Efficiency Criteria in Economics

Ram Singh

Microeconomic Theory

Lecture 16

The Social Welfare Function I

Let

$$W(u^1, u^2, \alpha_1, \alpha_2) = \alpha_1 u^1(x_1, y_1) + \alpha_2 u^2(x_2, y_2)$$

be a Utilitarian SWF.

$$\max_{x_1, x_2, y_1, y_2, x, y} W(u^1, u^2, \alpha_1, \alpha_2) \tag{1}$$

s.t. production technology

$$\begin{aligned} \text{production of x: } x_1 + x_2 &= f(l_x, t_x) \\ \text{production of y: } y_1 + y_2 &= g(l_y, t_y) \\ \text{full employment of l: } l_x + l_y &= \bar{l} \\ \text{full employment of t: } t_x + t_y &= \bar{t} \end{aligned}$$

Let $(x_1^*, x_2^*, y_1^*, y_2^*, x^*, y^*)$ be a solution to (1), where

$$x^* = f(l_x^*, t_x^*) \;\&\; y^* = f(l_y^*, t_y^*)$$

The Social Welfare Function II

Proposition

$(x_1^*, x_2^*, y_1^*, y_2^*, x^*, y^*)$ solves (1) only if $(x_1^*, x_2^*, y_1^*, y_2^*, x^*, y^*)$ is Pareto optimum.

Suppose $(x_1^*, x_2^*, y_1^*, y_2^*, x^*, y^*)$ is NOT Pareto optimum. Then there is feasible $(\hat{x}_1, \hat{x}_2, \hat{y}_1, \hat{y}_2)$ such that: s.t.

$$\begin{aligned} \hat{x}_1 + \hat{x}_2 &= f(\hat{l}_x, \hat{t}_x) \\ \hat{y}_1 + \hat{y}_2 &= g(\hat{l}_y, \hat{t}_y) \\ \hat{l}_x + \hat{l}_y &= \bar{l} \\ \hat{t}_x + \hat{t}_y &= \bar{t} \end{aligned}$$

$$\begin{aligned} \hat{u}^1(\hat{x}_1, \hat{y}_1) = \hat{u}^1 &\geq u^1(x_1^*, y_1^*) = u_1^* \\ \hat{u}^2(\hat{x}_2, \hat{y}_2) = \hat{u}^2 &\geq u^2(x_2^*, y_2^*) = u_2^* \end{aligned}$$

The Social Welfare Function III

with at least one inequality. That it,

$$[\alpha_1 \hat{u}^1 + \alpha_2 \hat{u}^2 = \hat{W}] > [\alpha_1 {u^*}^1 + \alpha_1 {u^*}^2 = W^*]$$

That is, $(x_1^*, x_2^*, y_1^*, y_2^*, x^*, y^*)$ does not solve (1), a contradiction.

Generally, $(x_1^*, x_2^*, y_1^*, y_2^*, x^*, y^*)$ is called the First Best outcome.

The First Best: An example

Consider a buyer and a seller: Let

- $U^B() = U(q) - p$
- $U^S() = p - C(q)$

Utilitarian (SWF) will solve:

$$max_q\{U(q) - p + p - C(q)\}$$

$$max_q\{U(q) - C(q)\}$$

FOC:

$$U'(q) - C'(q)$$

The solution will necessarily be Pareto optimum, regardless of the price.

Wealth Maximization I

Definition

When allocations can be described in perfectly divisible money units, Kaldor-Hicks Efficient allocation is wealth maximizing.

$$\mathbf{x} = (50, 100, 150) \quad , i.e., \quad \sum_{i=1}^{3} x^i = 300$$

$$\mathbf{y} = (90, 90, 90) \quad , i.e., \quad \sum_{i=1}^{3} y^i = 270$$

$$\mathbf{z} = (80, 250, 250) \quad , i.e., \quad \sum_{i=1}^{3} z^i = 580$$

Which of the above alternatives is efficient according to Kaldor-Hicks Efficient and Wealth maximization criterion?

Wealth Maximization II

Proposition

When allocations have money equivalent, a Kaldor Hicks efficient allocation is wealth maximizing. That is, an allocation is Kaldor Hicks efficient only if it is wealth maximizing

Consider

- $w^1(.)$ and $w^2(.)$ wealth levels of two individuals: $i = 1, 2$
- one FOP, y, is be used to produce wealth/income

Consider the OP for wealth maximization:

$$\max_y \{w^1(y) + w^2(y)\} \tag{2}$$

$$FOC \quad : \quad \frac{dw^1(y)}{dy} + \frac{dw^2(y)}{dy} = 0 \tag{3}$$

Wealth Maximization III

Next, consider the OP for Pareto :

$$\max_{y,t}\{u^1(w^1(y) - t)\} \tag{4}$$

$$\text{subject to} \quad : \quad u^2(w^2(y) + t) \geq \bar{u}^2 \tag{5}$$

Using Lagrangian technique, we get FOCs:

$$u'^1 \frac{dw^1(y)}{dy} + \lambda u'^2 \frac{dw^2(y)}{dy} = 0 \tag{6}$$

$$-u'^1 + \lambda u'^2 = 0 \tag{7}$$

the above FOCs give us

$$\frac{dw^1(y)}{dy} + \frac{dw^2(y)}{dy} = 0 \tag{8}$$