Social Choices Rules

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Microeconomic Theory

Lecture 18

Is Majority rule a SWF?

Proposition

There exists a SCR that satisfies conditions U, P, I, and ND.

Let

- $N(xPy)$ number of individuals who strictly prefer x over y
- $N(xRy)$ number of individuals who weakly prefer x over y

Definition

A Method of Majority Rule is a SWR such that:

$$(\forall x, y \in \mathbb{X})[x\mathfrak{R}y \Leftrightarrow [N(xPy) \geq N(yPx)], \text{ or}$$

$$(\forall x, y \in \mathbb{X})[x\mathfrak{R}y \Leftrightarrow [N(xRy) \geq N(yRx)].$$

MMR satisfies all conditions but $\mathfrak{R} \notin \mathbb{O}$.

Concentrated Power I

Definition

Almost Decisive Set. Let $V \subseteq \mathbb{N}$ be non-empty set, and $x, y \in \mathbb{X}$ be an ordered pair. Set V is almost decisive for x against y if

$$[(\forall i \in V)(xP_i y) \;\&\; (\forall j \in \mathbb{N} - V)(yP_i x)] \Rightarrow [x\mathcal{P}y].$$

In that case, we say: V is $D(x, y)$.

Definition

Decisive Set. Let $V \subseteq \mathbb{N}$, and $x, y \in \mathbb{X}$ be an ordered pair. Set V is decisive for x against y if

$$(\forall i \in V)(xP_i y) \Rightarrow [x\mathcal{P}y].$$

In that case, we say: V is $\bar{D}(x, y)$. Note:

$$[V \text{ is } \bar{D}(x, y)] \Rightarrow [V \text{ is } D(x, y)].$$

Concentrated Power II

Proposition

Suppose a SWF satisfies conditions U, P, I and ND.

$$[V \text{ is } D(x,y) \text{ for some } x,y \in \mathbb{X}] \Rightarrow [V \text{ is } \bar{D}(u,v) \text{ for all } u,v \in \{x,y,z\}].$$

Proof: Suppose, $\exists x, y \in \mathbb{X}$ such that V is $D(x,y)$. Take any $z \in \mathbb{X}$ such that $x \neq z$ and $y \neq z$. Consider the following profile:

$$(\forall j \in V)(xP_j y \ \& \ yP_j z) \qquad \text{and}$$
$$(\forall i \in \mathbb{N} - V)(yP_i x \ \& \ yP_i z)$$

So, we get

$$x\mathcal{P}y \ \& \ y\mathcal{P}z \qquad , i.e.,$$
$$x\mathcal{P}z \qquad , i.e., \text{ (Why?)}$$

Concentrated Power III

$$[V \text{ is } D(x,y)] \Rightarrow [V \text{ is } \bar{D}(x,z)] \quad (1)$$

Next, let

$$(\forall j \in V)(zP_jx \;\&\; xP_jy) \qquad \text{and}$$
$$(\forall i \in \mathbb{N} - V)(zP_ix \;\&\; yP_ix). \qquad \text{This gives}$$

$$z\mathcal{P}x \;\&\; x\mathcal{P}y \qquad, i.e., z\mathcal{P}y, i.e.,$$

$$[V \text{ is } D(x,y)] \Rightarrow [V \text{ is } \bar{D}(z,y)] \quad (2)$$

Interchanging y and z in (2), in view of (1), we get

$$[V \text{ is } D(x,z)] \Rightarrow [V \text{ is } \bar{D}(y,z)] \quad (3)$$

Concentrated Power IV

Consider the following replacements in (1): $x \to y$, $y \to z$ and $z \to x$. Now, we get

$$[V \text{ is } D(y,z)] \Rightarrow [V \text{ is } \bar{D}(y,x)] \tag{4}$$

To sum up, we have

$$\begin{aligned} \text{V is } D(x,y) &\Rightarrow \text{V is } \bar{D}(x,z), \text{ from (1)} \\ &\Rightarrow \text{V is } D(x,z), \text{ from the above defn} \\ &\Rightarrow \text{V is } \bar{D}(y,z), \text{ from (3)} \\ &\Rightarrow \text{V is } D(y,z), \\ &\Rightarrow \text{V is } \bar{D}(y,x), \text{ from (4).} \end{aligned} \tag{5}$$

Interchanging x and y in (1), (2) and (5), we get

$$[\text{ V is } D(y,x)] \Rightarrow [\text{ V is } \bar{D}(y,z), \text{ V is } \bar{D}(z,x), \text{ and V is } \bar{D}(x,y)] \tag{6}$$

Concentrated Power V

From (5) and (6), we get

$$[\text{ V is } D(x,y)] \Rightarrow [\text{ V is } \bar{D}(y,z), \text{ V is } \bar{D}(z,x), \text{ and V is } \bar{D}(x,y)] \tag{7}$$

(1), (2), (5) and (7) together implies

$$\text{V is } \bar{D}(u,v) \text{ for all } u,v \in \{x,y,z\}$$

Question

- *How many conditions we have used so far?*
- *What is the size of V?*

Proposition

Suppose a SWF satisfies conditions U, P, and I.

$$[V \textit{ is } D(x,y) \textit{ for some } x,y \in \mathbb{X}] \Rightarrow [V \textit{ is } \bar{D}(u,v) \textit{ for all } u,v \in \mathbb{X}].$$

Impossibility Result I

Question

Can $\sharp V = 1$*?*

Theorem

There is no SWF that satisfies conditions U, P, I and ND simultaneously.