004: Macroeconomic Theory

Micro Foundations of Macroeconomic Systems

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Micro-foundations of various Macroeconomic Systems:

- So far we have discussed various macroeconomic systems (Classical, Keynesian and their different extensions).
- Recall that we have presented each system as a bunch of 'adhoc' equations that are supposed to define the macroeconomy as a whole.
- We made no attempt to derive the underlying micro-behaviour of **optimizing** agents that would generate these aggregative equations for the macro-economy.
- Question is: Can these aggregative behavioural equations be reconciled with some kind of optimal behaviour of firms/households at the micro level?
- Put differently, can we provide some micro-foundation to these aggregative equations?

Micro Foundations:

- We now attempt to develop the micro-foundations of some of these equations. In the process we also demonstrate some of these micro-foundations would work *only* in a dynamic set up; hence there is a need to move from the static to a dynamic framework.
- Micro-foundations are thought to be necessary for two reasons:
  - There are many implicit assumptions that are made in formulating the aggregative relationships. A precise discussion of the micro-foundations allows us to highlight these assumptions and also check for their validity;
  - As Robert Lucas pointed out, many of the constants in the aggregative systems are not parameters in the true sense of the term; they capture equilibrium behaviour under certain conditions (e.g. ceratin expectations about the policies, environment etc). As those conditions change, people optimally change their equilibrium behaviour; hence these 'reduced form' terms also change their values. So predictions/forecasts about the economy based on these constants could go wrong unless one actually derives these optimal values from the underlying micro-foundations.

Micro Foundations: Classical/Keynesian Production Side Story

- We start with the micro-foundations of the production relations.
- Recall that the production side story for Classical and Keynesian systems are identical: both assume that firms operate in a *perfectly competetive market structure* with a production technology which exhibits the following properties:

$$Y_i = F(N_i, K_i)$$

  - Constant returns to scale (CRS)/Homegenous of degree one: $F(\lambda N_i, \lambda K_i) = \lambda F(N_i, K_i)$
  - Positive but diminishing returns in each factor: $F_{N_i}, F_{K_i} > 0$; $F_{N_i N_i}, F_{K_i K_i} < 0$
  - Both inputs are essential: $F(0, K_i) = F(N_i, 0) = 0$
  - Inada conditions:

$$\lim_{N_i \to 0} F_N(N_i, K_i) = \infty; \lim_{N_i \to \infty} F_N(N_i, K_i) = 0;$$

$$\lim_{K_i \to 0} F_K(N_i, K_i) = \infty; \lim_{K_i \to \infty} F_K(N_i, K_i) = 0$$

Classical/Keynesian Production Side Story (Contd.)

- Any production function that satisfies all the above properties is called a '*Neoclassical*' production function.
- Example: Cobb Douglas Technology: $Y_i = N_i^{\alpha} K_i^{1-\alpha}; 0 < \alpha < 1$.
- Sometimes the production function is also written as

$$Y_i = AF(N_i, K_i)$$

  where A is an index of technology that captures the economy-wide productivity level or *total factor productivity* (TFP). Under the *neoclassical* technology, the TFP term is assumed to be exogenously determined.
- In a static framework, A is typically assumed to be a constant.(Notice that A is not firm-specific; it relates to the entire economy).

Classical/Keynesian Production Side Story (Contd.)

- Suppose there are S firms in the economy, all having access to an identical '*Neoclassical*' technology given by:

$$Y_i = AF(K_i, N_i).$$

- Recall that the firms take decisions about:
  - How much final output to produce;
  - How much labour to employ.
- The firms also decide how much capital to employ. However in this static framework, we shall assume here that this decision is trivial: they employ the total capital stock available in the economy $(\bar{K})$ equally such that

$$\bar{K}_i = \frac{\bar{K}}{S}$$

Classical/Keynesian Production Side Story (Contd.)

- Indeed if the firms are perfectly competitive (which means they are price-takers), then a firm's choice is rather straight-forward.
- Given W and P, the optimization problem of the i-th firm is defined as:

$$\underset{\{N_i\}}{Max}.PAF(\bar{K}_i, N_i) - WN_i - R\bar{K}_i.$$

- This generates the following FONC from the i-th firm:

$$AF_N(\bar{K}_i, N_i) = \frac{W}{P}.$$

- The above equation implicitely defines the labour demand of the i-th firm as a function of the real wage rate ($\frac{W}{P}$), its capital share ($\bar{K}_i$) and the aggregate TFP Index (A):

$$N_i = \hat{f}\left(\frac{W}{P}, \bar{K}_i, A\right).$$

Classical/Keynesian Production Side Story (Contd.)

- Aggregating over all firms, we get the corresponding aggregate output supplied:

$$Y^S : Y = S.Y_i = S\ .AF\left(\bar{K}_i, N_i\right) = AF\left(S\bar{K}_i, SN_i\right) \text{ (by CRS)}$$

- As long as $S.N_i = N$ (which we are going to prove shortly), the latter describes the aggregate production function for the economy.
- Notice that the assumption of CRS plays a very important role in generating the equivalence between $S.Y_i$ and $AF(\bar{K}, N)$.
- Also recall that we have assumed that total capital is divided *equally* across all firms.

  Question: will such equivalence still hold when capital is divided arbitrarily across the firms? (We shall come back to this question soon).

- We still have to prove that $S.N_i = N$, i.e., the labour demand that is generated from the aggregate production function is identical to the labour demand function generated by summing up the labour demands coming from all the firms.
- The labour demand function in the Classical (and Keynesian) system was earlier defined in terms of the aggregate production function:

$$N^D : AF_N(\bar{K}, N) = \frac{W}{P}$$

- On the other hand, the labour demand function generated by aggregating all the firms' demand for labour is given by

$$\begin{aligned} N^D &= S.N_i \ \text{ where } N_i = \hat{f}\left(\frac{W}{P}, \frac{\bar{K}}{S}, A\right) \ \text{ such that} \\ N_i : \quad & AF_N\left(\frac{\bar{K}}{S}, N_i\right) = \frac{W}{P} \end{aligned}$$

- **Question: How do we know that these two expressions are equivalent?**

Classical/Keynesian Production Side Story (Contd.)

- In proving this equivalence, we can directly apply the following property of a homogenous function:
  "Let $f(x_1, x_2, ..., x_{n)}$ be a differentiable function of n variables that is homogeneous of degree k. Then each of its partial derivatives $\frac{\partial f}{\partial x_i}$ (for $i = 1, ..., n$) is homogeneous of degree $k - 1$."
- Here however we follow a more circuitous route.
- We exploit another characteristic of the production function which follows from its CRS property:
  - Consider *any* function: $Y = AF(N, K)$.
  - If it is CRS, then $\frac{Y}{N} = AF\left(1, \frac{K}{N}\right) = Af(k)$, where $k \equiv \frac{K}{N}$. Thus another way to write the production function is: $Y = N.Af(k)$.
  - From this latter specification,

$$\begin{aligned}\frac{\partial Y}{\partial N} &= A\left[f(k) + Nf'(k)\frac{\partial k}{\partial N}\right] = A\left[f(k) + Nf'(k)\left(-\frac{K}{N^2}\right)\right] \\ &= A\left[f(k) - kf'(k)\right]\end{aligned}$$

Micro Foundations: Production Side Story (Contd.)

- In other words, **for a CRS production function, the marginal product of labour is a function of the capital-labour ratio employed**.
  - (**Question:** How about the marginal product of capital? Can we derive a similar relationship between $\frac{\partial Y}{\partial K}$ and k through the function $f(k)$ and/or its derivatives? Try this as a homework.)

Classical/Keynesian Production Side Story (Contd.)

- Given this property, let us now go back to the two questions asked earlier.
- Let us take up the second question first:
- Are $N^D : AF_N(N, \bar{K}) = \dfrac{W}{P}$ and $N^D = SN_i : AF_N\left(N_i, \dfrac{\bar{K}}{S}\right) = \dfrac{W}{P}$ equivalent?
- Notice that $AF_N(N, \bar{K})$ is the marginal production labour associated with the aggregate production function. Hence applying the above property, the N^D equation from the aggregate production function reduces to:

$$A\left[f(k) - kf'(k)\right] = \frac{W}{P} \tag{i}$$

where the relevant k here is the aggregate capital-labour ratio: $\dfrac{\bar{K}}{N}$.

Classical/Keynesian Production Side Story (Contd.)

- On the other hand, $AF_N\left(N_i, \frac{\bar{K}}{S}\right)$ is the marginal product of labour relevant for the i-th firm. Hence applying the above property, the labour demand equation for each firm i reduces to :

$$A\left[f(k_i) - k_i f'(k_i)\right] = \frac{W}{P} \tag{ii}$$

  where the relevant k_i here is the firm-specific capital-labour ratio: $\frac{\bar{K}_i}{N_i}$.

- Since equation (i) and equation (ii) are identical equation (though the variable is different) the solutions to these two equations must also be the same, i.,e

$$k_i = k = C \text{ (some constant)}$$

Classical/Keynesian Production Side Story (Contd.)

- In other words, even though firms are employing different *levels* of capital and labour than the aggregate economy, the capital-labour *ratio* in all cases **must** be the same, denoted by some constant C. (This property is called **Scale-Neutrality**).
- Given this relationship, we can now write the labour demand coming out of the aggregate profuction function as $N^D : \frac{\bar{K}}{N} = C \Rightarrow N = \frac{\bar{K}}{C}$.
- On the other hand, labour demand coming from an individual firm can be written as $N_i : \frac{\bar{K}_i}{N_i} = C \Rightarrow N_i = \frac{\bar{K}_i}{C}$.
- Using the above solution, the labour demand coming out of aggregation over all firms is: $N^D = SN_i = S\frac{\bar{K}_i}{C} = S\frac{\bar{K}}{SC} = \frac{\bar{K}}{C}$.
- Thus that the two labour demand functions will be exactly the same.
- This result has of course been derived here under the assumption that $\bar{K}_i = \frac{\bar{K}}{S}$.
- What if total capital stock is NOT divided equally?

Classical/Keynesian Production Side Story (Contd.)

- The 'Scale-Neutrality' property also gives us the clue as to what happens when firms differ in terms of their share of capital.
- To see that, let us now assume that capital is **not** distributed equally across all the firm.
- For simplicity, let us assume that there are two sets of firms: one set of firms (S_1 in number) is given a capital stock of K_1 while the other set (S_2 in number) is given a capital stock of $K_2 < K_1$ such that

$$\begin{aligned} S_1 + S_2 &= S, \\ \text{and } S_1 K_1 + S_2 K_2 &= \bar{K}. \end{aligned}$$

Classical/Keynesian Production Side Story (Contd.)

- As we have seen earlier firms belonging to the first group will have a labour demand equation given by:

$$\begin{aligned} N_i \quad &: \quad AF_N(K_1, N_i) = \frac{W}{P} \\ \text{i.e., } A\left[f(k_i) - k_i f'(k_i)\right] \quad &= \quad \frac{W}{P};\ k_i \equiv \frac{K_1}{N_i} \qquad \text{(i)} \end{aligned}$$

- On the other hand, firms belonging to second group will have a labour demand equation given by:

$$\begin{aligned} N_j \quad &: \quad AF_N(K_2, N_j) = \frac{W}{P} \\ \text{i.e., } A\left[f(k_j) - k_j f'(k_j)\right] \quad &= \quad \frac{W}{P}; k_j \equiv \frac{K_2}{N_j} \qquad \text{(ii)} \end{aligned}$$

- Since equations (i) and (ii) are identical, their solution set must also be exactly the same. Moreover, since $f(k) - kf'(k)$ is a *monotonic* function of k taking values bewteen $(0, \infty)$ **(prove this),** we end up with a unique solution for both, given by $k_i = k_j = C$.

Classical/Keynesian Production Side Story (Contd.)

- Thus we can write the individual demand for labour functions coming from each set of firms as:

$$N_i = \frac{K_1}{C};$$
$$N_j = \frac{K_2}{C}.$$

- Aggregating over all firms, the aggregate labour demand function will now be given by:

$$\begin{aligned} N^D &= S_1 N_1 + S_2 N_2 \\ &= S_1 \frac{K_1}{C} + S_2 \frac{K_2}{C} \\ &= \frac{1}{C}\left[S_1 K_1 + S_2 K_2\right] \\ &= \frac{\bar{K}}{C} \end{aligned}$$

Classical/Keynesian Production Side Story (Contd.)

- Note that the labour demand function that we have obtained by aggregating over firms *with unequal distribution of capital* is exactly identical to the labour demand function that we had obtained earlier when *total capital stock was divided equally across all firms*.
- Moreover, it also coincides with the demand for labour that was derived by using the 'ad-hoc' *aggregate* production function.
- The upshot of this exercise is that when the production function is CRS and there is perfect competition, the size of the firm does not matter:
  - Different firms may employ different *levels* of capital and labour, but the capital-labour ratio for every firm is identical.
  - As a result **when we aggregate over all firms, the micro-founded labour demand function and the corresponding supply curve that we derive by aggregating optimal decisions of 'atomistic' firms operating under perfect competition would indeed be identical to those obtained from the aggregative relationship specified in the Classical/Keynesian system**.

Aggregate Production Function: Neoclassical or not?

- The above analysis also highlights the importance of the assumption of a 'Neoclassical' technology.
- Marvin Frankel (an American economist) pointed out (AER, 1962) that **even when each 'atomistic' micro firm faces a production function which is 'Neoclassical' in nature, there is no reason why the aggregate production function would also be strictly 'Neoclassical', especially if the TFP term is endogenously determined.**
- Consider an economy with S identical firms - each having access to an identical firm-specific technology:

$$Y_i = \bar{A}F(K_i, N_i) \equiv \bar{A}\left(K_i\right)^{\alpha}\left(N_i\right)^{1-\alpha};\ 0 < \alpha < 1.$$

- Note that the firm-specific production function exhibits all the neoclassical properties. The term $\bar{A}$ represents the current state of the technology in the entire economy, which is treated as exogenous by each 'atomistic' firm.

Production Function: Neoclassical or not? (Contd.)

- Frankel then relates the $\bar{A}$ term to the **aggregate capital labour-ratio** in the economy - due to '**knowledge spillovers**' and '**learning by doing**':

$$\bar{A} = g\left(\frac{K}{N}\right); \; g' > 0; \; \text{where } K = SK_i; \; N = SN_i.$$

- The idea is as follows:
  - Productivity depends on how quickly workers can adapt themselves to new machines. This is the process of learning by doing.
  - When the aggregate capital stock in the economy is very high in relation to its total labour stock, everybody gets greater opportunity to familiarise themselves with the machines; hence overall productivity is higher.
  - Moreover, there is knowlege spillovers - workers can learn from one another (without everybody spending time to go through the instruction manuals).
  - Both these factors would imply that $\bar{A}$ would be an increasing function of the aggregate capital-labour ratio.

Production Function: Neoclassical or not? (Contd.)

- Without much loss of generality, let us assume:

$$g\left(\frac{K}{N}\right) = B\left(\frac{K}{N}\right)^{\beta}; \ \beta > 0,$$

- Corresponding Aggregate Production Function (which can be obtained by summing over all firms):

$$\begin{aligned} Y &= \sum Y_i = S\left[\bar{A}\left(K_i\right)^{\alpha}\left(N_i\right)^{1-\alpha}\right] \\ &= \bar{A}\left(SK_i\right)^{\alpha}\left(SN_i\right)^{1-\alpha} \\ &= \bar{A}\left(K\right)^{\alpha}\left(N\right)^{1-\alpha}. \end{aligned}$$

- Notice however that $\bar{A}$ is the total factor productivity term - which is given for each firm, but not so for the aggregate economy.
- Replacing the value of $\bar{A}$ in the aggregate production technology:

$$Y = \bar{A}\left(K\right)^{\alpha}\left(N\right)^{1-\alpha} = B\left(K\right)^{\alpha+\beta}\left(N\right)^{1-\alpha-\beta}.$$

Production Function: Neoclassical or not? (Contd.)

- Notice that the aggregate production technology is indeed 'Neoclassical' only in the special case where $\alpha + \beta < 1$, but **not otherwise**!
- To carry forward the micro-founded production relations to the Classical/Keynesian aggregative production side story, we must therefore assume that there is no such spill over/externality at the aggregate level.
  (Is that empirically true? We shall attempt to answer this question later when we discuss the empirics of output dynamics).

Micro Foundations: Neo-Keynesian Production Side Story

- The production side story becomes a little more interesting for the Neo-Keynesian case.
- Recall that in this case the output supply function is perfectly elastic at some price level $\bar{P}$.
- What kind of micro-founded firm-side story would support this aggreagtive behaviour?
- It is obvious that we now have to move away from the perfectly competitive set up (i.e., price-taking behaviour by firms) and allow for some form of market imperfection (i.e., price-setting behaviour by firms).
- An obvious way to motivate this is to assume that the production function exhibits IRS (increasing returns to scale) such that perfect competition is not sustainable in equilibrium and monopoly emerges as the natural outcome.
- This is the route that we are going to follow now.

Neo-Keynesian Production Side Story: (Contd.)

- The simplest production function that exhibits IRS is a linear one with a fixed cost.
- Let the technology be represented by the following production function:

$$Y = \begin{cases} 0 & \text{if } N \leqq \bar{F} \\ \alpha \left(N - \bar{F} \right) & \text{if } N > \bar{F} \end{cases}$$

  where α is the constant marginal product of labour employed in the actual production process and $\bar{F}$ is the fixed cost defined in terms of units of labour.
- This production function implies that $\bar{F}$ quantity of labour is required to set up the production unit before actual production can take place. Thereafter every additional unit of labour employed produces α units of output.

  (We are ignoring the role of capital for the time being, but capital can be easily brought in either as a part of the fixed cost or the variable cost)
- **Question: Why is this production function IRS?**

Neo-Keynesian Production Side Story: (Contd.)

- Production is now carried out by a monopolist producer who knows the exact demand schedule.
- Given the demand function, the monopolist producer optimally chooses the price level to maximise his profit:

$$\underset{\{P\}}{Max}.\Pi = P.Y^D(P) - WN.$$

- Notice that to produce Y^D amount of output, the monopolist producer has to employ $\dfrac{Y^D}{\alpha}$ units of labour in actual production. In addition, he has to employ F units of labour to set up the production unit.
- Thus, $N = \dfrac{Y^D}{\alpha} + F$. Plugging this in the optimization problem of the monopolist:

$$\underset{\{P\}}{Max}.\Pi = P.Y^D(P) - W\left[\frac{Y^D(P)}{\alpha} + F\right].$$

Neo-Keynesian Production Side Story: (Contd.)

- Corresponding FONC:

$$\begin{aligned} \frac{d\Pi}{dP} &= Y^D(P) + \left[P - \frac{W}{\alpha}\right]\frac{dY^D}{dP} = 0 \\ \Rightarrow & \left[P - \frac{W}{\alpha}\right] = -\frac{Y^D(P)}{\frac{dY^D}{dP}} \\ \Rightarrow & \frac{\left[P - \frac{W}{\alpha}\right]}{P} = -\frac{Y^D(P)}{\frac{dY^D}{dP}P} = \frac{1}{\epsilon} \end{aligned}$$

where $\epsilon \equiv -\frac{dY^D}{dP}\frac{P}{Y^D}$ is the price elasticity of demand.

- Rearranging:

$$P = \frac{W}{\alpha}\left(\frac{\epsilon}{\epsilon - 1}\right)$$

Neo-Keynesian Production Side Story: (Contd.)

- In other words, if the nominal wage is constant and the demand function exhibits constant price elasticity of demand (which is greater than unity), then the monopolist producer will indeed optimally charge a constant price level $\bar{P}$ irrespective of the level of demand:

$$P = \frac{W}{\alpha}\left(\frac{\epsilon}{\epsilon - 1}\right)$$

- Notice however that the level of profit earned by the monopolist is:

$$\begin{aligned} \Pi &= P.Y^D(P) - W\left[\frac{Y^D(P)}{\alpha} + F\right] \\ &= \left(P - \frac{W}{\alpha}\right) Y^D(P) - WF \\ &= \left(\frac{1}{\epsilon - 1}\right)\frac{W}{\alpha} Y^D(P) - WF \end{aligned}$$

- Hence the monopolist will operate iff the demand is sufficiently high:

$$Y^D \geq \alpha(\epsilon - 1)F$$

Neo-Keynesian Production Side Story: (Contd.)

- From now on we shall assume that demand is high enough (presumably because G is sufficiently high); otherwise production process will crash to zero.
- Under that condition, the monopolist firm always earns a positive profit.
- But this generates an additional conceptual problem, which is at odds with the way we had specified our macro frameworks earlier.
- Recall that we had assumed that the entire output produced in the economy eventually goes back to the households who are the owners of all factors of production.
- That had completed the circular flow of income for the economy, which allowed us to write the consumption demand (coming from the households) as a function of the total income (Y).

Neo-Keynesian Production Side Story: (Contd.)

- It also fitted well with a perfectly competetive market structures where the firms in equilibrium were earning zero profits. After paying both the factors their respective returns, there was nothing left with the firms.
- This also explains why the firms had to borrow when the wanted to invest and hence the borrowing cost (r) was an important determinant of the investment demand.
- Now, with a monopolist producer which is earning positive profit, the neat chain of reasoning is broken:
  - Even if the households collective own the firm (as shareholders), what proportion of that profit is given back to households as dividend and how much is retained)?
  - If the firm can retain at least part its profit, then in order to invest why must it borrow from the market?
  - How can consumption be a function of aggregate output (Y) if a part of the output (retained profit) does no go back to the households?
  - How can investment be a function of the borrowing cost (r) alone?

Micro-Founadtions: Top Down vis-a-vis Bottom Up Approach

- It is not easy to answer these questions without simultaneously asking deeper questions (and making further assumptions about) the basic (ad-hoc) macro structure that we had assumed earlier.
- And if we try to do that, these micro-foundations begin to look just as 'ad-hoc' as the 'ad hoc' behavioural equations that we started with.
- Instead of trying to provide micro-foundations from top down, an internally consistent and more logical approch would be to build these models from the bottom (at the micro-level) leading all the to the top (at the macro level).
- And that is exactly what the more recent (DGE/DSGE) macro models do: they build the entire macro framework from first principles - specifying the market structure and optimization problem of the firms/households to simultneously arrive at their (and therefore the economy's) supply/demand, employment/leisure, consumption/savings decisions.

Micro-foundations: An Empirical Justification

- Dynamic General Equilibrium (DGE) models are also immune to the Lucas' Critique. In fact they originated as a response to Lucas' Critique.
- As we have already discussed, traditionally, macroeconomic analysis was based on some aggregative behavioural relationship (e.g., Keyensian Savings Function - which postulates a relationship between aggregate income and aggregate savings; Labour Demand curve which postulated a negative relationship between employment and real wages.).
- Often one would construct detailed behavioural equations for the macroeconomy and would try to estimate the parameters of these equations using time series data to come up with estimated values of the coefficients..
- To be sure some of these equations would be dynamic in nature, entailing some 'assumed' beliefs about futute. But **optimization** over time was not considered to be important or even relevant.

Lucas Critique: Optimization Comes to the Fore

- Lucas (1976) argued that aggregative macro models which are estimated to predict outcomes of economic policy changes are useless simply because the estimated parameters themselves depend on the existing policies. As the policy changes these coefficients themsleves would change, thereby generating wrong predictions!
- His solution was to build macroecnomic models with clear and specific microeconomic foundations - models that are explicitly based on households' optimization exercises.

Lucas Critique: Optimization Comes to the Fore (Contd.)

- Such models will be based on **true parameters** - primitives like tastes, technology etc - which are independent of the government policies.
- Moreover such models would explicitly take into account agents' expectations about government policies and how those expecations may change as policies change..
- Predictions based on such microfounded models would be more accurate than the aggregative models which club all these true parameters as well as other policy-dependent parameters together.

Micro Foundations: Household Side Story - The Consumption Function

- We now turn to the last example of providing ad-hoc micro foundations: the micro-foundations of the consumption function in the Classical/Keynesian system.
- In the process I shall also illustarte how Lucas' critique works.
- Recall that we had specified the aggregate consumption function as: $C(Y)$; $0 < C'(Y) < 1$. The marginal propensity to consume out of income $(C'(Y))$ less than unity implies that households don't consume their entire income; they save a part.
- Accordingly, the counterpart of the Keynesian consuption function is the Keynesian savings function: $S(Y) \equiv Y - C(Y)$; $0 < S'(Y) < 1$.
- Let us consider a linear from of the Keynesian savings function:

$$S_t = \alpha_1 + \alpha_2 Y_t + \epsilon_t$$

- An aggregative macro model would take the above behavioural relationship as given and would estimate the coefficients α_1 and α_2 from data.

Micro-foundation of Keynesian Savings Function:

- Let us now try to retrieve this postulated aggregative relationship from households' optimization exercise.
- In doing so notice that
  - Since we are modeling households' savings behaviour and typically savings are done for the purpose of future consumption, we must define the optimization problem over consumptions on at least two dates - current and future.
  - Since future consumption would also depend on future income and future prices which are currently unknown, expectations immediately enter into the picture.

- Assume that the economy consists of a finite number (H) of households. Each household is indexed by $h \in [1, 2, ..., H]$.
- Let us define a 2-period utility maximization problem of the h-th household as:

$$\underset{\left\{c_t^h, c_{t+1}^h\right\}}{Max.} \log(c_t^h) + \beta \log(c_{t+1}^h)$$

subject to,

$$\begin{aligned} \text{(i) } c_t^h + s_t^h &= \frac{I_t^h}{P_t}; \\ \text{(ii) } c_{t+1}^h &= (1 + r_{t+1}^e) s_t^h + \frac{\left(I_{t+1}^h\right)^e}{P_{t+1}^e}. \end{aligned}$$

where I_t^h and $\left(I_{t+1}^h\right)^e$ are the current and expected future income of the household *in nominal terms*.

Micro-foundation of Keynesian Savings Function: (Contd.)

- From (i) and (ii) we can eliminate s_t^h to derive the life-time budget constraint of the household as:

$$c_t^h + \frac{c_{t+1}^h}{(1 + r_{t+1}^e)} = \frac{I_t^h}{P_t} + \frac{\left(I_{t+1}^h\right)^e}{P_{t+1}^e(1 + r_{t+1}^e)}$$

- From the FONCs:

$$\frac{c_{t+1}^h}{\beta c_t^h} = (1 + r_{t+1}^e).$$

Solving we get:

$$c_t^h = \frac{1}{(1+\beta)}\left[\frac{I_t^h}{P_t} + \frac{\left(I_{t+1}^h\right)^e}{P_{t+1}^e(1 + r_{t+1}^e)}\right]$$

- Thus
$$s_t^h = \frac{\beta}{(1+\beta)} \frac{I_t^h}{P_t} - \frac{1}{(1+\beta)} \left[\frac{\left(I_{t+1}^h\right)^e}{P_{t+1}^e (1 + r_{t+1}^e)} \right]$$
- Aggregating over all households:
$$S_t \equiv \sum_{h=1}^{H} s_t^h = \frac{\beta}{(1+\beta)} \frac{\sum_{h=1}^{H} I_t^h}{P_t} - \frac{1}{(1+\beta)} \left[\frac{\sum_{h=1}^{H} \left(I_{t+1}^h\right)^e}{(1 + r_{t+1}^e)} \right]$$
- Noting that $\frac{\sum_{h=1}^{H} I_t^h}{P_t} \equiv Y_t$ (the aggregate current output in the economy) and $\frac{\sum_{h=1}^{H} \left(I_{t+1}^h\right)^e}{P_{t+1}^e} \equiv Y_{t+1}^e$, one can write the above micro-founded aggregate savings function as:
$$S_t = \frac{\beta}{(1+\beta)} Y_t - \frac{1}{(1+\beta)} \left[\frac{Y_{t+1}^e}{(1 + r_{t+1}^e)} \right]$$

Micro-foundation of Keynesian Savings Function: (Contd.)

- Note that the above micro-founded savings function would indeed resemble the postulated Keynesian savings behaviour provided households' expecations about future variables (as captured by $\frac{Y^e_{t+1}}{(1+r^e_{t+1})}$) remain constant over time.
- Indeed an aggregative model would equate $\frac{\beta}{(1+\beta)}$ to α_2 and $-\frac{1}{(1+\beta)}\left[\frac{Y^e_{t+1}}{(1+r^e_{t+1})}\right]$ to α_1.
- While the coefficient α_2 is based on true parameters (primitives) and would therefore be unaffected by policy changes, coefficient α_1 is not.
- Any policy that changes the household's expectations about its future income or future rate of interest rate would affect α_1.
- Thus predicting outcomes of such a policy change based on the estimated values of the aggregative equations (derived from data when the policy was not yet announced) would be wrong.

Ad Hoc Micro Foundations: A Critique

- In order to justify the aggregative macro systems (of either type) we have now constructed a variety of different micro-foundations, each of which would justify only a few equations of the aggregative system under special assumptions; they do not simultaneously consider all equations of the system.
- This approach to provide micro-foundations seems just as ad hoc as the aggregative macro systems themselves!!
- More importantly, each micro-foundation is based on certain set of assumptions and there is no obvious reason why all these different assumptions specified for different types of agents (households, firms) will be internally consistent with one another!
- As we had noted earlier, the micro-foundations for a neo-Keynesian AS equation (based on a single monopolist producer) may clash with the micro-foundations of an aggregate consumption function or investment function!!

Ad Hoc Micro Foundations: A Critique (Contd.)

- A logical apparoch is to specify an **internally consistent and unified general equilibrium set up** in which individual decisions of all agents are based on their respective optimization exercises and these individual decisions are then coordinated through the markets to characterize the macroeconomy.
- That is exactly the approach that modern dynamic general equilibrium (DGE) macroeconomic theory follows, which we take up as our next topic.