004: Macroeconomic Theory
Introduction to Macroeconomics (Static Macro Models)

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What is Macroeconomics?

- Macroeconomics is one particular field of Economics that pertains to the aggregate economy as opposed to a single microeconomic agent - a single household or firm. Even if actions of individual agents are the building blocks of our analyses, we are interested in the resulting aggregative behaviour of the economy as a whole.

- Macroeconomics therefore essentially deals with a general equilibrium set up where we simultaneously analyze the equilibrium scenarios in various markets, as determined by the behaviour of three sets of economic agents:
  - Households;
  - Firms;
  - Government.

- The functioning of these different set of agents are often interlinked and these interlinked actions generate certain outcomes for the aggregate economy. In Macroeconomics, we are concerned with these aggregative outcomes for the entire economy.
Modern Macroeconomics - The Central Issue:

- Macroeconomics is one field in Economics which carries utmost importance in the arena of policy making.
- Two central issues that motivate this field are as follows:
  - What causes aggregate output and employment levels in an economy to fluctuate/change over time?
  - How effective are various government policies in stabilizing the economy and/or in generating steady growth?
- Despite its tremendous importance from the policy perspective, macroeconomics is one field which is fraught with controversies and debates.
- The focal point of all the major debates in macroeconomics over the years has been an ideological issue: should government intervene in the functioning of a private market economy or should it not?
- Or, to put it differently, does government intervention in macroeconomic matters improve welfare or does government intervention create more problems than it solves?
There are various schools of thoughts - starting with Keynes and his predecessors (the Classics), and their modern reincarnations (Neo-classicals, Neo-Keynesians, New Keynesians, New Classicals) - each trying to make a case for its respective position using specific theoretical constructs (which we call the ‘macro models’).

These models differ in terms of the underlying assumptions about the functioning of the aggregate economy and about individual behaviour.

The primary objective of this course is to expose the students to these various schools of thoughts in terms of rigorous macro models and analyse the associated policy implications.
The secondary objective of the course is to familiarize the students with the major mathematical tools used in modern macro analyses across the world.

The core issues underlying these debates have remained unchanged, but the techniques used in expositing alternative schools of thoughts have undergone a dramatic change over the last two decades.

While the earlier theoretical expositions were based on ‘static’ models (involving a single time period), all the recent expositions use dynamic settings (involving tracing the economy/agents over a period of time) to analyse issues which also have dynamic implications (e.g. growth and fluctuations rather than commenting on just the current status of output, prices and employment).
Analysing macro issues in a dynamic setting is a key element of Modern Macroeconomics.

A part of this course would therefore entail some discussion of the basic dynamic tools (e.g., difference and/or differential equations) and dynamic optimization techniques (Dynamic Programming and/or Optimal Control)): both deterministic as well as stochastic versions.

These tools will then be applied to analyse the macro issues at hand.
Course Content:

- The course has two modules. The first module will be taught by Mausumi Das and the second module will be taught by Dibyendu Maiti.

- The first module begins with a brief discussion of the static macro models that are used to analyse very short run issues, when the economy/agents have not had time to adjust to various dynamic forces.

- These static models, being static in nature, take various time dependent variables (e.g., capital stock, population, technology) as exogenous and analyse the effectiveness of various policies under alternative assumptions but essentially in a static one-period framework. These static models are also aggregative in nature – often without any obvious micro-foundations.

- The first module then goes on to provide a micro-founded theoretical underpinning to these aggregative macro models and in the process also introduces a dynamic framework to analyse various medium and long run macroeconomic issues.
The first module focuses explicitly on issues pertaining to economic growth: the long run steady state/balanced growth path attained by an economy and its transition to the steady state/balanced growth path from any given initial condition. It uses dynamic techniques to analyse how the aggregate as well as per capita GDP growth rates in the long run respond to policy changes and other factors under alternative schools of thoughts.

The second module focuses on output fluctuations during short to medium run, i.e., on business cycle analyses. It uses dynamic tools to examine how business cycles are generated. In this context it also introduces various stochastic shocks that typically affect the macroeconomy and examines how the economy adjusts to these shocks. It further examines the efficacy of government policies to counter business cycles under alternative schools of thoughts.
Preliminary Reading:

- Any student of macroeconomics must start by reading the following article by Mankiw to get a broad perspective about the field: "The Macroeconomist as Scientist and Engineer": N. Gregory Mankiw, The Journal of Economic Perspectives, Vol. 20, No. 4 (Oct., 2006), pp. 29-46.

- Other (more specific) readings will be provided as we move from topic to topic.

- Exact chapters of various books and other supplementary readings will be specified during the course.

- Lecture notes on selected topics will be put up in the course folder at the department website and the department server.

- Problem sets will be circulated upon completion of various broad topics to help students apply the concepts taught in the class.
Mode of Evaluation:

- 2 midterms (of 15 marks each) will be held during the semester.
- A final examination (of 70 marks) will be held at the end of the semester.
Regular tutorials will be held by 2 tutors to facilitate understanding the concepts and techniques taught in the class and also to assist with the problem sets.

I am available for clarification outside the class during the following contact hours:

- Tuesdays: 3-4 pm
- Fridays: 3-4 pm

I can also be reached by e-mail at: mausumi@econdse.org
In the next few lectures, we shall briefly discuss the static, one-period macro models.

These models were very much in vogue in the decade of 1950s and 60s and majorly contributed to the academic and policy debates until 1970s.

Subsequently these theories were challenged on several grounds - both theoretical and empirical. We shall provide a critique of these static frameworks and then move on to more modern macro frameworks which are essentially dynamic in nature.
There are two main variants of the static macro models: (a) The Keynesian Framework; (b) The Classical/Neoclassical Framework.

We shall start by re-visiting these two basic static macro frameworks.

Both frameworks summarise the aggregate economy in terms of three markets:

- The Goods Market
- The Labour Market
- The Money (or alternatively, the Bond) Market

Each market is represented by a pair of equations that capture the demand and the supply side respectively. These equations represent aggregative behaviour that are not necessarily derived from any explicit micro-founded analysis.

The major difference between the frameworks arises from the description of the labour market.
Role of various agents in the macroeconomy:

- Recall that there are three sets of agents in this economy: households; firms and the government. Their respective roles are as follows:
  
  **Households:**
  - All factors of production (capital, labour) are owned and supplied by the households. They also are the share holders of the firms (implying if the firms earn any positive profit then that profit would be distributed back to the households in the form of dividends). Thus the entire flow of output (unless taxed) goes back to the households in the form of income. The households also decide how much to consume and how much to save out of their total income.

  **Firms:**
  - Firms are engaged in actual production. They employ the factors owned by the households to produce the final commodity and pay the households in the form of wages and rents. If the firms earn any positive profit, that also eventually goes back to the households as dividends. However, to begin with, we shall assume a perfectly competitive market structure and CRS technology (which means firms would earn zero profit). They may also undertake investment activities (borrowing resources from the households).
Government:

- We consider a decentralized market economy, not a socially planned (or centralized or command or socialist) economy. So the government is **not** actively engaged in the production process. But it can intervene into the goods market by imposing taxes and/or contributing to the demand for final goods (government consumption).
- It also directly operates in the money market, either by controlling the money supply or by setting the interest rate. To begin with, we shall assume that it controls the money supply and allows the interest rate to be determined by the market (an assumption that we shall change when we talk about interest rate targeting as opposed to money supply targeting).
Demand & Supply in the three markets:

- **In the goods market:**
  - demand comes from the households (consumption demand), firms (investment demand) and the government (government consumption);
  - supply comes from the firms.

- **In the labour market:**
  - demand comes from the firms;
  - supply comes from the household.

- **In the money market:**
  - demand comes from the households;
  - supply comes from the government.
The Classical/Neoclassical System (in equations):

- **The Goods Market:**
  - Supply Equation:
    \[ Y = F(N, \bar{K}); F_N, F_K > 0; F_{NN}, F_{KK} < 0 \]  
  - Demand Equation:
    \[ Y = C(Y) + I(r) + \bar{G}; \quad 0 < C'(Y) < 1; \quad l'(r) < 0 \]  

- **The Labour Market:**
  - Supply Equation:
    \[ W = Pg(N); \quad g'(N) > 0 \]  
  - Demand Equation:
    \[ W = Pf(N); \quad f'(N) < 0 \]  

- **The Money Market:**
  - Supply Equation:
    \[ M = \bar{M} \]  
  - Demand Equation:
    \[ M = PL(Y, r); \quad L_Y > 0; \quad L_r < 0 \]
The Classical System: Solution

- We have 6 equations in 6 variables \((P, Y, W, N, r, M)\) which define the classical macroeconomic system.

- Solution consists of
  - Equilibrium values of the Price & Quantity in the Goods Market: \(P^*, Y^*\)
  - Equilibrium values of the Price & Quantity in the Labour Market: \(W^*, N^*\)
  - Equilibrium values of the Price & Quantity in the Money Market: \(r^*, M^*\)

- The equations being **interdependent**, we cannot solve for the equilibrium values of quantities & prices in each market separately. So we follow a more roundabout method.
First we club the SS & DD equations in the Labour Market and the SS equation in the Goods Market together:

$$Y = F(N, K); F_N, F_K > 0; F_{NN}, F_{KK} < 0$$  \hspace{1cm} (1)

$$W = Pg(N); g'(N) > 0$$  \hspace{1cm} (3)

$$W = Pf(N); f'(N) < 0$$  \hspace{1cm} (4)

This sub-system involves four endogenous variables: $Y, N, W$ and $P$. We eliminate two of these variables to get a relationship between $P$ and $Y$ - which we call the aggregate supply curve (AS).

Let us now see how this AS curve looks for the Classical system.
Derivation of the AS Schedule under the Classical System: Graphical Method

- In deriving the AS curve, we first focus on the two labour market equations.
- Plot (3) and (4) in the $N-W$ plane (assuming some arbitrarily given value of $P$):

![Graph showing supply and demand curves in the N-W plane](image-url)
Now increase $P$ to a higher level, say $P'$:

- The $N^S$ curve shifts out proportionally - diverging away from the earlier curve for higher values of $N$ **(Why?)**
Likewise as $P$ increases to a higher level, say $P'$:

- The $N^D$ curve also shifts out proportionally - but it converges closer to the earlier curve for higher values of $N$ (Why?)
However despite the fact that the shifts in $N^S$ and $N^D$ are not parallel, the new point of intersection still remains the same at $N^*$ (Why?)

Correspondingly, the output supplied remains fixed at $Y^*$:
In other words, the **AS Schedule under the Classical System is Vertical:**
Let us now go back to the rest of the equations in the classical system.

Let us club the SS & DD equations in the Money Market and the DD equation in the Goods Market together:

\[ Y = C(Y) + I(r) + \bar{G}; 0 < C'(Y) < 1; l'(r) < 0 \]  
\[ M = \bar{M} \]  
\[ M = PL(Y, r); L_Y > 0; L_r < 0 \]

This sub-system involves four endogenous variables: \( Y, r, M \) and \( P \). We eliminate two of these variables to get another relationship between \( P \) and \( Y \) - which we call the aggregate demand curve (\( \text{AD} \)).

How does the \( \text{AD} \) look under the Classical system? We discuss that below.
Derivation of the AD Schedule under the Classical System: Graphical Method

- In deriving the AD Schedule, first notice that the Money Supply Function is constant. This allows us to write the Money Market Equilibrium condition as:

\[ \tilde{M} = P L(Y, r); \quad L_Y > 0; \quad L_r < 0 \]  

(7)

- This is the so-called LM curve, which represents a relationship between \( Y, r \) and \( P \).

- On the other hand the Demand Equation for the Goods market represents another relationship between \( Y \) and \( r \) :

\[ Y = C(Y) + I(r) + \tilde{G}; \quad 0 < C'(Y) < 1; \quad I'(r) < 0 \]  

(2)

- This is the so-called IS curve.
Plot the IS and the LM curve in the $Y$-$r$ plane (assuming some arbitrarily given value of $P$):

\[ IS: Y = C(Y) + I(r) + \bar{G} \]

\[ LM: \bar{M} = PL(Y, r) \]
Now increase $P$ to a higher level, say $P'$:
- The LM curve shifts up.
- The IS Curve remains unchanged.
Thus the new point of intersection shifts to the left:
In other words, the AD Schedule under the Classical System is Downward Sloping:
We then simultaneously plot the the AS and AD schedule in the $Y-P$ plane to determine the equilibrium price level $P^*$ and equilibrium output $Y^*$ in the Goods Market.

Once these two values are determined, other equilibrium values can be found by substituting these back in the other equations.

Equilibrium price and quantity in the Goods Market - $P^*$ and $Y^*$ - as determined simultaneously by the intersection of the AS and the AD schedule - are shown below:
Digression: A Heuristic Justification of the Classical System

Although we have specified all the equations here as ad hoc behavioural relationships without any explicit micro-foundations, they are not devoid of economic logic.

There are heuristic justifications for each of these equations that appeal to common sense; but they are not explicitly derived from agents’ optimization exercise.

We now examine some of these heuristic arguments. In the process we also point out the limitations of these arguments.

Let us first start with the labour market story underlying the classical system.
In fact there is indeed a micro-founded labour market story implicit in the labour demand equation (4) of the classical system.

Notice that labour is demanded by firms who are also engaged in the production and supply of the final commodity. Thus their labour demand decisions and production decisions are interrelated.

The assumption implicit here is that all firms operate in a perfectly competitive market structure such that they take all prices as given.

Profit maximization under perfect competition implies:

$$\max_{\{N\}} PF(N, \bar{K}) - WN$$
From the FONC, we get

\[ F_N(N, \bar{K}) = \frac{W}{P}; \quad F_{NN} < 0 \]

which can be indeed re-written as the labour demand function specified under the Classical system, given by (4):

\[ W = Pf(N); \quad f'(N) < 0. \]

But the above specification is not a complete description of the optimization problem of a firm!

Typically, in microeconomics, the optimization problem of a firm under perfect competition is written as follows:

\[
\begin{align*}
\text{Max} & \quad PF(N, K) - WN - RK \\
\{N, K\}
\end{align*}
\]

where for given \( P, W \) and \( R \), the firm optimally chooses both how much labour to employ as well as how much capital to employ and those optimal decisions depend both on \( W \) and \( R \).
As $W/P$ increases, whether the firm will employ more labour or less labour will depend crucially on whether $R/P$ also increases simultaneously or not!

Indeed it is conceivable that as $W/P$ increases, $R/P$ also increases - either proportionally or more than proportionally.

In the latter case, a rise in $W/P$ would be actually associated with an increase in labour demand - which will go completely against the postulated labour demand curve of the classical system, as represented by (4)!

The classical system implicitly assumes away this possibility by postulating that firms always employ $\bar{K}$ units of capital no matter what the corresponding $R$ is. In other words, it assumes away the possibility of substituting labour for capital and vice versa.

But that assumption is rather odd and does not sync well with the standard results of micro-theory. Obviously it will not be consistent with a micro-founded macro framework!
There are other implicit assumptions hidden behind the postulated firm side story of the neoclassical system.

Recall that the production side story captures the functioning of the aggregate economy, not individual firms.

But the profit maximization exercise that we have written above is valid for an individual firm. To get the aggregative relationship we have to sum it over all firms.

What is the guarantee that the aggregative relationship will exactly mimic the individual firm’s behaviour?

Indeed, if firms differ in their scale of operation, then at the margin different firms will respond differently to changes in the real wage rate and hence the aggregate behaviour would not mimic the action of any single firm!

The only in which the aggregative behaviour will exactly mimic the individual firms’ action is if the technology is scale neutral, i.e., it exhibits constant return to scale, which is another implicit assumption of the neoclassical system.
The upshot is: the firm side story of the classical system, which otherwise looks plausible, has many hidden assumptions which do not become clear unless we attempt to dig up the underlying micro-foundations of these postulated relationships.

This tells us the need to build these macro models brick by brick from the underlying micro structures. Otherwise the macro models might be inconsistent with the micro behaviour of agents. Moreover, the underlying assumptions will also remain hidden and one cannot verify them directly.

In fact possible inconsistency of the postulated macro relationships with micro behaviour is not limited only to the firm side story.

We provide below a few more examples where the postulated macro relationships may not be consistent with micro theory, unless we make some very specific assumptions.
For example, consider the labour supply equation in the Classical system (equation (3)):

$$ W = P g(N); g'(N) > 0 $$

Inverting, we get:

$$ N^S : N = \hat{g} \left( \frac{W}{P} \right); \hat{g} \equiv g^{-1}. $$

The underlying logic here is that workers look at the real wage rate when they decide how much labour to supply. If the real wage rate goes up, then they are willing to supply more labour.

Needless to say, this story presupposes an implicit labour-leisure choice of a household which positively responds to income. Does this necessarily hold? Again, without precise microfoundations we cannot say. (Remember the backward bending labour supply curve?)
The classical labour market story also presupposes that workers know the real wage rate when they decide about their labour supply, although prices are determined in the goods market. This implies some role of expectations (since goods prices would presumably not be known when wages were set in the labour market) which would influence the optimal labour supply decisions of the households. Once again without a precise theory of expectation formation, we do not know whether this assumption is justified or not.
Next, consider the goods market demand equation:

\[ Y = C(Y) + I(r) + \bar{G}; 0 < C'(Y) < 1; I'(r) < 0 \]  

(2)

The consumption demand function merely states that as income goes up, households consume a part of their increased income.

- Can one optimally derive a consumption/savings behaviour which depends only on income? Why does not savings depend on the rate of interest as well? The answers lie in the precise micro-foundations.

Investment demand depends negatively on the interest rate. The heuristic argument here is that firms have to borrow to invest and a higher rate of interest means a higher cost of borrowing; hence investment demand falls.

- But why do firms invest at all when they don’t earn any profit or don’t get to keep it (everything goes back to the households as dividends)? Again we need precise micro-foundations to answer these questions.
Finally let us look at the money market relationship.

The money demand is given by:

\[ M = PL(Y, r); L_Y > 0; L_r < 0 \]

This equation combines two motives for holding money:

- Money that is held for transaction purposes: \( M^d = KPY; K > 0 \)
- Money that is held as an asset: \( M^d = L(r); L' < 0 \)

Why should people hold money as an asset (and not bonds) when money yields zero real return?
The answer (actually provided by Keynes) is that people will hold money (vis-a-vis bond) if they expect bond prices to fall in future so that they can buy it cheap. Thus they can make some speculative gain out of it.

When bond prices are already very low (which means the interest rate is already very high) then people believe that it is unlikely to fall any further; hence they will be less willing to hold money. Opposite happens when bond prices are high (and consequently interest rate is low).

This would generate a negative relationship between demand for money (held as an asset) and the rate of interest. (Which interest rate - nominal or real? Why?)

What kind of optimizing behaviour under risk/uncertainty and expectation would generate this speculative behaviour of the households? Only a micro-founded story can answer that question.
Equilibrium in the Classical System:

- As we have seen before, the equilibrium in the classical system is determined by the intersection of the AS and the AD schedule:

- Once we know \( Y^* \) and \( P^* \), we can find out the corresponding equilibrium values of \( N^* \), \( W^* \), \( r^* \) and \( M^* \).
Recall that the equilibrium level of employment in this model is defined by the point of intersection between the $N^S$ and $N^D$ curves drawn for the equilibrium price level $P^*$. The equilibrium nominal wage rate ($W^*$) is also simultaneously determined.

This equilibrium level of employment represents the *market clearing* level of employment. It implies that given the equilibrium *real* wage rate ($\frac{W^*}{P^*}$), everybody who is willing to supply labour indeed finds employment.

In other words, there is no ‘*involuntary*’ unemployment.

But to call it ‘full employment’ could be misleading!

What is full employment?

Suppose there exists a maximum limit on the labour supply - say $\bar{N}$, such that it is not feasible to increase the labour supply beyond this level.

This maximum feasible level is what is typically referred to as the ‘*full employment*’ level of labour supply.
If there indeed exists such a maximum limit such that labour supply cannot be increased beyond this level - no matter how high the wage rate is, then the $N^S$ curve becomes vertical at this point.

But there is no *apriori* reason why the point of intersection between $N^S$ and $N^D$ will happen precisely at this vertical stretch.

Thus the classical model is perfectly consistent with a scenario where there is no *full-employment* of the entire labour force. But any such unemployment must be *voluntary*.

To differentiate this equilibrium unemployment level under the classical system from *full employment*, we shall call the former the *natural level of employment*. 
**Question:** Let us begin with an AS-AD configuration such that at the corresponding equilibrium $N^* < \bar{N}$. In that case, will the equilibrium price level ($P^*$) re-adjust so that in equilibrium you always end up with $N^* = \bar{N}$?

The answer is "no"! So the complete price flexibility in the Classical system does not necessarily lead to ‘full employment’; it only eliminates ‘involuntary unemployment’.

Can ‘full employment’ be attained by appropriate government policies?
The effectiveness of government policies under the Classical System:

- We shall primarily focus on two kinds of government policies:
  - Fiscal Policy - which usually changes the amount of government expenditure ($\tilde{G}$)
  - Monetary Policy - which usually changes the amount of money supply ($\tilde{M}$)
- There could be other forms of government policies - e.g. taxes; government borrowing; government directly influencing the wage rate or price level in the goods/labour market or the interest rate in the money market. We shall talk about some of these latter policies as special cases.
Notice that $\bar{G}$ enters only in the IS equation, while $\bar{M}$ enters only in the LM equation.

In particular, an increase in $\bar{G}$ shifts the IS curve up while an increase in $\bar{M}$ shifts the LM curve down.

Both policies lead to a rightward shift of the AD curve but leave the AS curve unchanged.

Therefore the equilibrium output does not change.
Effectiveness of Government Policies under the Classical System (contd.):

- Thus the standard Fiscal and Monetary Policies (which affect only the demand side of the economy) are **completely ineffective** in raising the equilibrium output and employment under the classical system:
So far we had not introduced taxes in our model. Let us now introduce a **proportional income tax** \((t)\) which is imposed **at the household level**.

This changes the disposable income -available to the household for consumption:

\[
C^d = C(Y - tY) = C(Y^d); \quad 0 < C'(Y^d) < 1
\]

Thus the demand equation in the Goods Market now becomes:

\[
Y = C(Y^d) + I(r) + \ddot{G}; \quad 0 < C'(Y^d) < 1; \quad I'(r) < 0
\]

The corresponding IS curve (representing the demand condition in the Goods Market in the \(Y\)-\(r\) plane) still looks the similar.

But a change in a tax rate will now shift the IS curve, but not the LM curve. Thus the AD schedule gets affected.
Consider a tax cut such that the tax rate decreases from $t$ to $t'$.
As before the AD shifts to the right (because the IS curve has shifted up due to the tax cut).

But this is not the end of the story!!

Recall that the labour is supplied by the households.

If all household incomes are taxed then so would be wage income! Hence the effective wage rate - relevant for the households (and only for the households) is now \((1 - t)W\) - not \(W\)!

In other words, the supply equation in the labour market now becomes:

\[
W = \frac{P}{(1 - t)} g(N); \quad g'(N) > 0
\]

The demand equation in the labour market however remains unchanged (Why?).
A tax cut shifts the labour supply schedule (in the $W-N$ plane), but not the labour demand schedule. Hence the AS schedule gets affected too!
Thus a proportional income tax on household income - in particular a tax cut - could be effective in raising the equilibrium output and employment under the classical system:

This tax-cut policy augers well with the ‘minimalist government’ (neoliberal?) philosophy (as exemplified by the Reagan-Thatcher era in the 1980s).
Emergence of the Keynesian System: The Great Depression (1929-1941)

- The western capitalist economies were more or less cruising along the Classical/Neo-classical path till the late 1920s - with a laissez faire government allowing private initiatives to flourish.
- Then came the Great Depression which created havoc in the western capitalist world - especially in the US economy.
“The Great Depression of 1929 devastated the U.S. economy. Half of all banks failed. Unemployment rose to 25 percent and homelessness increased. Housing prices plummeted 30 percent, global trade collapsed by 60 percent and prices fell 10 percent. ....The economy shrank 50 percent in the first five years of the Depression. In 1929, economic output was $105 billion, as measured by gross domestic product. By 1933, the country had suffered five years of losses. It only produced $57 billion, half what it produced in 1929. New Deal spending boosted GDP growth 10.8 percent in 1934. It grew another 8.9 percent in 1935, a whopping 12.9 percent in 1936 and 5.1 percent in 1937. Unfortunately, the government cut back on New Deal spending in 1938, and the depression returned.” (K. Amadeo (2017)).

Shift in ideology: The dominant economic belief shifted from a pure free market economy to a mixed economy with much more emphasis on government spending for its stability. This new school of thought owed its origin to Keynes’ General Theory (1936).
The Keynesian System:

- The benchmark Keynesian system that we shall consider here will be almost analogous to the Classical system characterized above, except for the labour supply equation.
- The Keynesian model assumes that the labour market is unionized and the supply of labour is therefore determined by the rules/norms set by the labour union.
- The union sets the nominal wage rate at some level $\bar{W}$ by collective bargaining and once the wage is set, all workers supply their entire labour stock at this wage rate. (Why all workers would comply to such a rule is a different story and would require precise modelling of the union’s and the workers’ optimization problem(s). We shall come back to this point when we discuss the microfoundations of these assumptions).
- Thus the labour supply schedule now becomes perfectly elastic (a flat line) at the union-determined wage rate: $\bar{W}$. 
The Keynesian System (in equations):

- **The Goods Market:**
  - Supply Equation:
    \[ Y = F(N, K); F_N, F_K > 0; F_{NN}, F_{KK} < 0 \]  
    (8)
  - Demand Equation:
    \[ Y = C(Y) + I(r) + \bar{G}; 0 < C'(Y) < 1; I'(r) < 0 \]  
    (9)

- **The Labour Market:**
  - Supply Equation:
    \[ W = \bar{W} \]  
    (10)
  - Demand Equation:
    \[ W = Pf(N) \]  
    (11)

- **The Money Market:**
  - Supply Equation:
    \[ M = \bar{M} \]  
    (12)
  - Demand Equation:
    \[ M = PL(Y, r); L_Y > 0; L_r < 0 \]  
    (13)
As we have explained before, the only equation that differs between the two systems is the labour supply equation.

The Keynesian System assumes that labour supply is perfectly elastic at a given wage rate $\bar{W}$.

**The Labour Market:**
- Supply Equation:
  \[ W = \bar{W} \] (14)
- Demand Equation:
  \[ W = Pf(N) \] (15)
Equilibrium in Keynesian Labour Market & the corresponding AS Schedule:
Equilibrium in Keynesian Labour Market & the corresponding AS Schedule:

- So the AS schedule is **upward sloping** under the Keynesian system.
- Notice however that as far as equations (2), (5) and (6) are concerned, nothing has changed, which means the AD curve in the Keynesian system is identical to the AD curve of the Classical System.
- The equilibrium in the Keynesian system is determined by the intersection of the (now upward sloping) AS and the AD schedule:
Effectiveness of Government Policies under the Keynesian System:

**Question:** What does this tell you about the effectiveness of the standard monetary and fiscal policies (↑ in $\tilde{G}$ or $\tilde{M}$)?
Equilibrium Employment: Comparing Keynes with Classics

- How does $\tilde{N}$ (equilibrium level of employment under the Keynesian System) compare with $N^*$ (equilibrium level of employment under the Classical system)?

- Or equivalently: how does $\tilde{Y}$ (equilibrium output under the Keynesian System) compare with $Y^*$ (equilibrium output under the Classical system)?

- To answer this question, we shall consider two cases:
  (a) $\tilde{W} > W^*$
  (b) $\tilde{W} < W^*$
Let us start with the case when $\bar{W} > W^*$. Let us first diagrammatically depict the Classical Equilibrium ($P^*, Y^*, N^*, W^*$) and then see whether this can still be an equilibrium when the nominal wage rate is arbitrarily fixed at some $\bar{W} > W^*$:
When labour supply is perfectly elastic at $W = \bar{W}$, the classical labour demand function (at $P^*$) intersects $\bar{W}$ at some $N'$ - resulting in $Y'$ amount of output being supplied. In other words, $(P^*, Y')$ constitute a point on the Keynesian AS curve. But this cannot be an equilibrium point since at this point $AD > AS$. 
This tells us that when $\bar{W} > W^*$:

- The Keynesian equilibrium price ($\hat{P}$) must be *higher* than the Classical equilibrium price level ($P^*$).
- But since higher $P$ means lower demand (along a downward sloping AD curve), this implies that the Keynesian equilibrium output ($\hat{Y}$) must be *less* than the Classical equilibrium output level ($Y^*$).
- Consequently, equilibrium employment level under the Keynesian system ($\hat{N}$) must be lower than the equilibrium employment level under the Classical system ($N^*$).
When $\bar{W} > W^*$, the relative position of $\bar{N}$ vis-a-vis $N^*$ is shown in the diagram below:
Is the reduction in employment from $N^*$ to $\tilde{N}$ involuntary or involuntary?

A flat Keynesian labour supply schedule at $W = \tilde{W}$ does not allow us to properly identify the extent of voluntary vis-a-vis involuntary unemployment.

To differentiate between voluntary and involuntary unemployment, we have to draw the classical labour supply schedule (that captures household’ willingness to work) for the given $\tilde{W}$ and $\tilde{P}$. 

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Once we draw this schedule, it is easy to see that indeed the entire gap of \((N^* - \tilde{N})\) represents involuntary unemployment.

In fact, at \(\bar{W}\) and \(\bar{P}\), the level of involuntary unemployment is even greater than \((N^* - \tilde{N})\), as shown by the difference \((N'' - \tilde{N})\) in the figure below. (Note however that \(N''\) is not an equilibrium configuration).
What happens is the Keynesian System if trade union fixes the nominal wage rate at a level such that $\bar{W} < W^*$?

We can analyse as before and show that now $\tilde{N} > N^*$. But notice that now part of this labour supply is ‘forced labour’!!

If is not clear why a worker would be part of trade union if the union forces him to work beyond his optimal choice. So from now on we shall focus only on the case where $\bar{W} > W^*$. 

\[ \text{Diagram showing the relationship between wage, employment, and output} \]
Suppose now the labour union becomes stronger and is able to negotiate a higher nominal wage $\bar{W}'$.

What happens to equilibrium output in the Keynesian System if the nominal wage rate changes (increases) from $\bar{W}$ to $\bar{W}'$?

In particular, would the households be better off in terms of income?
What happens when $\bar{W}$ goes up:

- The nominal wage rate $W$ increases.
- The labor demand $N^D$ decreases with the price level $P$.
- The output $Y$ increases.

Graphically, this is represented by the downward shift in the labor demand curve $N^D$ and the upward shift in the output curve $Y$. The new equilibrium is at $W'$, $N''$, and $Y''$. 
The AS schedule shifts to the left - reducing the equilibrium level of output and increasing the equilibrium price level.

**Question:** What about the real wage rate? Would the real wage rate be higher/lower or remain the same in the new equilibrium?
As it turns out, in the new equilibrium the real wage rate actually increases. (In other words, although the price level in the new equilibrium will be higher, it will not be high enough to completely outweigh the increase in the nominal wage rate). (Why?)

Are the households better off? The answer is not clear! While real wage rate is indeed higher than before, total income has actually gone down. So the households for whom wage component is high will be better off but at the expense of others.

Thus there might be some political economy justifications for maintaining a fixed wage-flexi price Keynesian macroeconomic structure compared to a flexi wage-flexi price Classical/Neoclassical set up, even though the latter (without government intervention) leads to lower equilibrium output.
What happens to the real wage rate when there is an exogenous rise in the aggregate demand either due to some exogenous demand shocks (when the autonomous component of the demand increases) or due to expansionary government policies (when \( \bar{G} \) or \( \bar{M} \) increases)? This leads to a shift in the AD curve (with unchanged AS curve).

So the price level rises in the new equilibrium, and with a constant \( \bar{W} \), the real wage rate surely falls.

Notice that the real wage rate in this model behaves in a **counter-cyclical** fashion: In periods of high demand or economic boom, we have higher equilibrium output and employment, but it is associated with lower real wages. Opposite happens in periods of low demand or slump.

In other words, in this version of the Keynesian model real wage rate and aggregate output (and employment) are *negatively* correlated.
This feature of the model is not supported by the empirical studies carried out after World War II.

It has been observed that real wage rate moves mildly in a **pro-cyclical** manner in tandem with employment/output. There certainly has been no evidence of the real wage moving in a counter-cyclical manner, as the Keynesian model discussed above would suggest.

"The correlation between changes in real wages and changes in output and employment is usually slightly positive but often statistically insignificant" – Blanchard and Fischer (1993)
An extension of the general Keynesian structure was later proposed, which was able to address this issue, while retaining the other basic Keynesian features. This is the Neo-Keynesian extension.

This extension assumes that not only the nominal wage is rigid, but so is the nominal price level.

Sticky prices mean that the aggregate supply curve is horizontal at some \( \bar{P} \).

Notice that a horizontal AS schedule means that equilibrium output in this system is completely demand-determined. At \( \bar{P} \) whatever output demanded is always supplied. (Thus this set up is diametrically opposite to the supply-determined Classical System discussed earlier).
The sticky price scenario is often justified by the assumption that there is imperfect competition in the final goods market; firms can set their own prices. Typically facing a constant nominal wage cost they set a price which is a mark up ($\lambda$) over the nominal cost such that $P = (1 + \lambda)W$.

But often firms do not adjust the price level immediately in response to an increase in $W$.

This could be because of a variety of reasons:

- There could be adjustment costs associated with price change (menu cost) due to which it may not be optimal for the firms to change their prices immediately;
- The firms might have limited bargaining power vis-a-vis the unions so that the union is able to extract a higher real wage (in which case $\lambda$ adjusts keeping $P$ unchanged).
In what follows, we shall however refrain from providing any firm-side justification to sticky prices.

We shall simply assume that $\bar{P}$ does not respond to a change in the wage rate for some exogenous reasons, while keeping the earlier (Keynesian) firm-side story in the background.

It is as if the government or some exogenous regulatory authority has fixed the price level at some $\bar{P}$ and firms have no choice but to operate within that regulatory framework.
In the sticky price scenario, the aggregate supply curve is horizontal. The equilibrium output is now completely determined by the position of the demand curve - in particular by the level of aggregate demand at the price level \( P = \bar{P} \).
The crucial question is: At what level would the price be set?

Notice that we can plot the Keynesian (upward-sloping) AS schedule in the backdrop and identify two regions with reference to the Keynesian equilibrium price level ($\hat{P}$):

- The region above $\hat{P}$ (above the intersection point of the Keynesian AS & AD schedules) which is **demand-constrained**;
- The region below $\hat{P}$ (below the intersection point of the Keynesian AS & AD schedules) which is **supply-constrained**.

Can we ever have a Neo-Keynesian scenario where the $\bar{P}$ is set below $\hat{P}$?
Well, we can...but in that case the AS supply curve will no longer be infinitely elastic. In fact it will be vertical for any $\bar{P} < \bar{P}$.

Why? The reason is as follows:

Notice that for any arbitrary $(\bar{P}, \bar{W})$, the profit-maximizing level of output $Y^*(\bar{P}, \bar{W})$ actually coincides with a point on the AS curve under the Keynesian system:

This being the profit-maximization point (given $\bar{P}, \bar{W}$), firms would actually like to produce upto this level - *if they could!*

But they cannot do so *if the economy is demand constrained* (since at this price level, the aggregate demand falls short of the profit maximising level of output).
But this argument does not hold if at this price level, the economy is actually supply constrained!

When the economy is supply-constrained, the producers can choose output level that maximizes their profit (given \( \bar{P}, \bar{W} \)).

That is, they can actually pick a point on the Keynesian AS curve (given \( \bar{P}, \bar{W} \)).

Indeed facing a wage-price combination of (\( \bar{P}, \bar{W} \)), a firm would never have any incentive to produce beyond \( Y^*(\bar{P}, \bar{W}) \).

This implies that we have to draw the Neo-Keynesian AS schedule (with sticky prices and sticky wages) a little differently that we did before:

- At the given \( \bar{P} \), it is no longer horizontal for all \( Y \); it becomes vertical precisely at the point \( Y^*(\bar{P}, \bar{W}) \).
Labour Market under Sticky Prices:

- Let us now assume that $\bar{P}$ is such that the economy is indeed demand-constrained.

- We have seen that in this case, stickiness of the price level implies quantity adjustment by the firms: they produce exactly as much output as is demanded.

- This quantity adjustment will have implications for the labour demand function as well.

- When prices are sticky and the economy is demand-constrained, the labour demand function is given by:

$$N^D = \hat{N}(\bar{P}) : F(\hat{N}, \bar{K}) = \hat{Y}(\bar{P}).$$

- Since the labour supply is horizontal at $\bar{W}$, $\hat{N}(\bar{P})$ also represents the equilibrium employment level under the Neo-Keynesian system.
It is important to recognise here that due to the stickiness of the price that is set at a region where there is excess supply, the producers are not operating on their AS schedule.

This implies that they are not supplying their profit-maximising level of output, which in turn means they are unable to operate on their optimal labour demand curve (which presupposes profit maximising behaviour). Thus the labour demand schedule now becomes irrelevant in determining equilibrium level of employment.
The following diagram characterizes the equilibrium in the Neo-Keynesian system:
Now let’s see what happens when the nominal wage rate when $\bar{W}$ goes up:

Since $\bar{P}$ does not respond to a change in $\bar{W}$ (by assumption), nothing changes in the Goods Market. The equilibrium output is still given by $\hat{Y}(\bar{P})$. Hence so is equilibrium employment: $\hat{N}(\bar{P})$.

In fact nothing changes except that now the real wage is higher which implies that there is a re-distribution of income from profit-earners to wage-earners.
Now consider a positive demand shock: suppose for some reason the aggregate demand schedule shifts to the right. This could be policy-induced (e.g., a change in $\bar{G}$ or $\bar{M}$) or could be due to an independent shift in parameters. However let us assume that the economy is still demand-constrained.

Notice that with sticky prices and rigid nominal wages, real wage rate remains the same, while output and employment goes up in the new equilibrium.

So even though real wage now is not exactly pro-cyclical, at least it does not move in the opposite direction to output and employment! (In fact the total wage bill would actually go up.)
There are other variants of the Neo-Keynesian framework that assume that union’s bargaining power increases as level of employment goes up.

In such a case, a positive aggregate demand shock (with sticky prices) would increase the real wage rate as well. (Aggregate profit would also rise, although the mark up/profit margin would go down).

Thus real wage and output (as well as employment) would now move in the same direction, making the real wage rate pro-cyclical as is consistent with the empirical evidence.
Other Extensions of Keynes & the Classics:

- **Quantity Theory of Money** (A Special Case of the Classical System):
  - Money Demand Equation now becomes:

    \[ M = PkY; \]

    where \( k \) is a positive constant (related to the velocity of circulation of money)
  - Notice that this still generates a downward sloping AD schedule (**why?**), while the AS schedule remains vertical as before.
  - Nothing much changes in this special case of the Classical System in terms of equilibrium output/employment.

- **Question**: What is the role of the IS curve here?
Liquidity Trap/Interest Rate Targeting (A Special Case of the Keynesian System)

- Equation of the LM curve now becomes:

\[ r = \bar{r} \]

(At this interest rate money supply is perfectly elastic.)
- The aggregate demand schedule in the Y-P plane is now vertical.
- In this special Keynesian System, output is completely demand determined. Since the level of demand is now independent of the price level, there is only quantity adjustment - even though prices are fully flexible.

Question: What happens if we import this assumption of liquidity trap/interest rate targeting to an otherwise Classical System?
Autonomous Investment (Another Special Case of the Keynesian System):

- Equation of the IS curve now becomes:

\[ Y = C(Y) + \bar{I} + \bar{G} \]

- The aggregate demand schedule in the $Y$-$P$ plane is once again vertical.
- In this special Keynesian System once again output is completely demand-determined. Again, there is only quantity adjustment - even though prices are fully flexible.

Question: What happens if we import this assumption of autonomous investment to an otherwise Classical System?
We have now seen different variants of the Keynesian and the Classical System.

No matter which specific set of assumptions one takes, the starkest difference between these two categories of models (in terms of characterization of the equilibrium) is as follows:

- In the Keynesian system, demand plays a crucial role in determining the equilibrium output.
- In the Classical model, demand plays no role in determining the equilibrium output; it is completely supply driven.

To the extent that government policies affect the demand side (and only the demand side) of the economy, such policies would work in the Keynesian system but would not work in the Classical System.

Notice however that any policy that affects the supply side of the economy will work in the Classical system as well as in the Keynesian system (but may not work in the two special cases of the Keynesian system).
So far we have assumed that workers (households) as well as firms have complete information about the prices and wages that would prevail in the actual economy.

In fact, the classical system that we have discussed represents the ideal scenario - there is no market imperfection or rigidity in any market, nor is there any incomplete information.

We could treat this as our benchmark case - the best possible scenario (what would have happened if everything were perfect!)

This ideal scenario - the benchmark case - is also somewhat unrealistic. The real world is characterized by various kinds of market imperfections or rigidities as well as incomplete information.

We have already seen what happens if there are various kinds of rigidities.

But even in the absence of market imperfections, things could be far from perfect simply because agents have incomplete information.

We now turn to one such case.
Incomplete Information and Role of Expectations:

- Let us consider a modified version of the classical system, where the firms have full information about the wages and prices, but workers do not have complete information.

- In particular, let us assume that workers do not have complete information about the price level. (This is the Lucas model of incomplete information.)

  - The underlying logic is that since prices are determined in the goods market while nominal wages are set in the labour market, workers often do not have perfect knowledge about the price behaviour.

- If the workers do not have perfect knowledge about the price level that prevails, then they would make their calculations on the basis of their expectations about the price level.

- In other words, in this model with incomplete information, workers determine their labour supply on the basis of the ‘expected’ real wage.
Notice that till now we have deliberately kept expectations out of the picture. But the moment we bring in incomplete information, expectations start playing a major role. In this modified Classical System, workers’ labour supply schedule now depends on the expected real wage rate. If we incorporate this in our standard Classical system, then the AS schedule under the classical system may change its character - as we will see in a moment.
AS Schedule in the Classical System when Labour Supply depends on Expected Real Wage:

- When the workers determine their labour supply on the basis of the ‘expected’ real wage, the labour supply equation is given by:

\[ N^S : W = P^e g(N); \quad g' > 0 \]

- The labour demand equation remains unchanged (because producers’ are assumed to have complete information about the price of the product that they themselves would be selling):

\[ W = PF_N(N, \bar{K}) \]

- The labour market equilibrium now depends crucially on how price expectations are formed.

- If workers can perfectly anticipate the actual price level, then \( P^e = P \) and we are back to the good old Classical world with a vertical AS curve.
If, on the other hand, $P^e$ gets determined quite independent of the current price level (e.g., by past prices), then AS completely changes its character.

The AS schedule is now upward sloping - just as it was in standard the Keynesian system!

Thus standard fiscal and monetary policies would be effective in this ‘modified’ Classical system - although there is no wage or price rigidity!
How are Expected Prices Determined?

- It seems a little unrealistic to assume that the agent’s expectations would be completely independent of the actual value of the variable.
- But to see exactly how they are related we have to look for some theories of expectation formation, which we shall discuss now.
Various Theories of Expectation Formation:

- **Static Expectations:**
  Today’s expected value of the variable \((x)\) depends on previous period’s actual value. In particular:
  \[
x^e_t = x_{t-1}
  \]

- **Adaptive Expectations:**
  Today’s expected value of the variable \((x)\) depends on previous period’s actual value and previous period’s expected value. In particular:
  \[
x^e_t = x^e_{t-1} + \lambda [x_{t-1} - x^e_{t-1}]; \quad 0 < \lambda < 1
  \]

- Notice that Static Expectations is a special case of Adaptive Expectations (when \(\lambda = 1\))
Various Theories of Expectation Formation (Contd.):

- **Perfect Foresight:**
  Agents make a **guess** about the value of the variable and (by some divine power) the guess exactly matches its actual value. In particular:
  \[ x_t^e = x_t \]
  Notice that guessing is **NOT** knowing!

- **Rational Expectations:**
  Agents apply statistical tools of expectation formation, using the available information set, to come up with the expected value of the variable. In particular:
  \[ x_t^e = E[x_t | I_{t-1}] \]
  Notice that under complete information and complete certainty, Perfect Foresight and Rational Expectations are equivalent (but their underlying mechanisms are still different).
We have now specified 4 different expectation formation rules.

There are more sophisticated rules of expectation formation which explicitly incorporate a learning process (e.g., Bayesian Learning).

More recent theories of expectation formation also talk about ‘optimally’ pegging one’s expectation so as to minimize the (expected) loss from forming ‘wrong’ expectations, subject to costly information gathering.

However in this course we shall limit ourselves to the 4 simple rules specified above.

We shall now apply these rule one by one to a particular macroeconomic system.

A natural choice is Lucas’ Incomplete Information model which brings in a role of expectations in the labour market.
Recall that in the Classical system, when both workers and producers base their supply/demand decisions on the actual real wage \( \left( \frac{W_t}{P_t} \right) \), then the labour market equilibrium is given by:

\[
\tilde{N}^* : g(N) = F_N(N, \tilde{K})
\]

This \( \tilde{N}^* \) - which we defined earlier as the ‘**Natural Level of Employment**’ - is independent of the current price level \( (P_t) \).
On the other hand, when workers base their supply decisions on the expected real wage \( \left( \frac{W_t}{P^e_t} \right) \), then the actual level of employment differs from the natural rate \( (\bar{N}^*) \) in the following way:
An Application of Various Expectation Formation Rules (Contd.):

- In other words,
  \[ N_t \geq N^* \quad \text{according as} \quad P^e_t \leq P_t. \]

- Define \( \bar{Y}^* \) as the ‘natural level of output’ such that
  \[ \bar{Y}^* = F(\bar{N}^*, \bar{K}). \]

- When workers base their supply decisions on the expected real wage, then the actual output supplied differs from the natural level (\( \bar{Y}^* \)) in the following way:
  \[ Y^*_s \geq \bar{Y}^* \quad \text{according as} \quad P^e_t \leq P_t. \]

- This allows us to write the Aggregate Supply schedule in the following way:
  \[ Y^*_t : Y_t = \bar{Y}^* + \hat{f}(P_t - P^e_t); \quad \hat{f}(0) = 0; \quad \hat{f}' > 0. \]
This representation of the aggregate supply schedule is called the **Lucas Supply Function** (after Robert Lucas, who postulated that (in the short run) workers’ may not have complete information about the price behaviour and hence their expectations may differ from the actual.)

Without any loss of generality, let us assume that the Lucas Supply Function (i.e., the AS schedule under incomplete information) is linear:

\[
Y_t^s : Y_t = \bar{Y}^* + \alpha [P_t - P_t^e]; \quad \alpha > 0.
\]  

The equilibrium price level will of course depend on aggregate demand function, which we now now turn to.

**Aside:** The Lucas Supply Function has a close relationship with the **Expectation-augmented Phillips Curve**. We shall come back to this point later.
An Application of Various Expectation Formation Rules (Contd.):

- we know that the Aggregate Demand schedule is a decreasing function of the price level:

\[ Y_t^d : Y_t = h(P_t); \quad h' < 0 \]

- We also know that aggregate demand increases corresponding to any increase in the policy parameters \( \bar{G}, \bar{M} \). Thus

\[ Y_t^d : Y_t = h(P_t; \bar{G}; \bar{M}); \quad h' < 0; \quad \frac{\partial Y}{\partial \bar{G}} > 0; \quad \frac{\partial Y}{\partial \bar{M}} > 0 \]

- Without any loss of generality, let us again assume that the AD schedule is linear:

\[ Y_t^d : Y_t = -\mu P_t + \gamma \bar{G} + \mu \bar{M}; \quad \gamma, \mu > 0 \] (II)

**Question:** In the AD schedule written above, why have we attributed the same coefficient (\( \mu \)) to both \( P_t \) and \( \bar{M} \)?
From the AS and the AD schedule (given by (I) and (II) respectively, we can solve for the equilibrium price level at time $t$ as:

$$P_t : \bar{Y} + \alpha [P_t - P^e_t] = -\mu P_t + \gamma \bar{G} + \mu \bar{M}$$

$$\Rightarrow P_t = \frac{1}{\alpha + \mu} [\gamma \bar{G} + \mu \bar{M} - \bar{Y} + \alpha P^e_t] \quad (III)$$

Equation (III) gives us the precise relationship between actual price level and expected price level in this economy at every point of time.

Let us now apply the different theories of expectation formation to equation (III) and see how the behaviour of the aggregate economy changes (if at all) over time.
An Application of Various Expectation Formation Rules
(Contd.):

- We know that the equilibrium price level at time $t$ is determined by the following equation:

$$P_t = \frac{1}{\alpha + \mu} \left[ \gamma \bar{G} + \mu \bar{M} - \bar{Y}^* + \alpha P^e_t \right] \quad (III)$$

- **Under Static Expectations:**
  - $P^e_t = P_{t-1}$
  - Plugging this value of $P^e_t$ in equation (III) we get a single difference equation in $P_t$, which will determine the movement of equilibrium price level over time:

$$P_t = \frac{\alpha}{\alpha + \mu} P_{t-1} + \frac{1}{\alpha + \mu} \left[ \gamma \bar{G} + \mu \bar{M} - \bar{Y}^* \right]$$
An Application of Various Expectation Formation Rules (Contd.):

- **Under Adaptive Expectations:**

  - \( P_t^e = P_{t-1}^e + \lambda \left[ P_{t-1} - P_{t-1}^e \right] \)

  - Plugging this value of \( P_t^e \) in equation (III) we get a system of two difference equations in two variables, \( P_t \) and \( P_t^e \), which will simultaneously determine the movement of equilibrium price level as well as expected price level over time:

    \[
    P_t = \alpha \lambda \frac{P_{t-1}}{\alpha + \mu} + \alpha \frac{(1 - \lambda)}{\alpha + \mu} P_{t-1}^e + \frac{1}{\alpha + \mu} \left[ \gamma \tilde{G} + \mu \tilde{M} - \tilde{Y}^* \right] (1)
    \]

    \[
    P_t^e = \lambda P_{t-1} + (1 - \lambda) P_{t-1}^e\quad (2)
    \]
An Application of Various Expectation Formation Rules (Contd.):

- **Under Perfect Foresight:**
  - $P_t^e = P_t$
  - Plugging this value of $P_t^e$ in equation (III) we get a unique solution for the equilibrium price ($P_t$), which must be the ‘perfect foresight’ solution to the system:

\[
\left(1 - \frac{\alpha}{\alpha + \mu}\right) P_t = \frac{1}{\alpha + \mu} \left[\gamma \bar{G} + \mu \bar{M} - \bar{Y}^*\right]
\]

\[
\Rightarrow P_t = \frac{1}{\mu} \left[\gamma \bar{G} + \mu \bar{M} - \bar{Y}^*\right] = P_t^e
\]

- Notice that perfect foresight does not suggest that any *arbitrary* expectation held by the agent would necessarily be fulfilled.
- The expectations of economic agents would have to be **consistent** with the models used to explain their behavior.
An Application of Various Expectation Formation Rules (Contd.):

- **Under Rational Expectations:**
  - \( P_t^e = E[P_t | I_{t-1}] \)
  - Notice that under perfect certainty, the information set, \( I_{t-1} \), would include the information that the equilibrium price level in every period is determined by:
    
    \[
    P_t = \frac{1}{\alpha + \mu} \left[ \gamma \bar{G} + \mu \bar{M} - \bar{Y}^* + \alpha P_t^e \right]
    \]
  - Hence when agents form their expectations, they will utilize this information. In other words:
    
    \[
    E(P_t) = E \left[ \frac{1}{\alpha + \mu} \left[ \gamma \bar{G} + \mu \bar{M} - \bar{Y}^* + \alpha E(P_t) \right] \right]
    \]
    
    \[
    = \frac{1}{\alpha + \mu} \left[ \gamma \bar{G} + \mu \bar{M} - \bar{Y}^* \right] + \frac{\alpha}{\alpha + \mu} E(P_t)
    \]
    
    \[
    \Rightarrow E(P_t) = \frac{1}{\mu} \left[ \gamma \bar{G} + \mu \bar{M} - \bar{Y}^* \right]
    \]
Notice that once we replace this value of $E(P_t)$ in the equilibrium price determination equation (III), we get

$$P_t = \frac{1}{\mu} \left[ \gamma \bar{G} + \mu \bar{M} - \bar{Y}^* \right] = E(P_t)$$

Notice that under rational expectation hypothesis, expectations of economic agents are model-consistent by construction. The expectations are derived from the model itself.

**Point to note**: The rational expectation solution and the perfect foresight solution are identical, although the underlying mechanisms of arriving at the two solutions are different!
Difference between Perfect Foresight and Rational Expectation Solutions Under Uncertainty:

- Let us now introduce some uncertainty in the system such that the price determination equation is given by:

\[
P_t = \frac{1}{\alpha + \mu} \left[ \gamma \bar{G} + \mu \bar{M} - \bar{Y}^* + \alpha P_t^e \right] + \epsilon_t
\]  

(IIIa)

where \( \epsilon_t \) is a random variable with an expected value of \( \bar{\epsilon} \).

- In this case the perfect foresight solution is given by:

\[
P_t = \frac{1}{\mu} \left[ \gamma \bar{G} + \mu \bar{M} - \bar{Y}^* \right] + \frac{\alpha + \mu}{\mu} \epsilon_t = P_t^e.
\]

- On the other hand the Rational Expectation solutions for expected price and actual price are given by:

\[
E(P_t) = \frac{1}{\mu} \left[ \gamma \bar{G} + \mu \bar{M} - \bar{Y}^* \right] + \frac{\alpha + \mu}{\mu} \bar{\epsilon} \\
P_t = \frac{1}{\mu} \left[ \gamma \bar{G} + \mu \bar{M} - \bar{Y}^* \right] + \frac{\alpha + \mu}{\mu} \epsilon_t
\]
Notice that since the realized value of the random term $\epsilon_t$ may not be equal to its expected value $\bar{\epsilon}$,

- the solution for expected price level under perfect foresight and under rational expectation now differ.
- In fact, under perfect foresight, the expected price level still coincides with its actual value.
- However, under rational expectations, the expected price level now differs from the actual price level due to the existence of a random surprise term $(\epsilon_t - \bar{\epsilon})$. 

Existence of No or Multiple Perfect Foresight/Rational Expectation Solutions:

Food for Thought:

- In this model (because the functional forms are assumed to be linear) we end up with a ‘unique’ perfect foresight/rational expectation solution.
- It is conceivable that even with these linear equations, a economically meaningful solution does not exist. In that case perfect foresight/rational expectation becomes a vacuous concept; there is no model-consistent solution possible!
- It is also conceivable that these equations are not linear at all. Then there may exist multiple solutions to the same equation.
- In such a scenario, would agents’ expectations be necessarily met even under the assumption of perfect foresight/rational expectation (with complete certainty and perfect information)?
- The answer is "no" - unless all agents coordinate and everybody picks the same value among these multiple solutions! This in fact is one of the problematic areas of the perfect foresight/rational expectation hypothesis, which we shall come back to later in the course.
Lucas Supply Function and The Phillips Curve:

- Recall that the Lucas Supply function was derived from the Imperfect Information Classical model (of Lucas), which generated the following relationship:

\[ N_t \gtrless N^* \quad \text{according as} \quad P^e_t \lesseqqgtr P_t. \]

- Now define the "natural level of unemployment" as the difference between the "Full-employment" level and the "natural" level of employment:

\[ U^* = \bar{N} - N^* \]

- The corresponding natural rate of unemployment can be defined as:

\[ u^* = \frac{\bar{N} - N^*}{\bar{N}} = 1 - \frac{N^*}{\bar{N}} \]

- Notice that for any given \( \bar{N} \) (total available labour force), \( u^* \) and \( N^* \) are negatively related.

- Hence we can write the above relationship as follows:

\[ u_t \lesseqqgtr u^* \quad \text{according as} \quad P^e_t \lesseqqgtr P_t. \]
In other words,

\[ u_t = u^* + g(P_t - P^e_t); \quad g(0) = 0; \quad g' < 0. \]

Assuming a linear form,

\[ u_t = u^* - \beta [P_t - P^e_t]; \quad \beta > 0 \]

\[ \Rightarrow P_t = P^e_t - \frac{1}{\beta} [u_t - u^*] \]

\[ \Rightarrow (P_t - P_{t-1}) = (P^e_t - P_{t-1}) - \frac{1}{\beta} [u_t - u^*] \]

This indicates that for a given \( P_{t-1} \),

\[ \pi_t = \pi^e_t - \gamma [u_t - u^*]; \quad \gamma > 0. \]
This is the so-called Phillips Curve, which postulates a negative relationship between inflation rate and unemployment rate (for any given value of expected rate of inflation).

One can now bring in various expectation formation rules and see how the nature of the Phillips curve relationship changes under various assumptions about expectation formation.

One can also verify the standard macroeconomic proposition that the Phillips curve is downward sloping in the short run (or when agents have adaptive expectations) but vertical in the long run (or when agents have rational expectations).

I leave all these exercises to you as homework.
Expectation Formation: What is the "right" theory?

- How do people form their expectations in reality?
- Which of the expectation formation rules explains data better?
- What is the policy implication of each of these expectation formation rules?
- These are all important questions. Unfortunately there is no clear answer to any of these questions. Even data do not help since it is difficult to collect data on how people form their expectations.

"... like utility, expectations are not observed, and surveys cannot be used to test the Rational Expectations hypothesis. One can only test if some theory, whether it incorporates Rational Expectations or for that matter, irrational expectations, is or is not consistent with observations." - Edward Prescott (1977)

- There are some recent studies in experimental economics where researchers have tried to identify whether people have rational expectations or not; But the conclusions are still tentative and at a very nascent stage.
As we have mentioned at the beginning of the class, these static (one period) macro models have two major short-comings:

- They are all based on ‘ad-hoc’ assumptions. Prima facie it is not obvious that all these assumptions can be substantiated by explicit optimizing behaviour of agents;
- They ignore all intertemporal (dynamic) issues, even when we know that savings, investment etc. will necessarily have implications for future and therefore completely ignoring the future in current decision making process does not seem right.

In the next few topics, we shall try to address each of these criticisms one by one.
A word of caution: I do not follow any particular textbook *ad verbatim*. Thus the references are only suggestive; they are meant to be read as a supplementary reading and not as a substitute for the lecture notes.

The book that I have followed more closely than any other:

- William Scarth: "Macroeconomics: The Development of Modern Methods for Policy Analysis", Edward Elgar, 2014, Chapter 1. (Scanned copy of the chapter is available in the course web page)

For a broad overview of the current topic, you may also consult the following book:

- Wendy Carlin & David Soskice: "Macroeconomics: Imperfections, Institutions & Policies", Oxford University Press, 2006, Chapter 2. (Scanned copy of the chapter is available in the course web page)