ECONOMICS 001
Microeconomic Theory (Part 2)
Summer 2018-19

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Problem Set 2

This problem set is available with the Photocopy shop. Watch out for typos!

1. Problems 15.B.1-15.B.4 and 15.B.8 from MWG.

2. Problems 5.10, 5.11, 5.14, 5.15 and 5.17 from Jehle and Reny.

3. Consider a two person two goods pure exchange economy. The goods are; x and y. The utility functions are $u^1(.) = x_1^2.y_1$ and $u^2(.) = x_2.y_2$, and the initial endowments are $\mathbf{e}^1(.) = (25, 75)$ and $\mathbf{e}^2(.) = (25, 75)$. Assuming $p_1 = 1$, compute competitive equilibria for this economy.

4. Consider a two person two goods exchange economy. The goods are; x and y. The utility functions are $u^1(.) = x_1.y_1$ and $u^2(.) = x_2 + y_2$, and the initial endowments are $\mathbf{e}^1(.) = (4, 2)$ and $\mathbf{e}^2(.) = (2, 3)$. For this economy:

   (a) Find out the competitive equilibrium consumption and price vectors. Draw the equilibrium allocations in the Edgeworth box.

   (b) Consider the allocation $\mathbf{z} = (\mathbf{z}^1(.), \mathbf{z}^1(.))$, where $\mathbf{z}^1(.) = (1, 1)$ and $\mathbf{z}^2(.) = (5, 4)$. Find out if this allocation Pareto optimum?

   (c) Find out the transfer payments T_1 and T_2 such that $T_1 + T_2 = 0$, and the allocation $\mathbf{z}^1(.) = (1, 1)$ and $\mathbf{z}^2(.) = (5, 4)$ is a competitive equilibrium with transfers.

5. Consider the following 2×2 pure exchange economy: The goods are; x and y. Individual utility functions are $u^1(.) = x^{\alpha}.y^{1-\alpha}$ and $u^2(.) = x^{\beta}.y^{1-\beta}$, $\alpha, \beta \in (0, 1)$ and $\alpha \neq \beta$. The initial endowments are $\mathbf{e}^1(.) = (\bar{x}_1, \bar{y}_1) >> (0, 0)$ and $\mathbf{e}^2(.) = (\bar{x}_2, \bar{y}_2) >> (0, 0)$. For this economy:

   (a) Does an increase in endowment of person 1 always have welfare enhancing effect for her?

   (b) Does an increase in endowment of person 1 have welfare decreasing effect for person 2?

   (c) Does an increase in endowment of person 2 always have welfare enhancing effect for him?

   (d) Does an increase in endowment of person 2 have welfare decreasing effect for person 1?

6. Consider the economy as in the above question. Suppose $\alpha = \beta$. Is equal division of endowment between the individuals necessarily a Pareto optimum allocation?

7. Does equal division of endowment among the individuals constitutes a Pareto optimum allocation, in general?

8. Consider the following 3×3 pure exchange economy. The goods are; x, y and z. Individual utility functions are: $u^1 = 3x_1 + 2y_1 + z_1$; $u^2 = 2x_2 + y_2 + 3z_2$ and $u^3 = x_1 + 3y_1 + 2z_1$. The initial endowments are $\mathbf{e}^1 = \mathbf{e}^2 = \mathbf{e}^3 = (1, 1, 1)$. Do initial endowments

   (a) constitute an 'equal division' allocation ?

   (b) constitute a non-envious allocation?

   (c) constitute a Pareto Efficient allocation?

9. Let $\mathbf{y} = \sum_j^K \mathbf{y}^k$, $\mathbf{y}^k \in \mathbb{Y}^k$, where $\mathbf{y}$, $\mathbf{y}^k$, $\mathbb{Y}^k$, etc., are as defined in the class. Now, consider the following two statements:

   Statement 1: $\mathbf{y}$ maximizes the aggregate profit, for a price vector $\mathbf{p} = (p_1, ..., p_J)$.

   Statement 2: There is no $\mathbf{z} = \sum_j^K \mathbf{z}^k$, such that $\mathbf{z}^k \in \mathbb{Y}^k$ and $\mathbf{z} \geq \mathbf{y}$.

   (a) Does Statement 1 implies Statement 2?

   (b) Does Statement 2 implies Statement 1?

   Prove your claims.

10. Is it possible to find out allocations of factors of production under a Walrasian equilibrium, without explicitly modeling the factor prices? Explain in detail.

11. For a 'regular' economy, prove that there can be only finitely many equilibria. (Optional question)

12. For a 2×2 economy, prove that under the standard assumptions on utility functions and the excess demand function, the number of equilibria will be odd.

13. Consider a $N \times M$ pure exchange economy. Suppose the initial endowments are such that: For all $i = 1, 2..., N$, $\mathbf{e}^i = (\bar{e}_1, ..., \bar{e}_M)$. Which of the following statements is/are necessarily correct?

   (a) A Walrasian equilibrium allocation will satisfy the equal division property, i.e., will give same quantity of a good to each individual.

   (b) A Walrasian equilibrium allocation will be non-envious.

14. Consider a two person two goods production economy. The goods are x_1 and x_2. The utility functions are:

$$u^1(x_1^1, x_2^1) = x_1^1 + \sqrt[4]{x_2^1} \text{ and } u^2(x_1^2, x_2^2) = x_1^2 + \sqrt[2]{x_2^2} \ ;$$

where x_1^i and x_2^i is the quantity consumed by person i of good x_1 and x_2, respectively; $i = 1, 2$. The initial endowments are $\mathbf{e}^1(.) = (4, 12)$ and $\mathbf{e}^2(.) = (8, 8)$. The production sector is competitive. It uses x_1 to produce x_2, using production technology $x_2 = \alpha x_1$, subject to $x_1 \geq 0$ and $\alpha > 0$. Let $\theta^1 \geq 0$ and $\theta^2 \geq 0$ denote the shares of persons 1 and 2, respectively, in the profits of the production sector; $\theta^1 + \theta^2 = 1$. For this economy,

(a) Find out competitive equilibrium prices and allocations, assuming that the production sector will meet the entire demand as long as profits are non-negative.

(b) Can there be a competitive equilibrium involving no production of x_2 at all? If your answer is 'No', discuss the reasons why. If your answer is 'Yes', fully state the conditions for existence of such an equilibrium.