The Land Assembly with Pooling

Public Economics: 602

Lecture 5

Land Pooling: Basics I

Let

- n be the number of total land parcels, each of size 1.
- Different parcels are owned by different individuals.
- A project can be taken up on parcels $n' \leq n$
- $B(.)$ be the benefits if all these parcels are taken for a project; $B' > 0$.
- $B = B(n)$
- Different owners might value their properties differently
- There are two types of owners: $v \in \{\underline{v}, \overline{v}\}$

Land Pooling: Basics II

Reservation (Itatus-quo) utility of a property owner is given by

$$u_0(l, v) = vu_0(l) = vu_0(1) = v$$

where $l = 1$, $u_0(1) = 1$ and $v \in \{\underline{v}, \overline{v}\}$, and $\underline{v} < \overline{v}$.

Assume, the pooling works like this:

- land of all owners is taken
- An owner is returned back a plot of size $l < 1$ (land for land)
- Also receives a payment/transfer of t
- The returned land, for each owner is valuable after the project

If an owner participate in the pooling scheme her utility is given by

$$u(l, v) = vu(l) + t$$

where $u(l) > u_0(l)$ for all $l > 0$.

Land Pooling: Asymmetric Information I

Suppose

- v is privately known to the owner
- developer/government knows that the proportions of $\underline{v}$ [of $\overline{v}$] is $\underline{\pi}$ [$\overline{\pi}$]
- the low types are offered $(\underline{l}, \underline{t})$
- the high types are $(\bar{l}, \bar{t})$

The Individual Rationality (IR) constraints for the owners are:

$$\underline{v}u(\underline{l}) + \underline{t} \geq \underline{v} \qquad (1)$$

$$\overline{v}u(\bar{l}) + \bar{t} \geq \overline{v} \qquad (2)$$

and Incentive Compatibility (IC) constraints:

$$\underline{v}u(\underline{l}) + \underline{t} \geq \underline{v}u(\bar{l}) + \bar{t} \qquad (3)$$

$$\overline{v}u(\bar{l}) + \bar{t} \geq \overline{v}u(\underline{l}) + \underline{t} \qquad (4)$$

Land Pooling: Asymmetric Information II

Note, if the scheme is successfully implemented, under scheme

- total transfers are

$$n[\underline{\pi}\,\underline{t} + \overline{\pi}\,\overline{t}]$$

- total land area returned to the owners is

$$n[\underline{\pi}\,\underline{l} + \overline{\pi}\,\overline{l}]$$

Let
$\phi(\cdot)$ be the cost for the principal/delveoper for returning land for land;$\phi' > 0, \phi'' > 0$

Now, the prinicipal's expected utility under incomplete information is -

$$U_0 = B - n[\underline{\pi}\,\underline{t} + \overline{\pi}\,\overline{t}] - \phi(n(\underline{\pi}\,\underline{l} + \overline{\pi}\,\overline{l})) \quad (5)$$

The principal's problem is to maximise (5) subject to constraints (1) - (4)

Land Pooling: Asymmetric Information III

A profit maximizing developer will choose offer such that:

- (1) and (4) hold with equality,
- and given these two, (2) is automatically satisfied, with strict inequality since $\overline{v} > \underline{v}$.
- We ignore (3) and show that it is indeed satisfied later.

We can re-write (1) as

$$\begin{aligned} \underline{t} &= -\underline{v}[u(\underline{l}) - 1], i.e., \\ -\underline{t} &= \underline{v}[u(\underline{l}) - 1] \end{aligned}$$

and (4) as

$$\begin{aligned} \bar{t} &= (\overline{v} - \underline{v})u(\underline{l}) - \overline{v}u(\bar{l}) + \underline{v}, i.e., \\ -\bar{t} &= \overline{v}u(\bar{l}) - (\overline{v} - \underline{v})u(\underline{l}) - \underline{v} \end{aligned}$$

Land Pooling: Asymmetric Information IV

Replacing $\underline{t}$ and $\bar{t}$ in (5),

$$U_0 = B + n\underline{\pi}\,\underline{v}[u(\underline{l}) - 1] + n\bar{\pi}[\bar{v}u(\bar{l}) - (\bar{v} - \underline{v})u(\underline{l}) - \underline{v}] - \phi(n(\underline{\pi}\,\underline{l} + \bar{\pi}\bar{l})) \qquad (6)$$

using the first order conditions, we get the following conditions.

$$\begin{aligned} n[\underline{\pi}\,\underline{v} - \bar{\pi}(\bar{v} - \underline{v})]u'(\underline{l}) &= n\underline{\pi}\phi'(n(\underline{\pi}\,\underline{l} + \bar{\pi}\bar{l})) \\ n\bar{\pi}\bar{v}u'(\bar{l}) &= n\underline{\pi}\phi'(n(\underline{\pi}\,\underline{l} + \bar{\pi}\bar{l})) \end{aligned}$$

Let $(\underline{l}^{SB}, \underline{t}^{SB})$ and $(\bar{l}^{SB}, \bar{t}^{SB})$ be the solution.

$$[\underline{\pi}\,\underline{v} - \bar{\pi}(\bar{v} - \underline{v})]u'(\underline{l}^{SB}) = \underline{\pi}\phi'(n(\underline{\pi}\,\underline{l}^{SB} + \bar{\pi}\bar{l}^{SB})) \qquad (7)$$

$$\bar{v}u'(\bar{l}^{SB}) = \phi'(n(\underline{\pi}\,\underline{l}^{SB} + \bar{\pi}\bar{l}^{SB})) \qquad (8)$$

Clearly

$$\bar{l}^{SB} > \underline{l}^{SB}$$

Land Pooling: Asymmetric Information V

- $\underline{t}$ and $\overline{t}$ are chosen to make 1 and 4 bind.
- Now, it can be shown that (3) holds. Thus, the above represents a truth telling equilibrium.
- Under some conditions, interior solution might not hold
- Pooling can take place

The First Best I

As a benchmark case, assume the government/developer knows the owners' types.
In this case, the developer can extract the full surplus from each type.
So the problem is to maximise -

$$U_0^* = B - n[\underline{\pi}\,\underline{t} + \overline{\pi}\bar{t}] - \phi(n(\underline{\pi}\,\underline{l} + \overline{\pi}\bar{l})) \tag{9}$$

Subject to

$$\underline{v}[u(\underline{l}) - 1] + \underline{t} = 0 \tag{10}$$

$$\overline{v}[u(\bar{l}) - 1] + \bar{t} = 0 \tag{11}$$

From the First Order Conditions, we get

$$\underline{v}u'(\underline{l}) = \phi'(n(\underline{\pi}\,\underline{l} + \overline{\pi}\bar{l})) \tag{12}$$

$$\overline{v}u'(\bar{l}) = \phi'(n(\underline{\pi}\,\underline{l} + \overline{\pi}\bar{l})) \tag{13}$$

The First Best II

Let $(\underline{l}^*, \underline{t}^*)$ and $(\bar{l}^*, \bar{t}^*)$ be the solution.

It can be see that

$$\underline{l}^* \quad < \quad \bar{l}^*$$

Given $\underline{l}^{SB}$,

$$\bar{l}^{SB} = \bar{l}^*$$

The developer's payoffs

$$U_0^* = B + n\underline{\pi}\,\underline{v}[u(\underline{l}^*) - 1] + n\bar{\pi}\bar{v}[u(\bar{l}^*) - 1] - \phi(n(\underline{\pi}\,\underline{l}^* + \bar{\pi}\bar{l}^*)) \tag{14}$$

Show that:

- The high-type land owners are better-off under asymmetric information
- The developer is better off under complete information