Competitive Equilibria: Uniqueness and Stability

Ram Singh

Microeconomic Theory

Lecture 7

Questions

- Is Competitive/Walrasian equilibrium unique?
- Why is a unique equilibrium helpful?
- If WE is not unique, how many WE can be there?
- What are the conditions, for a unique WE?
- Do these conditions hold in the real world?
- Is Competitive/Walrasian equilibrium stable?
- Why is stability of an equilibrium important?

Background Readings:

Arrow and Hahn. (1971), Jehle and Reny* (2008), MWG*(1995)

Multiple WE: Example

Example

From MWG*(1995): Two consumers:

- $u^1(.) = x_1^1 - \frac{1}{8}\frac{1}{\left(x_2^1\right)^8}$ and $u^2(.) = -\frac{1}{8}\frac{1}{\left(x_1^2\right)^8} + x_2^2$
- $e^1 = (2, r)$ and $e^2 = (r, 2)$; $r = 2^{\frac{8}{9}} - 2^{\frac{1}{9}}$
- The equilibria are solution to

$$\left(\frac{p_2}{p_1}\right)^{-\frac{1}{9}} + 2 + r\left(\frac{p_2}{p_1}\right) - \left(\frac{p_2}{p_1}\right)^{\frac{8}{9}} = 2 + r, i.e.,$$

there are three equilibria:

$$\frac{p_2}{p_1} = \frac{1}{2}, 1, \text{ and } 2.$$

Unique WE: Conditions I

Consider 2×2 economy:

- Two goods: food and cloth
- Let (p_f, p_c) be the price vector.
- We know that for all $t > 0$: $\mathbf{z}(t\mathbf{p}) = \mathbf{z}(\mathbf{p})$.
- Therefore, we can work with

$$\mathbf{p} = (\frac{p_f}{p_c}, 1) = (p, 1).$$

From Walras's Law, we have $pz_f(\mathbf{p}) + z_c(\mathbf{p}) = 0$. That is,

$$z_f(\mathbf{p}) = 0 \Leftrightarrow z_c(\mathbf{p}) = 0$$

When utility functions are continuous, strongly monotonic and strictly quasi-concave:

Unique WE: Conditions II

- $z_i(\mathbf{p})$ is continuous for all $\mathbf{p} >> \mathbf{p}$, i.e., for all $p > 0$.
- there exists small $p = \epsilon > 0$ s.t. $z_f(\epsilon, 1) > 0$, and
- there exists another $p' > \frac{1}{\epsilon}$ s.t. $z_c(p', 1) > 0$.

Question

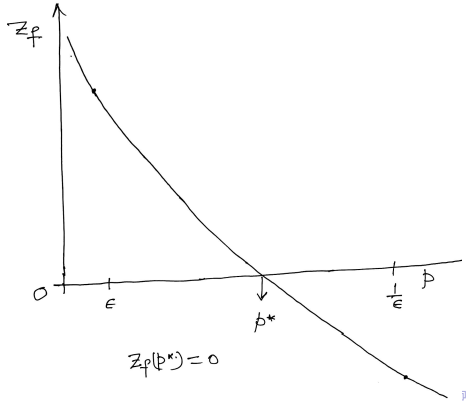
- *Do the above assumptions guarantee unique WE?*
- *Under what conditions the WE be unique?*

An additional assumption can ensure uniqueness of WE:

- $z_f^{'}(\mathbf{p}) < 0$ for all $p > 0$.

Uniqueness of Equilibrium: 2×2 Economy

Do the above assumptions on utility functions ensure $z_f'(\mathbf{p}) < 0$ for all $p > 0$?



Normal Goods and Number of Equilibria I

Let,

- there be two goods - food and cloth.
- $\mathbf{e}^1 = (e^1_f, e^1_c)$ and $\mathbf{e}^2 = (e^2_f, e^2_c)$ be the initial endowment vectors
- $\mathbf{p} = (p, 1)$ be a price vector.

Assumption: Assume utility functions to be

- continuous, strongly monotonic and strictly quasi-concave

From Walras Law we have $\mathbf{p}.\mathbf{z}(\mathbf{p}) = 0$, i.e.,

$$pz_f(\mathbf{p}) + z_c(\mathbf{p}) = 0.$$

By definition:

$$\begin{aligned} z_f(\mathbf{p}) &= z^1_f(\mathbf{p}) + z^2_f(\mathbf{p}) \\ &= [x^1_f(\mathbf{p}) - e^1_f] + [x^2_f(\mathbf{p}) - e^2_f] \end{aligned}$$

Normal Goods and Number of Equilibria II

- Let $\mathbf{p}^*$ denote an equilibrium price vector.
- We know that for the above economy at least one $\mathbf{p}^*$ exists. Why?

Let

- $I^1(\mathbf{p}, \mathbf{e}^1) = \mathbf{p}.\mathbf{e}^1 = pe_f^1 + e_c^1$
- $I^2(\mathbf{p}, \mathbf{e}^2) = \mathbf{p}.\mathbf{e}^2 = pe_f^2 + e_c^2$

Note

$$\underbrace{\frac{dz_f(\mathbf{p})}{dp}}_{Price\ effect\ (total)} = \underbrace{\frac{dz_f^1(\mathbf{p})}{dp}}_{Price\ effect\ (total)} + \underbrace{\frac{dz_f^2(\mathbf{p})}{dp}}_{Price\ effect\ (total)}$$

Normal Goods and Number of Equilibria III

Since endowments are fixed, we get

$$\underbrace{\frac{dz_f(\mathbf{p})}{dp}}_{Price\ \ effect\ \ (total)} = \left(\frac{\partial x_f^1(\mathbf{p})}{\partial p}\right)_{du^1=0} - (x_f^1(\mathbf{p}) - e_f^1)\left(\frac{\partial x_f^1(\mathbf{p})}{\partial I^1}\right)$$
$$+ \left(\frac{\partial x_f^2(\mathbf{p})}{\partial p}\right)_{du^2=0} - (x_f^2(\mathbf{p}) - e_f^2)\left(\frac{\partial x_f^2(\mathbf{p})}{\partial I^2}\right) \qquad (1)$$

Normal Goods and Number of Equilibria IV

WLOG assume that

- in equilibrium (at $\mathbf{p}^*$). Person 1 is net buyer of food; i.e., $x_f^1(\mathbf{p}^*) - e_f^1 > 0$.
- In equi. (food) market clears. So,

$$x_f^2(\mathbf{p}^*) - e_f^2 = -[x_f^1(\mathbf{p}^*) - e_f^1].$$

At equilibrium price, $\mathbf{p}^*$, we have

$$\begin{aligned} \frac{\partial z_f(\mathbf{p}^*)}{\partial p} &= \left(\frac{\partial x_f^1(\mathbf{p}^*)}{\partial p}\right)_{du^1=0} - (x_f^1(\mathbf{p}^*) - e_f^1)\left(\frac{\partial x_f^1(\mathbf{p}^*)}{\partial I^1}\right) \\ &+ \left(\frac{\partial x_f^2(\mathbf{p}^*)}{\partial p}\right)_{du^2=0} - (x_f^2(\mathbf{p}^*) - e_f^2)\left(\frac{\partial x_f^2(\mathbf{p}^*)}{\partial I^2}\right) \end{aligned} \quad (2)$$

We can rearrange (2) to get

Normal Goods and Number of Equilibria V

$$\frac{\partial z_f(\mathbf{p}^*)}{\partial p} = \left(\frac{\partial x_f^1(\mathbf{p}^*)}{\partial p}\right)_{du^1=0} + \left(\frac{\partial x_f^2(\mathbf{p}^*)}{\partial p}\right)_{du^1=0}$$
$$+ \quad (x_f^1(\mathbf{p}^*) - e_f^1)\left(\frac{\partial x_f^2(\mathbf{p}^*)}{\partial I^2} - \frac{\partial x_f^1(\mathbf{p}^*)}{\partial I^1}\right),$$

Now, even if both goods are normal,

- Person 2 might have large income effect that can offset the negative substitution effects.
- $\frac{\partial z_f(\mathbf{p}^*)}{\partial p} < 0$ might not hold.
- So, we cannot be sure of uniqueness of WE.

Gross Substitutes I

Suppose,

- There are two goods
- Consider three price vectors: $\mathbf{p} = (p_1, p_2) = (2, 1)$, $\mathbf{p}' = (p'_1, p'_2) = (3, 1)$ and $\bar{\mathbf{p}} = (\bar{p}_1, \bar{p}_2) = (3, 2)$.

Let, $\mathbf{x}^i(\mathbf{p})$, be the demand function for individual i.

Question

Suppose, the above goods are 'gross substitutes' for individual i.

- *How will $x_2^i(\mathbf{p}')$ compare with $x_2^i(\mathbf{p})$?*
- *How will $x_2^i(\bar{\mathbf{p}})$ compare with $x_2^i(\mathbf{p})$?*

- Let $\lambda = \max_j\{\frac{\bar{p}_j}{p_j}\}$, $j = 1, 2$.
- Note here $\lambda = \frac{\bar{p}_2}{p_2} = 2$

Gross Substitutes II

- Also, $\lambda\mathbf{p} \geq \bar{\mathbf{p}}$. Since $(4,2) \geq (3,2)$.

Question

What can we say about the individual demand for the two goods at these two price vectors $\lambda\mathbf{p} = (4,2)$ and $\bar{\mathbf{p}} = (3,2)$?

Question

What can we say about the individual demand for the two goods at the price vectors $\mathbf{p} = (2,1)$ and $\lambda\mathbf{p} = (4,2)$?

Gross Substitutes III

Next, consider two price vectors

- $\mathbf{p} = (p_1, p_2, p_3) = (3, 2, 1)$ and $\bar{\mathbf{p}} = (\bar{p}_1, \bar{p}_2, \bar{p}_3) = (5, 1, 4)$

Question

What can we say about the excess demand at these two price vectors?

- Let $\lambda = \max_j\{\frac{\bar{p}_j}{p_j}\}$, $j = 1, .., 3$.
- Note here $\lambda = \max\{\frac{5}{3}, \frac{1}{2}, \frac{4}{1}\} = \frac{\bar{p}_3}{p_3} = 4$
- Also, $\lambda\mathbf{p} \geq \bar{\mathbf{p}}$. Since $(12, 8, 4) \geq (5, 1, 4)$.

Remark

$\mathbf{z}(\lambda\mathbf{p}) = \mathbf{z}(\mathbf{p})$, i.e., $\mathbf{z}(4\mathbf{p}) = \mathbf{z}(\mathbf{p})$.

Gross Substitutes IV

Consider the following price vectors

- $\mathbf{p} = (p_1, p_2, p_3) = (3, 2, 1)$, $\hat{\mathbf{p}} = (\hat{p}_1, \hat{p}_2, \hat{p}_3) = (12, 8, 4)$ and $\bar{\mathbf{p}} = (\bar{p}_1, \bar{p}_2, \bar{p}_3) = (5, 1, 4)$.

Question

- *What can we say about the excess demand for 3rd good at prices* $\hat{\mathbf{p}}$ *and* $\bar{\mathbf{p}}$*? That is,*
- *How is* $z_3(\hat{\mathbf{p}})$ *expected to compare with* $z_j(\bar{\mathbf{p}})$*?*

Note:

- $\hat{\mathbf{p}} = \lambda \mathbf{p}$ and $\hat{\mathbf{p}} \geq \bar{\mathbf{p}}$
- $\hat{p}_3 = \lambda p_3 = \bar{p}_3$.