# Land and Property Taxes 

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Lecture 2

## Stamp Duty: Tax Evasion and Remedy I

Suppose

- a buyer has already engaged in a property transaction at price $p^{*}$.
- A stamp duty tax needs to be paid on the "reported price" $p$.
- $t^{S}$ denote the stamp duty tax/rate
- the circle rate $c$.
- $\pi($.$) is the probability of being investigated by the tax department and$ getting caught if and only if he under-reports
- $F$ is the fine


## Stamp Duty: Tax Evasion and Remedy II

The buyer solves the following maximisation problem:

$$
\max _{p}\left\{t^{s} p^{*}-t^{s} \max (p, c)-\pi(p) F\right\}
$$

Assume:

$$
\begin{gather*}
\pi^{\prime}(p)<0, \quad \pi^{\prime \prime}(p)>0, \quad \pi(0)=1, \quad \pi\left(p^{*}\right)=0, \quad F>0  \tag{0.1}\\
-t^{s}-\pi^{\prime}(c) F>0 \tag{0.2}
\end{gather*}
$$

The buyer compares the benefits of tax evasion against the costs. That is, $t^{s}\left(p^{*}-p\right)$ against the costs $\pi(p) F$
The solution to to this maximisation satisfies the following FOC:

$$
\begin{equation*}
-t^{s}=\pi^{\prime}(p) F \tag{0.3}
\end{equation*}
$$

## Property Tax I

## Property Tax

- Annual tax on Annual Property Value (APV).
- In Delhi the unit area method is used to calculate the annual property value
- is calculated by multiplying unit area value assigned to the colony/locality by the covered area of the property
- depends on factors such as age, structure of the property and nature of use.

Let

- $L$ be the land area
- Property value be $V=p K^{\alpha} L^{1-\alpha}$, where $K$ is the amount of capital investment.


## Property Tax II

- $p=1$. So, value of property is $V=K^{\alpha} L^{1-\alpha}$
- $t^{p}$ be the property tax
- $r$ be the cost of capital
- $r$ be the same for all types of properties

In the absence of property tax, the OP of the landowner :

$$
\max _{K} \quad K^{\alpha} L^{1-\alpha}-r K
$$

The profit maximizing investment solves :

$$
K^{*}=\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} L
$$

## Property Tax III

In the presence of property tax, the OP of the landowner :

$$
\max _{K}\left(1-t^{p}\right) K^{\alpha} L^{1-\alpha}-r K
$$

The profit maximizing investment solves :

$$
K^{* *}=\left(\frac{\alpha\left(1-t^{p}\right)}{r}\right)^{\frac{1}{1-\alpha}} L
$$

Clearly, under tax the optimal level of value improving activity $K^{* *}$ is less than $K^{*}$.

## Property Taxes: Tax Revenue

Let

- $T=t^{p} V$ denote the tax revenue collected from a property

That is,

$$
T=t^{p}\left(\left(\frac{\alpha\left(1-t^{p}\right)}{r}\right)^{\frac{1}{1-\alpha}} L\right)^{\alpha} L^{1-\alpha}
$$

Hence,

$$
T=t^{p}\left(\frac{\alpha\left(1-t^{p}\right)}{r}\right)^{\frac{\alpha}{1-\alpha}} L
$$

$T$ is increasing function of tax rate, $t^{p}$, only when:

$$
\left(1-t^{p}\right)^{\frac{\alpha}{1-\alpha}}-t^{p} \frac{\alpha}{1-\alpha}\left(1-t^{p}\right)^{\frac{2 \alpha-1}{1-\alpha}}>0
$$

That is,

$$
1-t^{p}>\alpha
$$

Or,

$$
t^{p}<1-\alpha
$$

## Property Tax based on Location I

Let

- a denote the index of locational amenities where the property is located

Now the value of a house is now:

$$
V=p(a) K^{\alpha} L^{1-\alpha}
$$

where $p^{\prime}(a)>0$
In the absence of tax, the OP for the landowner is: The FOC yields:

$$
K^{*}=\left(\frac{\alpha p(a)}{r}\right)^{\frac{1}{1-\alpha}} L
$$

Let

- $t^{p}$ be tax rate (per-unit) of property value


## Property Tax based on Location II

Now, the OP of the landowner becomes:

$$
\max _{K}\left\{\left(1-t^{p}\right) p(a) K^{\alpha} L^{1-\alpha}-r K\right\}
$$

The FOC yields:

$$
K^{* *}=\left(\frac{\alpha p(a)\left(1-t^{p}\right)}{r}\right)^{\frac{1}{1-\alpha}} L
$$

- An increase in the tax rate $t^{p}$ leads to a decrease in the equilibrium level of investment
- An increase in the index of locational amenities $a$ and hence $p(a)$ leads to an increase in the equilibrium level of investment.

Clearly, for given $a$,

$$
K^{* *}<K^{*}
$$

## Property Tax based on Location III

Note that for given a,

- $K^{*}-K^{* *}=\left(\frac{\alpha p(a)}{r}\right)^{\frac{1}{1-\alpha}} L\left[1-\left(1-t^{p}\right)^{\frac{1}{1-\alpha}}\right]$
- $\frac{K^{* *}}{K^{*}}=\left(1-t^{p}\right)^{\frac{1}{1-\alpha}}$

The tax collected is given by

$$
T^{P}\left(t^{p}, a, L\right)=t^{p} p(a)\left(K^{* *}\right)^{\alpha} L^{1-\alpha}
$$

## Locally Funded Public Good

## Assume

- there is only one property owner
- provision for $a$ is funded by property tax
- $g(a)$ be the cost of $a$

Consider the following social welfare

$$
\max _{t p, a}\left\{\left(1-t^{p}\right) p(a) K^{\alpha} L^{1-\alpha}-r K+t^{p} p(a) K^{\alpha} L^{1-\alpha}-g(a)\right\}
$$

s.t. $t^{p} p(a) K^{\alpha} L^{1-\alpha}=g(a)$.

Question:

- Find out optimal mix of $a$ and $t^{p}$
- Is $K=K^{* *}=\left(\frac{\alpha p(a)\left(1-t^{p}\right)}{r}\right)^{\frac{1}{1-\alpha}} L$ ?
- Is $K><K^{*}$ ?


## Land Tax I

The maximisation problem of the landowner now looks like :

$$
\max _{K} \quad p(a) K^{\alpha} L^{1-\alpha}-r K-\left(t^{L}\right) L
$$

The FOC for optimization yields :

$$
K^{L}=p(a)\left(\frac{\alpha}{r}\right)^{\frac{1}{1-\alpha}} L=K^{*}
$$

Clearly, the optimal level of value improving activity $K^{L}$ equals its efficient level $K^{*}$.
The tax revenue is given by

$$
T^{L}\left(t^{L}, a, L\right)=t^{L} L
$$

## Land Tax II

## Consider

- two localities- 1 and 2 with different locational advantages, $a_{1}$ and $a_{2}$;
- $a_{1}>a_{2}$. So $p\left(a_{1}\right)>p\left(a_{2}\right)$,
- two parcels of same size - one in each locality
- $t^{L}$ as land tax

Surplus accruing to landowners in the two regions are:

$$
\begin{aligned}
& \pi_{1}=p\left(a_{1}\right)\left(K^{\alpha} L^{1-\alpha}\right)-r K-t_{1}^{L} L \\
& \pi_{2}=p\left(a_{2}\right)\left(K^{\alpha} L^{1-\alpha}\right)-r K-t_{2}^{L} L
\end{aligned}
$$

## Land Tax III

The optimization exercise of the landowners in the two regions yields:

$$
\begin{aligned}
& K_{1}^{*}=\left(\frac{\alpha p\left(a_{1}\right)}{r}\right)^{\frac{1}{1-\alpha}} L \\
& K_{2}^{*}=\left(\frac{\alpha p\left(a_{2}\right)}{r}\right)^{\frac{1}{1-\alpha}} L
\end{aligned}
$$

It can be seen that $\pi_{1}\left(K_{1}^{*}\right)>\pi_{2}\left(K_{2}^{*}\right)$. Consider

$$
\begin{aligned}
& \pi_{1}\left(K_{1}^{*}\right)=p\left(a_{1}\right)\left(K_{1}^{* \alpha} L^{1-\alpha}\right)-c K_{1}^{*}-t_{1}^{L} L=0 \\
& \pi_{2}\left(K_{2}^{*}\right)=p\left(a_{2}\right)\left(K_{1}^{* \alpha} L^{1-\alpha}\right)-c K_{2}^{*}-t_{2}^{L} L=0
\end{aligned}
$$

That is, the tax rates are given by:

$$
\begin{equation*}
t_{1}^{L}>t_{2}^{L} \tag{0.4}
\end{equation*}
$$

