ECONOMICS 001
General Equilibrium Analysis (Part 1)
Summer 2018-19

Instructor: Ram Singh

Problem 1

Please watch out for typos!

1. Consider a two-person two-goods economy. Suppose individual utility functions are: $u^i(x^i_1, x^i_2) = x^i_1.x^i_2$, $i = 1, 2$. Let initial endowment of the first person be the vector $\mathbf{e}^1 = (1, 9)$, i.e, to start with the first person has one unit of good one and nine units of good two. Let, $\mathbf{e}^2 = (9, 1)$. Now, consider an allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2)$, where $\mathbf{x}^1 = (3, 3)$ and $\mathbf{x}^2 = (7, 7)$.
   (a) Is allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2)$ Pareto optimum?
   (b) Does it belong to the Core?

2. Let us replicate the above economy. Suppose, now there are four individuals but only two goods as above. Utility functions are: $u^i(x^i_1.x^i_2) = x^i_1.x^i_2$, $i = 1, ..., 4$. Let initial endowments be $\mathbf{e}^1 = (1, 9)$, $\mathbf{e}^2 = (9, 1)$, $\mathbf{e}^3 = (1, 9)$, and $\mathbf{e}^4 = (9, 1)$. That is, person 3 is a twin of person 1, and 4 is twin of person 2. Next, consider an allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3, \mathbf{x}^4)$, where $\mathbf{x}^1 = (3, 3) = \mathbf{x}^3$ and $\mathbf{x}^2 = (7, 7) = \mathbf{x}^4$.
   (a) Is allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3, \mathbf{x}^4)$ Pareto optimum?
   (b) Does it belong to the Core?

3. Consider a pure exchange economy; $(u^i(.), \mathbf{e}^i)_{i \in I}$. Suppose $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, ..., \mathbf{x}^I)$ is a feasible allocation, and $S \subseteq \{1, ..., I\}$ is a blocking coalitions for $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, ..., \mathbf{x}^I)$. Which of the following is necessarily true?
   (a) Allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, ..., \mathbf{x}^I)$ is not Pareto optimum
   (b) Allocation $\mathbf{e} = (\mathbf{e}^1, \mathbf{e}^2, ..., \mathbf{e}^I)$ is not Pareto optimum

   Explain your answer.

4. Answer the above question assuming that allocation $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, ..., \mathbf{x}^I)$ is Pareto superior to $\mathbf{e} = (\mathbf{e}^1, \mathbf{e}^2, ..., \mathbf{e}^I)$.

5. Consider a two-person two-goods pure exchange economy. The initial endowment vectors are $\mathbf{e}^1 = (1, 0)$ and $\mathbf{e}^2 = (0, 1)$. Utility function is $u^i(x, y) = x^\alpha y^\beta$ for individual $i = 1, 2$, where x is the quantity of the first good and y is the quantity of the second good consumed; $x, y \geq 0$. For this economy:
   (a) Draw the core assuming $\alpha = \beta = 1$.

(b) Draw the core assuming $\beta = 1 - \alpha$ and $\alpha \in (0, 1)$.

(c) Find out the competitive equilibrium price and consumption vectors assuming $\alpha = \beta = 1$.

(d) Find out the competitive equilibrium price and consumption vectors assuming $\beta = 1 - \alpha$ and $\alpha \in (0, 1)$.

6. Consider a two-person two-goods pure exchange economy. The initial endowment vectors are $\mathbf{e}^1 = (1, 0)$ and $\mathbf{e}^2 = (0, 1)$. Utility function for person 1 is:

$$u^1(x, y) = \begin{cases} 1, & \text{when } x + y < 1 \\ x + y, & \text{when } x + y \geq 1, \end{cases}$$

where x is the quantity of the first good and y is the quantity of the second good. Utility function for person 2 is: $u^2(x, y) = x + y$. $x, y \geq 0$. For this economy:

(a) Find out a competitive equilibrium, assuming $p_1 = p_2 = 1$. (You can take it that there is at least one competitive equilibrium)

(b) Is the competitive equilibrium Pareto efficient? Why or why not? Explain the finding.

7. Consider a two-person two-goods pure exchange economy. The initial endowment vectors are: $\mathbf{e}^1 = (1, \frac{1}{2})$ and $\mathbf{e}^2 = (0, \frac{1}{2})$. The utility functions are: $u^1(.) = x_1^1 + 2x_2^1$ and $u^2(.) = x_1^2 x_2^2$; $x, y \geq 0$. For this economy:

(a) Find out the competitive equilibrium vectors of consumptions and prices. Denote the equilibrium vector of prices by $\mathbf{p}^* = (p_1^*, p_2^*)$.

(b) Find out the competitive equilibrium with 'cash transfers', assuming that person 1 is required to pay $\frac{p_1^*}{2}$ to person 2.

8. Consider a two-person two-goods pure exchange economy. The initial endowment vectors are: $\mathbf{e}^1 = (2, r)$ and $\mathbf{e}^1 = (r, 2)$, where $r = 2^{\frac{8}{9}} - 2^{\frac{1}{9}}$. The utility functions are: $u^1(.) = x_1^1 - \frac{1}{8}\frac{1}{(x_2^1)^8}$ and $u^2(.) = -\frac{1}{8}\frac{1}{(x_1^2)^8} + x_2^2$. For this economy:

(a) Show that there are three competitive equilibria: with price vector $\mathbf{p} = (p_1^*, p_2^*)$ $\frac{p_2^*}{p_1^*} = \frac{1}{2}, 1,$ and 2.

(b) Which of the conditions necessary for uniqueness of competitive equilibrium is/are violated by the above economy?