Organizations and Agents: Multi-tasks

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Lecture 11

Multiple Tasks I

So far, we modeled production wherein

- Agent performed only one task;
- There was only one output q.

In real world,

- employees at work perform multi-tasks
- produce several outputs

For example,

- Workers
  - Produce output (using firm's assets)
  - Maintain assets

Multiple Tasks II

- Managers/CEO
  - Supervise existing workers/employees
  - Train existing workers/employees
  - Hire new workers/employees
- Salespersons
  - Promote sale with existing customers
  - Make new customers
  - Launch sale of new products
- Teachers
  - Teach
  - Research
  - Serve on administrative committees

Multiple Tasks III

The output is also multi-dimensional

- Workers output
  - Quantity/units of output
  - Residual value of assets
- Managers/CEO
  - Current profits
  - Value of stocks/shares of company
- Teachers
  - Teaching quality and quantity
  - Research output

Model I

Holmstrom and Milgrom (1991, J Law Eco and Organizations)
A simple version:

- Two tasks; $i = 1, 2$
- Two signals/outputs: $q_i(e_i, \epsilon_i) = e_i + \epsilon_i$, $i = 1, 2$. Specifically, $q_i(e_i, \epsilon_i) = e_i + \epsilon_i$, where

$$\begin{aligned} q_1(e_1, \epsilon_1) &= e_1 + \epsilon_1 \\ q_2(e_2, \epsilon_2) &= e_2 + \epsilon_2, \end{aligned}$$

  $\epsilon = (\epsilon_1, \epsilon_2) \sim N(0, \Sigma)$, where Σ

- $\epsilon \sim N(0, \Sigma)$, where Σ is variance-covariance matrix;

$$\Sigma = \begin{pmatrix} \sigma_1^2 & R \\ R & \sigma_2^2 \end{pmatrix}$$

Model II

Payoffs:

- Contract: $w(q_1, q_2) = t + \sum_{i=1}^{2} s_i x_i = t + \mathbf{s}^T \mathbf{q}$, where $s_i \geq 0$
- Principal is risk-neutral with expected payoff $V = V(\mathbf{q}, w) = V(q_1, q_2, w)$, i.e., $V = V(\mathbf{e}, w) = V(e_1, e_2, w)$
- Agent is risk-avers: $u(w, \mathbf{e}) = -e^{-r(w-\psi(\mathbf{e}))}$, $r > 0$, where
- $r = -\frac{u''}{u'} > 0$, i.e., CARA, and
- Principal's payoff: $V(q_1, q_2, w) = E(q_1 + q_2 - w) = e_1 + e_2 - E(w)$
- $\psi(e) = \frac{1}{2}c_1 e_1^2 + \frac{1}{2}c_2 e_2^2 + \delta e_1 e_2$
- $\psi_1(.) = \frac{\partial \psi(e_1, e_2)}{\partial e_1} = c_1 e_1 + \delta e_2$ and $\psi_2(.) = \frac{\partial \psi(e_1, e_2)}{\partial e_2} = c_2 e_2 + \delta e_1$. So

Model III

- $\begin{cases} \delta = 0 & \text{tasks are independent;} \\ \delta > 0 & \text{tasks are technological substitutes;} \\ \delta < 0 & \text{tasks are technological complements.} \end{cases}$

  imperfect substitutes if $0 < \delta < \sqrt{c_1 c_2}$
- Contract: $w(q_1, q_2) = t + s_1 q_1 + s_2 q_2$, where $s_i \geq 0$. Note
- $$\begin{aligned} E(w(q_1, q_2)) &= E(t + s_1(e_1 + \epsilon_1) + s_2(e_2 + \epsilon_2)) \\ &= t + s_1 e_1 + s_2 e_2. \end{aligned}$$
- $Var(t + s_1(e_1 + \epsilon_1) + s_2(e_2 + \epsilon_2)) = s_1^2 \sigma_1^2 + s_2^2 \sigma_2^2 + 2R s_1 s_2$
- $\bar{w}$ = Certainty equivalent of the reservation wage (the outside option for the agent)

First Best

The first best is solution to

$$\max_{e_i} E(\sum q_i - w)$$

s.t. $-e^{-r[w-\psi(e_1,e_2)]} = -e^{-r\bar{w}}$, i.e., $w - \psi(e_1, e_2) = \bar{w}$, i.e.,

$$w = \bar{w} + \psi(e_1, e_2).$$

Therefore, the first best is solution to

$$\max_{e_1, e_2} E(e_1 + \epsilon_1 + e_2 + \epsilon_2 - \bar{w} - \psi(e_1, e_2)), i.e.,$$

$$\max_{e_1, e_2}\{e_1 + e_2 - [\frac{1}{2}c_1 e_1^2 + \frac{1}{2}c_2 e_2^2 + \delta e_1 e_2]\}$$

Therefore, the first best efforts, e_1^* and e_2^*, solve the following foc

$$\psi_1(e) = c_1 e_1 + \delta e_2 = 1 \quad (0.1)$$

$$\psi_2(e) = c_2 e_2 + \delta e_1 = 1. \quad (0.2)$$

Second Best I

e is not contractible but q is. As before, the agent solves

$$\max_{e_1, e_2}\{\hat{w}(e_1, e_2)\},$$

where

$$\underbrace{\hat{w}(e_1, e_2)}_{certainty-equivalent\ \ net-wage} = \underbrace{E[w(e_1, e_2)]}_{expected\ \ wage} - \underbrace{\psi(e_1, e_2)}_{effort\ \ cost} - \underbrace{\frac{r}{2} Var[w(e_1, e_2)]}_{risk-premium}, i.e.,$$

$$\begin{aligned}\underbrace{\hat{w}(e_1, e_2)}_{certainty-equivalent\ \ net-wage} &= \underbrace{t + s_1 e_1 + s_2 e_2}_{expected\ \ wage} \\ &- \underbrace{[\frac{1}{2}c_1 e_1^2 + \frac{1}{2}c_2 e_2^2 + \delta e_1 e_2]}_{effort\ \ cost} \\ &- \frac{r}{2}\underbrace{[s_1^2\sigma_1^2 + s_2^2\sigma_2^2 + 2Rs_1 s_2]}_{risk-premium}\end{aligned}$$

Second Best II

The foc w.r.t. e_1 and e_2 are

$$s_1 = c_1 e_1 + \delta e_2 \tag{0.3}$$
$$s_2 = c_2 e_2 + \delta e_1 \tag{0.4}$$

That is,

$$\mathbf{s}(\mathbf{e}) = \nabla \psi(\mathbf{e}).$$

IR is given by

$$u(\hat{w}(e_1, e_2)) \geq u(\bar{w}), i.e., \;\; \hat{w}(e_1, e_2) \geq \bar{w}, i.e.,$$

$$t + s_1 e_1 + s_2 e_2 - [\frac{1}{2} c_1 e_1^2 + \frac{1}{2} c_2 e_2^2 + \delta e_1 e_2] - \frac{r}{2}[s_1^2 \sigma_1^2 + s_2^2 \sigma_2^2 + 2R s_1 s_2] \geq \bar{w} \tag{0.5}$$

The principal solves $\max_{e_1, e_2, t, s_1, s_2} E[q_1 + q_2 - w(q_1, q_2)]$, i.e.,

$$\max_{e_1, e_2, t, s_1, s_2} E[q_1 + q_2 - (t + s_1 q_1 + s_2 q_2)], i.e.,$$

Second Best III

$$\max_{e_1,e_2,t,s_1,s_2} E[e_1 + (1-s_1)\epsilon_1 + e_2 + (1-s_2)\epsilon_2 - (t + s_1 e_1 + s_2 e_2)]$$

s.t. (0.3) – (0.5) hold. Clearly, (0.5) will bind. Therefore, the Principal's problem can be written as

$$\max_{e_1,e_2,s_1,s_2} \{e_1 + e_2 - [\frac{1}{2}c_1 e_1^2 + \frac{1}{2}c_2 e_2^2 + \delta e_1 e_2] - \frac{r}{2}[s_1^2\sigma_1^2 + s_2^2\sigma_2^2 + 2Rs_1 s_2]\},$$

s.t. (0.3) and (0.4) hold.

Note that the Principal programme can be written as

$$\max_{\mathbf{e}} \{V(\mathbf{e}) - \psi(\mathbf{e}) - \frac{r}{2}\mathbf{s}^T\Sigma\mathbf{s}\}$$

s.t. $\mathbf{e} = \arg\max\{\mathbf{s}^T\mu(e) - \psi(\mathbf{e})\}$

where $\mathbf{s}^T = (s_1, s_2)$.

Special Case 1: R=0

From (0.3) and (0.4) we get

$$e_1 = \frac{s_1 c_2 - \delta s_2}{c_1 c_2 - \delta^2} \tag{0.6}$$

$$e_2 = \frac{s_2 c_1 - \delta s_1}{c_1 c_2 - \delta^2} \tag{0.7}$$

The FOC w.r.t to s_1 is

$$s_1 = \frac{c_2 - \delta + \delta s_2}{c_2 + r\sigma_1^2[c_1 c_2 - \delta^2]} \tag{0.8}$$

By symmetry FOC w.r.t. s_2 gives

$$s_2 = \frac{c_1 - \delta + \delta s_1}{c_1 + r\sigma_2^2[c_1 c_2 - \delta^2]}, i.e., \tag{0.9}$$

Second Best V

From (0.8) and (0.9), we can see that $\frac{\partial s_i}{\partial \sigma_i} < 0$ and $\frac{\partial s_i}{\partial \sigma_j} < 0$.

Further, in view of (0.8)

$$s_2^{SB} = \frac{1 + r\sigma_1^2(c_1 - \delta)}{(1 + r\sigma_1^2 c_1)(1 + r\sigma_2^2 c_2) - \delta^2\sigma_1^2\sigma_2^2 r^2} \tag{0.10}$$

Similarly,

$$s_1^{SB} = \frac{1 + r\sigma_2^2(c_2 - \delta)}{(1 + r\sigma_1^2 c_1)(1 + r\sigma_2^2 c_2) - \delta^2\sigma_1^2\sigma_2^2 r^2} \tag{0.11}$$

From (0.10) and (0.11), it can be checked that

$$\frac{ds_i}{d\sigma_i} < 0 \text{ and } \frac{ds_i}{d\sigma_j} < 0.$$

Second Best VI

That is, if it is hard to measure a task,

- low incentive is provided for that talk
- low incentive is provided even for the measurable task

Moreover, if production is very noisy, $\sigma_2^2 \Rightarrow \infty$ implies

$$\begin{aligned} s_2 &\Rightarrow 0 \\ s_1 &\Rightarrow \frac{r(c_2 - \delta)}{(1 + r\sigma_1^2 c_1) r c_2 - \delta^2 \sigma_1^2 r^2} \end{aligned}$$

That is,

- if it is impossible to measure a task, no incentive is provided for that talk