Efficiency Criteria in Economics

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Microeconomic Theory

Lecture 15

Efficiency as Social Choice Criterion

Question

- *How to choose from the feasible set of alternatives?*
- *Do societies have preference relations similar to the ones assumed for individuals?*
- *Is Pareto criterion helpful here?*
- *What are the other approaches possible in a social context?*

The Setting

Let

- $\mathbb{N}$ be the set of individuals.
- $\mathbb{S}$ be the set of feasible alternatives.
- u^i utility fn for i the individual
- $\mathbb{U}$ be the set of possible utilities

$$\mathbb{U} = \{(u^1(\mathbf{x}), ..., u^n(\mathbf{x})) | \mathbf{x} \in \mathbb{S}\}$$

Remark: For an N individuals and M goods economy, let $\mathbf{e} = (\mathbf{e}^1, ..., \mathbf{e}^N)$ be the endowment vector. We had assumed that the set of feasible allocations is

$$\mathbf{F}(\mathbf{e}) = \{\mathbf{x} = (\mathbf{x}^1, ..., \mathbf{x}^N) | \sum_{i=1}^{N} x_j^i = \sum_{i=1}^{N} e_j^i \text{ for all j=1,...,M}\}$$

In general, $\mathbb{S}$ is different from $\mathbf{F}(\mathbf{e})$.

Example I

- There are 50 fisheries located on a water stream. A factory has come up upstream.
- The factory discharges pollutants (chemicals) in the water stream.
- The polluted water is bad for fisheries - the chemicals are injurious to health of fish.
- In the absence of any corrective measure, fisheries will suffer a harm of 10 each, that is, a total harm of 500.
- Factory generates a net profit of 600

Example II

Consider the following Scenarios:

- Scenario 1: The environmental regulation does not allow the factory to operate at all
- Scenario 2: The factory operates without any obligations to compensate the loss caused
- Scenario 3: The factory operates but has to compensate the fisheries for the loss caused.

Questions:

- Which alternative is/are Pareto optimum
- Which alternative will maximize the net social gains?

Example III

Suppose, the following corrective measure is available:

- A chemical treatment device can be installed at the factory at a cost of 150

Answer the above questions.

Note

- Scenario 3 is Pareto superior to Scenario 1
- Scenario 2 is (potentially) Pareto superior to Scenario 1

Kaldor-Hicks Efficiency

- Deals uses **potential** Pareto superiority as criterion
- Provides basis for wealth maximization criterion
- Scenario 2 is (potentially) Pareto superior to Scenario 1
- Scenario 2 is **Kaldor-Hicks** superior to Scenario 1

Kaldor-Hicks Criterion I

Let

- $\mathbf{x}$ and $\mathbf{y}$ be any two allocations
- $\mathbb{S}(\mathbf{x})$ be the set of allocations that are accessible from $\mathbf{x}$.
- $\mathbb{S}(\mathbf{y})$ be the set of allocations that are accessible from $\mathbf{y}$.

Definition

$\mathbf{x}$ is Kaldor superior to $\mathbf{y}$, i.e., $\mathbf{x}\mathcal{K}\mathbf{y}$ if there exists $\mathbf{z} \in \mathbb{S}(\mathbf{x})$ such that $\mathbf{z}\mathcal{P}\mathbf{y}$

$$(\forall i \in \mathbb{N})[\mathbf{z}R_i\mathbf{y}]$$
$$(\exists j \in \mathbb{N})[\mathbf{z}P_j\mathbf{y}]$$

However, it is possible that

$$\mathbf{x}\mathcal{K}\mathbf{y} \text{ and } \mathbf{y}\mathcal{K}\mathbf{x}.$$

Scitovsky Criterion I

Definition

x is Scitovsky superior to **y**, i.e., $\mathbf{x}\mathcal{S}\mathbf{y}$ if

$$\begin{aligned} \mathbf{x}\mathcal{K}\mathbf{y} \quad & \text{but} \\ \sim \mathbf{y}\mathcal{K}\mathbf{x} & \end{aligned}$$

$\mathbf{x}\mathcal{K}\mathbf{y}$ implies there exists $\mathbf{z} \in \mathbb{S}(\mathbf{x})$ such that $\mathbf{z}\mathcal{P}\mathbf{y}$. That is,

$$(\forall i \in \mathbb{N})[\mathbf{z}R_i\mathbf{y}]$$
$$(\exists j \in \mathbb{N})[\mathbf{z}P_j\mathbf{y}]$$

But, $\sim \mathbf{y}\mathcal{K}\mathbf{x}$ means that there should not exist any $\mathbf{t} \in \mathbb{S}(\mathbf{y})$ such that

$$(\forall i \in \mathbb{N})[\mathbf{t}R_i\mathbf{x}]$$
$$(\exists j \in \mathbb{N})[\mathbf{t}P_j\mathbf{x}]$$

Scitovsky Criterion II

Definition

All social states/alternatives are accessible from each other if

$$(\forall \mathbf{x}, \mathbf{y}, \mathbf{z})[\mathbf{z} \in \mathbb{S}(\mathbf{x}) \Rightarrow \mathbf{z} \in \mathbb{S}(\mathbf{y})]$$

Proposition

If all social states/alternatives are accessible from each other then $\mathbf{x}\mathcal{S}\mathbf{y}$ *if and only if* $\mathbf{x}$ *is P.O but* $\mathbf{y}$ *is not P.O*

Proposition

If all social states/alternatives are accessible from each other then $\mathbf{x}\mathcal{K}\mathbf{y}$ *if and only if* $\mathbf{y}$ *is not P.O.*

Samuelson Criterion

Definition

$\mathbf{x}$ is Samuelson superior to $\mathbf{y}$, i.e., $\mathbf{x}\bar{\mathcal{S}}\mathbf{y}$ if for any $\mathbf{z} \in \mathbb{S}(\mathbf{y})$

$$\mathbf{x}\mathcal{K}\mathbf{z}$$

That is, for any $\mathbf{z} \in \mathbb{S}(\mathbf{y})$, there exists $\mathbf{w} \in \mathbb{S}(\mathbf{x})$ such that $\mathbf{w}\mathcal{P}\mathbf{z}$, i.e.,

$$(\forall i \in \mathbb{N})[\mathbf{w}R_i\mathbf{z}]$$
$$(\exists j \in \mathbb{N})[\mathbf{w}P_j\mathbf{z}]$$

Social Choice Criteria: Compared

Consider

$$\mathbf{x} = (50, 100, 150) \quad , i.e., \quad \sum_{i=1}^{3} x^i = 300$$

$$\mathbf{y} = (90, 90, 90) \quad , i.e., \quad \sum_{i=1}^{3} y^i = 270$$

$$\mathbf{z} = (80, 250, 250) \quad , i.e., \quad \sum_{i=1}^{3} z^i = 580$$

Question

Which of the above alternatives is efficient according to Pareto, Rawlsian, Kaldor-Hicks Efficient, Scitovsky and Samuelson criteria?