mathbf{p}^*$ be the associated equilibrium price vector

Suppose, at $(\mathbf{x}^{*1}, ..., \mathbf{x}^{*N})$, some individual i envy another person j, i.e.,

$$(\exists i, j \in \{1, ..., n\})[u^i(\mathbf{x}^{*i}) < u^i(\mathbf{x}^{*j})].$$

Note: $(\mathbf{x}^{*1}, ..., \mathbf{x}^{*N})$ is a WE implies that

- person i can afford and demands $\mathbf{x}^{*i}$

Fairness under Markets II

- person j can afford and demands $\mathbf{x}^{*j}$
- but, purchasing power of i is the same as that of j
- so, person i can afford more preferred bundle $\mathbf{x}^{*j}$

This means that $(\mathbf{x}^{*1}, ..., \mathbf{x}^{*N})$ cannot be a WEA, a contradiction.
So, under a WE the following holds.

$$(\forall i, j \in \{1, ..., n\})[u^i(\mathbf{x}^{*i}) \geq u^i(\mathbf{x}^{*j})].$$

Pareto Criterion I

Let

- $\mathbb{N}$ be the set of individuals.
- $\mathbb{S}$ be the set of feasible alternatives.
- u^i utility fn for i the individual
- $\mathbf{x} = (\mathbf{x}^1, ..., \mathbf{x}^N) \in \mathbb{S}$ be an arbitrary allocation in $\mathbb{S}$
- $\mathbb{U}$ be the set of possible utilities

$$\mathbb{U} = \{(u^1(\mathbf{x}^1), ..., u^N(\mathbf{x}^N)) | \mathbf{x} = (\mathbf{x}^1, ..., \mathbf{x}^N) \in \mathbb{S}\}$$

Pareto Criterion II

Definition

Take any $\mathbf{x} = (\mathbf{x}^1, ..., \mathbf{x}^N)$, and $\mathbf{y} = (\mathbf{y}^1, ..., \mathbf{y}^N)$, $\mathbf{x}, \mathbf{y} \in \mathbb{S}$.
Suppose $\mathbf{x}$ is ‘Pareto as goods as’ $\mathbf{y}$, i.e., $\mathbf{x}\mathcal{R}\mathbf{y}$ if

$$(\forall i \in \mathbb{N})[\mathbf{x}R_i\mathbf{y}]$$

Definition

$\mathbf{x}$ is Pareto superior to $\mathbf{y}$, i.e., $\mathbf{x}\mathcal{P}\mathbf{y}$: if $\mathbf{x}\mathcal{R}\mathbf{y}$ but $\sim \mathbf{y}\mathcal{R}\mathbf{x}$. That is,

$$(\forall i \in \mathbb{N})[\mathbf{x}R_i\mathbf{y}]$$
$$(\exists j \in \mathbb{N})[\sim \mathbf{y}R_j\mathbf{x}]$$

- As a preference relation, is ‘Pareto-superior’ a complete relation?
- As a preference relation, is ‘Pareto-as good as’ a complete relation?

Rawls Criterion: Egalitarian World I

Consider

$$\mathbf{x} = (50, 100, 150) \quad , i.e., \quad \sum_{i=1}^{3} x^i = 300$$

$$\mathbf{y} = (90, 90, 90) \quad , i.e., \quad \sum_{i=1}^{3} y^i = 270$$

$$\mathbf{z} = (80, 250, 250) \quad , i.e., \quad \sum_{i=1}^{3} z^i = 580$$

1. Which of the above alternatives is socially desirable?
2. Is an Equal division allocation Pareto Efficient?
3. Is an Equal division a Rawls Best allocation?

Rawls Criterion: Egalitarian World II

Veil of ignorance:

- Consider various possible distributions of one good, say wealth, across N individuals.
- Assume individual preferences are monotonic in the good

Distribution $\mathbf{x} = (x^1, ..., x^N)$ is Rawls superior to distribution $\mathbf{y} = (y^1, ..., y^N)$ if

$$\min_{i \in \mathbb{N}} \{x^1, ..., x^N\} > \min_{i \in \mathbb{N}} \{y^1, ..., y^N\}$$

The Difference Principle:

Proposition

Let $\sum_{i=1}^{N} e^i = C$. *Distribution* $\mathbf{x} = (x^1, ..., x^N)$ *is Rawls Best if*

$$\{x^1, ..., x^N\} = \min\{\frac{C}{N}, ..., \frac{C}{N}\}$$

Rawls Criterion: Egalitarian World III

In general, suppose endowments are multi-dimensional, i.e., an allocation

$$\mathbf{x} = (\mathbf{x}^1, ..., \mathbf{x}^N)$$

where for $i = 1, ..., N$,

$$\mathbf{x}^i = (x_1^i, ..., x_M^i)$$

Definition

Distribution $\mathbf{x} = (\mathbf{x}^1, ..., \mathbf{x}^N)$ is Rawls superior to distribution $\mathbf{y} = (\mathbf{y}^1, ..., \mathbf{y}^N)$ if

$$\text{minimum}\{u^1(\mathbf{x}^1), ..., u^N(\mathbf{x}^N)\} > \text{minimum}\{u^1(\mathbf{y}^1), ..., u^N(\mathbf{y}^N)\}$$

Rawls' Criterion and Markets

Question

Suppose we start from a Rawls Best allocation as the endowment. Will competitive equilibrium allocation be egalitarian?

Proposition

When preferences are strongly monotonic and initial allocation is Rawls Best, the competitive equilibrium is non-envious and Pareto efficient.

Rawls Criterion: Limitations

In real world

- individual welfare has several components; $u^i(\mathbf{x}^i)$, where $\mathbf{x}^i$ has several components
- Implications for policy interventions are complex
- Individuals have different beliefs about desirability of the possible outcomes.

For example, consider m goods some of which are legal, economic and social entitlements.

Even under the **Veil of ignorance** person 1 may feel

$$\text{minimum}\{u^1(\mathbf{x}^1), ..., u^1(\mathbf{x}^N) > \text{minimum}\{u^1(\mathbf{y}^1), ..., u^1(\mathbf{y}^N)\}$$

But, person 2 may have

$$\text{minimum}\{u^2(\mathbf{x}^1), ..., u^2(\mathbf{x}^N) < \text{minimum}\{u^2(\mathbf{y}^1), ..., u^2(\mathbf{y}^N)\}$$